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100 Years of Superconductivity: The Past, The Present And The Future Quest for a Generalized Theory

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Abstract

This is a review paper to celebrate the first century (1911 – 2011) of the discovery of superconductivity. The emphasis is on the search for a generalized theory of superconductivity. It is observed here that though there are many theories currently in the literature, there are three domineering mechanisms for the Cooper pair formation (CPF) and their emergent theories of superconductivity. Two of these mechanisms, based on the quantum theory axiom of action-at-a distance, may be only an approximation of the third mechanism which is contact interaction of the wavepackets of the two electrons forming the Cooper pair as envisaged in hadronic mechanics. It is therefore suggested that the future of the search for the theory of superconductivity should be considered from the natural possible bonding in element that at short distances, the CPF is by a nonlinear, nonlocal and nonhamiltonian strong hadronic-type interaction due to deep wave-overlapping of spinning particles leading to a Hulthen potential that is attractive between two electrons in singlet couplings while at large distance the CPF is by superexchange interaction which is purely a quantum mechanical effect.

1.0 Introduction

This year, 2011, has been declared the centenary celebration of the discovery of superconductivity (SC) in Hg at about 4 degrees Kelvin (K) of absolute zero (zero Kelvin is equal to –458 degrees Fahrenheit (^{o}F) or –273 degrees Celsius (^{o}C)) by the Dutch physicist, Heike kamerlingh Onnes on April 8, 1911 [1, 2]. In 1908, Onnes succeeded in liquefying helium which was the only gas not yet liquefied in the first decade of the twentieth century. The low temperature range (below 4.2 K or –268.8 ^{o}C) at which the liquefaction occurred made him decide to use it to measure the resistivity of metals. While measuring a thread of mercury frozen in a glass capillary, he and his colleague, Gilles Holst observed that the resistivity fell abruptly at about 4 K to a negligible value. The temperature at which this transition took place is called the critical or transition temperature designated by the symbol T_c. Onnes concluded that mercury had passed into a new state, which on account of its remarkable electrical properties, might be called a "supra" conductivity which in modern parlance, is superconductivity.

The race for a theoretical explanation of the phenomena of SC began immediately after the discovery but quite a number of these theories failed to account for SC and these include those of some of the respected 20th century physics intellectuals such as Block, Einstein, Heisenberg, Pauli, Dirac, Feynman, etc (for a recent review of the failed theories, see Ref.[3]). The situation was so frustrating that Felix Block was quoted as saying then that, "The only theorem of SC that can be proved is that any theory of SC is refutable." Einstein on his part was more factually optimistic as he observed that the wide ranging ignorance of quantum mechanics of composite system makes it impossible to formulate a theory of SC and therefore predicted that the progress of SC could only be made by relying on experiments. Indeed this prediction has remained valid as all the new superconductors discovered till date have been made by experimentalists [4-8].

One of the early difficulties in developing a theory of SC is that it was assumed to be a product of perfect conductivity [9]. Therefore many of the theories based on this understanding attempt to formulate Ohm's law for SC: here their basic goal is to formulate a relation for perfect electric current in an electric field. A radical departure from this line of thinking came from the London brothers which are now acclaimed to have proposed the first major successful theoretical explanation in SC [10]. Their motivation came from the experimental observation of Meissner and Ochsenfeld in 1933 [11] that when a superconductor is cooled below its transition temperature in an applied magnetic field, it expels that magnetic field. As commonly known in textbooks today, the explanation for this phenomenon now known as the Meissner effect is that superconducting electrical currents flow around the superconductor in such a way as to shield the interior from the applied magnetic field so that deep within the superconductor, the magnetic field will be zero. The London brothers observed that this state known as perfect diamagnetism is crucial to understanding the physics of SC. In other words, the SC phenomena can be

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accounted for, by formulating a relation between the electric current and the magnetic field, rather than the electric field. Thus the London brothers developed such an equation which produces the required screening of the static magnetic field hence the Meissner effect. Their theory also predicted a length scale over which a magnetic field can penetrate through the surface of a superconductor and this became known as the London penetration depth [10].

