Journal of the Nigerian Association of Mathematical Physics
***Volume* 19 (November, 2011), pp 615 – 618**
© J. of NAMP

Binding Energy and Compression Modulus of Infinite Nuclear Matter Derived from Variational Calculation

J.O. Fiase and Frederick Gbaorun
Department of Physics,
Benue State University P.M.B 102119, Makurdi

Abstract

The determination of binding energy per nucleon of infinite nuclear matter and its compression modulus has been a great challenge for nuclear physicists for many decades. In this work we have calculated the binding energy and compression modulus k_∞ of infinite nuclear matter from a density-dependent potential derived from a variational approach. The density-dependent potential reproduces the binding energy of nuclear matter of approximately -16 MeV at the normal nuclear matter saturation density consistent with the best available density-dependent potentials derived from the G-matrix approach. The results of the incompressibility modulus, k_∞ is in excellent agreement with the results of other workers.

1.0 Introduction

The use of density-dependent effective interactions for inelastic scattering as well as ion-ion optical potentials has received a lot of attention for some time now [1, 2, 3]. The pioneering work in this direction has been the G-matrix calculations of Bertsch and co-workers [1]. An extension of this work [4] and the large number of papers based on such effective interactions has been phenomenon.

In a recent paper [5] we derived a similarly motivated effective interaction which was based on the lowest order constrained variation approach [5, 6, 7]. The results of that paper compared very favourably with the results obtained from the G-matrix calculations.

One of the stringent tests an effective interaction must pass among other tests is to reproduce the binding energy of nuclear matter of ≈ -16 MeV at the normal nuclear matter density of ≈ 0.17 fm^{-3}

The aim of this paper is to derive a density-dependent effective potential from the results of our work in Ref. [5] and then use it to calculate the binding energy and the compression modulus, k_∞ of nuclear matter.

The present paper is organized as follows:

In Section 2, we give a brief summary of the method used in deriving our effective interaction. In Section 3, we derive our density-dependent effective interaction. In Section 4, we present the results of nuclear matter calculations with the present effective interaction. The final Section is devoted to the conclusion of the paper.

2. Effective potential matrix elements.

The determination of the effective potential matrix elements of the nucleon-nucleon potential is not very easy as such potentials usually take on infinite values at very short inter-nucleon distances of < 0.3 fm. For this reason, the wave function must be correlated with a correlation function $f(ij)$ such that as $r \to 0,\ f(ij) \to 0$ while the two-body potential $V_{ij} \to \infty$ as $r \to 0$ and the matrix elements of the effective potential E^{eff} given below become finite [5]:

$$E^{eff} = \langle \varphi | \sum_{i>j} f(ij) V_{ij} f(ij) | \varphi \rangle \tag{1}$$

In eqn. (1) $|\varphi\rangle$ is taken to be the harmonic oscillator wave function, while V_{ij} here is taken to be the Reid soft-core potential [8]. In Ref. [5] we defined an effective interaction which is suitable for calculations of inelastic scattering and ion-ion optical potentials. This is taken to be the sum of Yukawa functions with different ranges to which the two-body matrix elements of eqn. (1) are fitted. Specifically this was defined for the central (*c*), spin-orbit (*ls*) and tensor (*t*) channels as

Corresponding author: E-mail: -, Tel. -

$$V_{ls} = \sum_p D_p \frac{e^{-(r_{ij}/R_p)}}{(r_{ij}/R_p)} \underline{L}\cdot\underline{S},$$
$$V_t = \sum_p D_p \frac{e^{-(r_{ij}/R_p)}}{(r_{ij}/R_p)} r_{ij}^2 S_{ij}, \quad (2)$$

where the D_p 's are the strengths of the effective interaction which were determined by fitting the oscillator matrix elements of eqn. (2) to those of eqn. (1). The ranges $p \le 4$ were 0.25, 0.4, 0.7 and 1.414 fm. These ranges were theoretically motivated by the one-boson exchanges. For example the longest range of 1.414 fm corresponds to the one pion exchanges while the shorter ranges correspond to heavier meson exchanges such as σ, ρ and ω.

In Ref. [5] Table V, the strengths of our effective interaction were determined by fitting the oscillator matrix elements of eqn. (2) to those of eqn. (1) as described above. The results were separated into various angular momentum channels which were; the singlet–even (SE), singlet-odd (SO), triplet-even (TE), triplet-odd (TO), the spin-orbit and the tensor channels. Here, we are interested only in the first four channels mentioned above and we shall use the results of Table V of Ref. [5] for these channels.

3. Density-Dependent Effective Interactions.

In this Section we present the form of our density-dependent effective nucleon-nucleon potential suitable for the calculation of nuclear matter properties. Here the direct and exchange potentials in the SE, SO, TE, and the TO channels can be recast into the spin-isopin formalism as [9]:

$$V^D = \frac{1}{16}\left(3V^{SE} + 3V^{TE} + V^{SO} + 9V^{TO}\right), \quad (3)$$

with

$$V^{EX} = V^D P^\sigma P^\tau, \quad (4)$$

where P^σ and P^τ are projection operators in the spin and isospin channels. Hence,

$$V^{EX} = \frac{1}{16}\left(3V^{SE} + 3V^{TE} - V^{SO} - 9V^{TO}\right). \quad (5)$$

Using the various ranges as defined in Table V of Ref. [5], we obtain

$$V^D(r) = 11012\,\frac{e^{-4r}}{4r} - 2359\,\frac{e^{-2.5r}}{2.5r}, \quad (6)$$

and

$$V^{EX}(r) = 1039.25\,\frac{e^{-4r}}{4r} - 1503.94\,\frac{e^{-2.5r}}{2.5r} - 7.847\,\frac{e^{-0.7072r}}{0.7072r}. \quad (7)$$

These results may be compared with the very popular M3Y effective interaction derived from the G-matrix approach given by [10]:

$$V^D(r) = 7999.0\,\frac{e^{-4r}}{4r} - 2134.85\,\frac{e^{-2.5r}}{2.5r},$$
$$V^{EX}(r) = 4631.38\,\frac{e^{-4r}}{4r} - 1787.13\,\frac{e^{-2.5r}}{2.5r} - 7.847\,\frac