The London theory of SC only tells us how SC works but not how it emerges. This information on the behavour of superconductors, in my view, is a crucial insight to the mechanism of SC since the relation between the electric current and magnetic field which the London brothers considered is the macroscopic quantum equivalent of the current quantum theory relating the interactions of the electrons and their spin orderings. Here macroscopic quantum means quantum phenomena on the scale of large objects since the London equations originated from the Maxwell equations. The current quantum theoretical approach to explain SC is how to first provide a mechanism for the Cooper pair formation (CPF) and then its coherent propagation. Here a Cooper pair is the bound state of two electrons which somehow have overcome their mutual electrostatic repulsion contrary to the prediction of quantum mechanics. Therefore many of the theories that have been proposed based on this quantum mechanical approach seek for approximate means to glue the electrons to form the Cooper pair. Though there are currently an infinite number of these approximate mechanisms for the CPF in the literature, there is wide acceptance that three of them are domineering [12,13] and these are the three to be discussed in this current work. I apologize for the other numerous mechanisms and their emergent theories which are not considered here because of space.

The plan of this paper is as follows. In the next section, the three domineering mechanisms for the CPF and the emergent theories will be discussed to highlight their strengths and weaknesses. It is argued that the weaknesses of the first two, electron pairing based on lattice vibrations and electron pairing based on correlations and spin fluctuations, emanate from the limitation of quantum mechanics. Consequently the third mechanism of the CPF emanates from a hadronic-type bond and the formulation of the emergent iso-superconducting theory is discussed. This is followed by a summary and conclusion in section 3, the last section.

2.0 The Domineering Theories

Early enough, experimental evidence pointed to the fact that in the transition of a metal to the superconducting state, the lattice and its properties were essentially unchanged, whereas some of the properties of the conduction electrons were changed radically [9]. This led to the very important but yet to be properly answered questions: what happens to the electrons? What kind of interacting mechanism do they undergo that can lead to a qualitative change in the properties of the system, obviously, the kind of interaction that is responsible for the transition is difficult to see beforehand without solving the problem completely. The difficulty in finding the right mechanism is due to small energy change between the normal and superconducting states [13]. After 100 years of search for this mechanism, the three generally accepted domineering ones are electron pairing based on lattice vibrations, electron pairing based on correlations and spin fluctuations and electron pairing based on deep overlapping wavepackets[12, 13].

2.1 Electron Pairing Based on Lattice Vibrations

The electron pairing based on lattice vibrations occurs when one moving electron distorts the lattice and thereby attracts positive charge. The next electron is then attracted to this positive charge, and so becomes paired with the first electron and this is the Cooper pair formation. The application of this mechanism of electron-electron interaction mediated by phonon also known as electron-phonon interaction (EPI) emanates from the observation of independent studies of Reynolds et al. [14] and Maxwell [15] that the T_c of superconductors depends on the isotopic mass of the lattice: this is now known as the isotopic effect in SC which suggested that the vibration of the lattice could be involved in the interaction. Frohlick [16] used perturbation method to investigate this pairing mechanism. Frohlich was able to obtain the correct isotopic effect and the critical field at $T = 0$. However, his theory could not produce a phase with superconducting properties. Moreover, the energy difference between the supposed normal and superconducting phases was too large (recall that I said in the preceding section that the energy difference is small). It is obvious today why the Frohlich attempt to account for SC by obtaining the superconducting state from the normal ground state failed: the superconducting state cannot be obtained from the normal state by perturbation [17]. In other words, SC is a non-adiabatic phenomenon.

In 1956, Neil Cooper [18] made a significant theoretical progress in the search for a theory of SC by showing that the EPI under favorable condition can result to the Cooper pair formation as shown in Fig. (1a). The CPF, however, does not change the parent material into a superconducting state: this state can only be formed if the paired electrons form a condensate that moves as a single entity. This is what Bardeen, Cooper and Schrieffer (BCS) achieved in 1957 using a variational approach [19]. Thus the salient feature of their theory (now commonly known as the BCS theory) is that the EPI will lead to the formation of an ensemble of Cooper pairs which can propagate coherently with negligible resistance which is the superconducting state of the parent material.

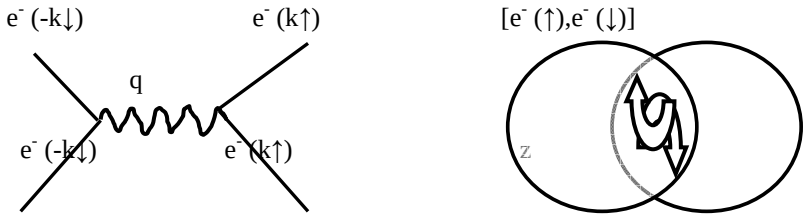

Fig. 1: Attractive interaction of the electron pair **(a)** due to virtual phonon exchange in the BCS model and **(b)** due to hadronic mechanical deep overlapping electron wavefunctions.

The BCS theory has been used to account for a number of metallic and intermetallic SC and all such materials are known as conventional superconductors and their transition temperatures are low [20]. The BCS theory has led to a number of important applications like superconducting magnets for laboratory use (spanning from small-scale laboratory experiments to the Large Hadronic collider (LHC)'s bending magnets), and importantly, for MRI systems as well as explanation of puzzling experimental data such as nuclear magnetic resonance (NMR) relaxation rate and Josephson tunneling [21]. However, the BCS failed to explain the Meissner effect which as discussed in the introduction, is a fundamental property of superconductors. Further, the BCS theory has been proven incapable of predicting high – temperature superconductors and providing the guidelines to search for new materials [4]. Therefore the BCS theory did not only fail to predict the relatively high T_c of the superconducting copper oxide compounds commonly known as the cuprates [7] but also failed to account for this class of superconductors [22]. In general, the BCS has failed in its application to a number of new classes of superconductors discovered since 1970 [23,24]. To account for these superconductors now collectively known as non-conventional superconducting materials, there has been a deluge of proposals of new theories which generalized the BCS theory by replacing the phonon with other bosons [24 – 26], introduce an interplay of the EPI and other mechanisms [27] or are formulated from non – EPI mechanisms (see Ref. [24] and references therein) as well as those that even question the validity of the BCS theory [23]. As stated above, there are many mechanisms already proposed either to generalize the EPI of the BCS theory or to replace it.

2.2 Electron Pairing Based on Correlation and spin fluctuations

Electron correlation is the state of matter in which many electrons are strongly interacting and thereby changing the magnetic, electrical and optical properties of the systems. Thus strong correlated electron systems is believed to exhibit some of the most intriguing phenomena in condensed matter physics [28,29]. One basic feature common to all these systems is that they are a collection of electrons that are neither fully itinerant nor fully localized on their atomic sites [30]. Consequently, neither band theory [31] which is successful for many itinerant systems nor localization theory [32] which is used to study systems with localized carriers has been successful for these materials [33]. The Hubbard model proposed in 1963 [34] is universally considered the simplest minimal description of the strongly correlated systems as it has a kinetic part that is hoped to account for the itinerancy and a Coulombic interaction to represent the localization of the electrons. For a simplified but very qualitative insight into the possible properties of the Hubbard model, see Ref.[35].

2.2.1 Hubbard and Hubbard-like models application to superconductivity

Following the discovery high T_c superconducting cuprates [7] and the failure of the BCS theory to account for their SC [26,27], P. W. Anderson [36] proposed that the Hubbard model could be used to explain the physics of SC in these materials by assuming that a slightly doped Mott insulator can be used in the description of the cuprates. This emanates from the early consensus that the key to understanding these materials is the CuO_2 planes common to all of them (see Ref. [37] and references therein). This was boosted by the experimental demonstration early enough that when the parent material is doped, the mobile holes reside on the O site of this plane. This led to the suggestion that a three-band Hubbard model (H_{3b}) in which the hole is mobile and carry a spin should be the starting point to investigate these materials [36].

A motivation for this concept can be found in the earlier work by Kohn and Luttinger (KL) [38], who showed that the Cooper pairing of fermions with hard-core repulsion (e.g., Hubbard U) is possible in a finite orbital momentum state. However, the same work showed that T_c of repulsive fermions is well below the mK scale, and moreover, the KL mechanism does not work for charged fermions with the realistic finite-range Coulomb repulsion [39].

Classically, the motion of a body can be described by its dynamic variables such as its velocity, mass and time. However, the Heisenberg principle restrict such description in quantum mechanics to variables such as momentum p, position r and time t (not often). This restriction makes it necessary to re-express the total energy of a quantum physical system in terms of p, r, and t only. This can be depicted mathematically by the conventional quantum mechanical equation in relative coordinates and reduced mass for two electrons in singlet coupling as

$$(p^2/m + e^2/r)\,\psi(r) = E\,\psi(r) \quad (1)$$

where m is the mass of the electron.

It is obvious that the potential in Eq.(1) is the repulsive Coulomb force between the point-like charges of the electrons. Thus the replacement of the classical variables in Eq. (1) by their corresponding quantum mechanical operators will result in a Hamiltonian for two repelling electrons. This is the origin of the limitation of quantum mechanics to account for SC since the repelling electrons are not expected to bind to form the Cooper pairs.

Expressing the Hamiltonian from Eq. (1) in quantum field theory for a many body problem usually involves the matrix elements of many operators which require tedious evaluation. An easy way to evaluate the matrix elements of the many body operators is to express all the operators in terms of a fundamental set called creation and annihilation operators and then develop a set of rules for this fundamental set. This is the method of canonical transformation which is also known as second quantization [40]. In this second quantization formulation, the standard Hubbard model is given by

$$H = -t_{ij} \sum_{<ij>\sigma} (C_{i\sigma} C_{j\bar{\sigma}} + \text{H.C.}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow} \quad (2)$$

where t is the hopping term, U is the Coulomb term while $C^{+}_{i\sigma}(C_{j\sigma^1})$ and $n_{i\sigma}$ are respectively the creation (annihilation) and number operators for an electron in the Wannier state on the ith (jth) lattice site with spin projection σ and H.C is the Hermitian conjugate.

In general, the Hubbard model attempts to explain SC from two basic assumptions: the first one is that the undoped material is antiferromagnetic (AFM) at low temperature and the second one is that doping eradicates the AFM hence the material becomes superconductive. It is not possible for a positive U in Eq. (2) to produce SC while a negative U is likely to produce a superconducting state [35] though the negative U Hubbard model is generally believed to be an unrealistic model [41]. The inability of the simplest Hubbard model to account for high-temperature SC [27] has also been recently demonstrated in a Monte Carlo (VMC) simulations with a (projected) BCS-type trial wave function, using an advanced sign-problem-free Gaussian-Basis Monte Carlo algorithm (GBMC). In order to manipulate the Hubbard model to account for SC, there has been numerous extensions of it commonly known as Hubbard-like models such as the t-U-h model [42], two-leg Hubbard ladder [43], and some other models in Ref. [44] for some others. The conclusion I reached from a review [44] of these models is that though they are able to account for some properties of the high T_c superconducting materials, they do not capture the essential origin of SC in these materials.

2.2.2 t-J and t-J-like models application to superconductivity

In a seminal paper [45], Zhang and Rice suggested that the mobile hole on the O site of the three-band Hubbard model (H_{3b}) will form a singlet state with a hole on the Cu site at the centre of each CuO_2 plaquette to form a single-band character. This is the mapping of the H_{3b} into the single-band Hubbard model (H_{1b}) when $J = 4t^2/U$ which is consistent with Anderson's proposal that a strong on-site Coulomb interaction among a partially filled band of Cu 3d levels should be the starting effective single-band model for the supeconducting cuprates. Since then, the researchers who follow this line of thinking believe the ground states of the CuO_2 are the Zhang–Rice singlet (ZRS) which are expected to become the Cooper pairs of the superconducting states when liberated from the insulating host material [37]. The resulting Hamiltonian of the Zhang and Rice proposal is the t-J model given in its simplest form by

$$H = -t_{ij} \sum_{<ij>\sigma} (C_{i\sigma} C_{j\sigma} + \text{H.C.}) + J \sum_{i} S_i . S_j \quad (3)$$

where t, $C^{+}_{i\sigma}(C_{j\sigma^1})$ and H.C. retain their earlier definitions and J is the exchange integral while S_i and S_j are spin operators.

In general, the t-J model attempts to explain SC from two basic assumptions: the first one is that the undoped material is antiferromagnetic (AFM) at low temperature like the first assumption of the Hubbard and Hubbard-like models but unlike their second assumption, the doped material of the t-J model is assumed to possess short range AFM and SC. Thus the t-J model which is based on spin fluctuation mechanism attempt to describe SC and magnetism on the same footing, that is, as a function of band filling [46]. Therefore in my view, the t-J model is the quantum theoretical equivalent of the London brothers' significant early proposal that SC is a relation between the electric field and the magnetic field discussed in the introduction. Interestingly, a number of researchers believe that the t-J model is more fundamental than the Hubbard model since under some approximations, the t-J model can be derived directly from the Cu-O Hamiltonian so that it can be independently analyzed [37,45,47]. In Ref. [37], I have derived the t-J model from the CuO_2 plaquettes from the superexchange interaction of the electron spins using the first electron removal approach [48]:

$$H_{t_{pd}} = -t_{pd} \left[\sum_{\{i\}} \sum_{<j,k>\in\{i\}} d^{+}_{i_{r'},\sigma} p^{+}_{j_{r'},\sigma} d_{i_r\sigma} p_{k_r\sigma} + \text{H.C.} \right] + J_{dp} \left[\sum_{\{i\}} \sum_{<j,k>\in\{i\}} d^{+}_{i_{r'},\sigma} p^{+}_{j_{r'},\bar{\sigma}} d_{i_r\bar{\sigma}} p_{k_r\sigma} \right] \quad (4)$$

where $d^+(d)$ is the creation (annihilation) of the carrier at the Cu $3dx^2 - y^2$ orbital and $p^+(p)$ is the creation (annihilation) of the carrier at the O $2p_x$ and O $2p_y$ orbitals and the t_{pd} denotes a hopping between a Cu and O in the same plane.

Using a highly simplified correlated variational approach (HSCVA), I was able to obtain the energy band of the CuO_2 plane in agreement with experiment and then account for other properties of the superconducting cuprates. In Ref. [48], we have obtained the t-J model from iso-superconductivity theory based on hadronic type electron pairing which I will discuss in the next section. The main difference between t-J model in Refs. [37] and [48] is that the former is developed purely from a quantum mechanical superexchange interaction and its application has been restricted to the high T_c superconducting cuprates while that of the latter which is based on hadronic mechanical electron pairing can possibly be applied to all superconducting materials. This limitation of the quantum mechanical t-J model which is also inherent in its extended versions known as the t-J-like models (see Ref. [44]) means it is not a candidate for a generalized theory of SC.

It is pertinent to mention also that there have been several attempts to explain superconductivity as an interplay between the EPI and electronic correlations [49,50] but the progress in this direction is limited [25,51].

2.3 Electron Pairing Based on Deep Overlapping Wavepackets

The electron pairing based on deep overlapping wavepackets emanated from Santilli's proposal in 1978 [52] to account for the appropriate bonding of the elements wherein a bound state of one electron and one positron at a short distance (< 1 fm ~ 10^{-13} cm) with non-local, non-linear and non-potential is due to deep overlapping of their wavepackets. This is the foundation of hadronic mechanics. Animalu observed that at such distances, the magnetically induced Hulthen potential which is an attractive force will dominate the Coulomb repulsion between two electrons to allow them to bond into singlet coupling as in the CPF in the cuprates [53-55] and the iron based superconducting compounds [24,47,56] and possibly in other superconducting materials [57]. The motivation for Santilli's proposal is the assumption that in nature, the electrons have extended wavepackets of the order of 1 fm as shown in Fig. (1b). Therefore there will be mutual overlap/penetration of the wavepackets of the two electrons which allows them to have a non-linear, non-local and non-potential interactions that will result to valence bond of the Cooper type. One possible way to achieve an invariant representation of these interactions is to exit from the class of unitary equivalence of quantum mechanics,

$$UU^+ = UU^\dagger = I \tag{5}$$

via an isounitary transformation by projecting into a conventional nonunitary form

$$UU^+ \neq I,\ UU^\dagger = I^* = 1/T\ . \tag{6}$$

Taking into account Eqs.(1) and (6), one can project out a different eigenvalue E' different from the one E in Eq. (1):

$$\begin{aligned} &U\,[(p^2/m + e^2/r)\,\psi(r)]\,U^+ \\ &= [\,(Up^2U^+)/m + (e^2/r)UU^+]\,(UU^+)^{-1}\,[U\psi(r)U^+] \\ &= [1/m)p^*\,T\,p^*\,T + e^2/r\,]\,\psi^*(r) = E'\,\psi^*(r). \end{aligned} \tag{7}$$

At this point, Santilli and Shillady [58] introduce the following realization of the non-unitary transform,

$$\begin{aligned} UU^\dagger = I^* = 1/T = \\ = e^{\{[\psi(r)/\psi^*(r)]\int \psi^\dagger{}_1(r)\,\psi_2(r)\,d3(r)\}} = 1 + [\psi(r)/\psi^*(r)]\int \psi^\dagger{}_1(r)\,\psi_2(r)\,d^3(r) + ... \end{aligned} \tag{8}$$

where ψ and ψ^* are the solutions of the unitary and nonunitary equations, and ψ_k, k = 1,2, are the conventional quantum mechanical wavefunctions of the two electrons.

It is evident that, as desired, the above isounit represents interactions that are: nonlinear, because dependent in a nonlinear way in the wavefunctions; nonlocal, because inclusive of a volume integral; and nonpotential, because not representable with a Hamiltonian. Additionally, for all mutual distances between the valence electrons greater than 1 fm, the volume integral of Eq. (8) is null with the crucial limit

$$\mathrm{Lim}_{r\ \underline{\text{bigger}} > 1\ \text{fm}}\ I^* = 1, \tag{9}$$

under which the quantum scenario can be identically and uniquely recovered from that of the hadronic.

Santilli and Shillady [58] solved the above equations in all details. First, by inserting isounit in Eq.(8) into Eq. (7), they obtained the isoequation here projected on a conventional Hilbert space

$$[p^2/2m' + e^2/r - V_o e^{-br}/(1 - e^{-br})]\,\psi^*(r) = E'\psi^*(r), \tag{10}$$

where m' represents the isorenormalization of the mass caused by nonpotential interactions, and one recognizes the emergence of the attractive Hulthen potential

$$V_{\text{Hulthen}} = V_o e^{-br}/(1 - e^{-br}). \tag{11}$$

But the Hulthen potential is known to behave like the Coulomb potential at short distances and be much stronger than the latter. Therefore, Eq. (10) admits the excellent approximation

$$[p^2/2m' - V'\,e^{-br}/(1 - e^{-br})]\,\psi^*(r) = E'\,\psi^*(r), \tag{12}$$

where the new constants V' reflects the "absorption" of the repulsive Coulomb potential by the much stronger attractive Hulthen potential.

Thus Eq.(12) depicts a hadronic mechanical equation in relative coordinates and reduced mass for two electrons in singlet coupling with a strongly attractive force capable of forming the Cooper pair, as requested by experimental evidence. This is the foundation of the iso-SC theory. It is pertinent to point out that at large distances (> 1 fm), the Hulthen potential no longer dominates and it has been suggested in Ref. [37] that the CPF is by superexchange interaction which naturally affects electrons that are close enough to have (no deep) overlapping wavefunctionis and this is purely a quantum mechanical affair [59]. It follows then that the results to be obtained taking into account the CPF at short distances can also be obtained as the approximation of the hadronic-type from the CPF by superexchange interaction at large distances. However, an important difference between the quantum mechanical t−J model and the iso-superconductivity model lies in the ability of the latter to predict T_c from an exact solution of the model. Thus T_c expressions have been obtained from the iso-superconductivity model for the cuprates, iron based compounds and magnesium diboride [47]. These T_c expressions depend on the effective valence z of the Cu, Fe and Mg respectively which is believed to be the natural condition to trigger the hadronic mechanical CPF in contrast to the case of the often ambiguous 'favourable condition' required for the CPF in the BCS theory. Interestingly, z can take values that will yield T_c values even at room temperature (297 K) when z is introduced into the BCS T_c expression variation [57]

$$T_c \approx \frac{68x}{k_\beta} e^{-1/x}, \tag{13}$$

with x = z = λ using a pre-exponential value of 68x motivated by the restriction of these values (see Fig. 2b) unlike the electron-phonon coupling constant of the BCS theory which is weak and consequently the BCS T_c expression is limited to a maximum temperature of 25 K. (see Fig.2a).

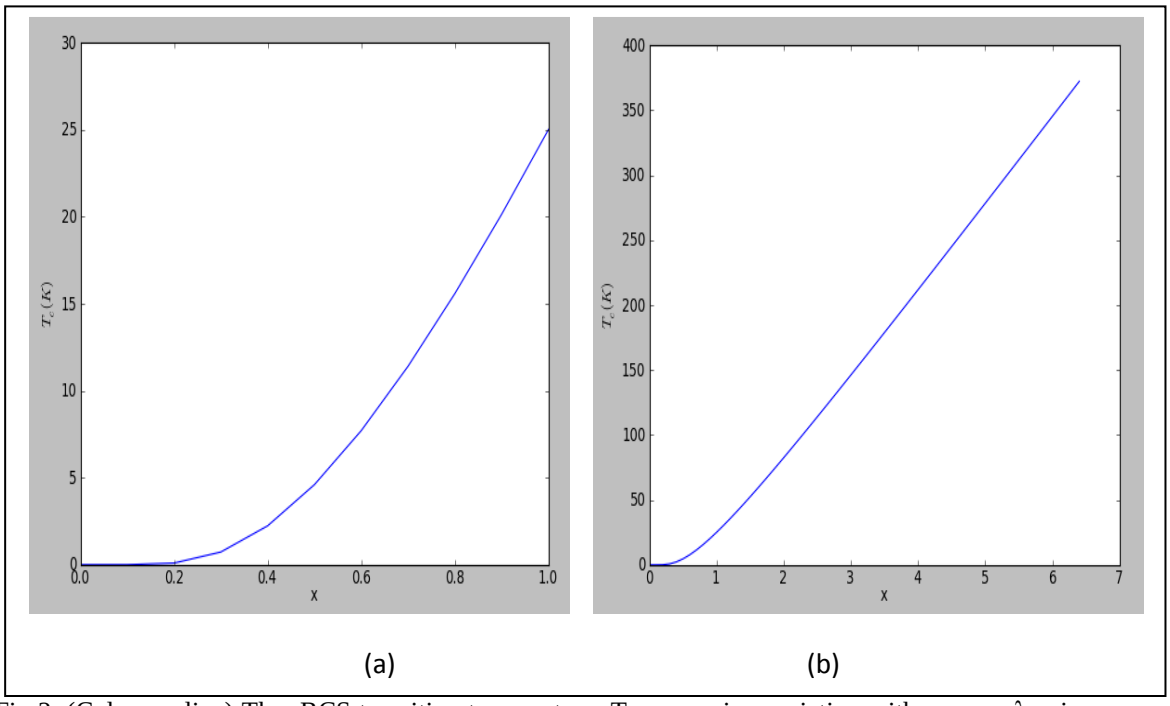

Fig 2. (Colour online) The BCS transition temperature, T_c expression variation with x = z = λ using a pre-exponential value of 68x motivated by the restriction of this values is used so that **(a)** the x = λ ranging from 0 to 1 with the highest T_c at 25 K which is the highest predicted by the BCS theory because the λ is restricted to small values ranging from 0 to 1 and **(b)** x = z ranging from 1 to 6 with the highest T_c > 297 K which is the room temperature.

3.0 Summary and Conclusion

SC is a fascinating but counterintuitive phenomenon. The inability to understand its origin hence how to design superconducting materials with the desired transition temperatures and other properties has adversely affected man's dream of remarkable technological harvest from it since its discovery in 1911 [2]. I have considered in this work three domineering mechanisms which are candidates for the origin of SC and these are the electron pairing based on lattice vibrations, electron pairing based on correlations and spin fluctuations and electron pairing based on deep overlapping wavepackets. The partial success of the first two mechanisms hence the theories emerging from them for the superconducting materials they are developed for, means the main properties of the superconducting phenomena are still poorly understood and new concepts are needed [12, 24,59]. This is the motivation for the third mechanism which holds more prospect to achieve a general theory of SC. Thus it is hoped that the future search for a general mechanism for the Cooper pair formation and the possible

propagation to achieve a general superconducting theory is the hadronic mechanical electron pairing mechanism.

To conclude, perhaps it is not superfluous to remind the scientific community that nature has sometimes made some of its phenomena so obscure that it looks daunting to explore them. However, science history has shown that man has somehow most often unraveled them at the end. A handy example is the breakthrough in the study of the atom which Mckenzie [60] summarized as follows, "The atom reveals its secrets only gradually, like a cleverly written play, allowing sparks of knowledge briefly to illuminate corners of the story, but jumbling scenes and inferences so that full realization came only at the end of, after many members of the audience had given up hope of understanding. Physicists are pleased too, that nature indeed is like a good playwright and has closed this particular play with a dramatic promise of unity." A review of the SC story so far indicates that nature is perhaps shooting another interesting drama and only those who diligently persevere will come to full realization of the origin of SC.

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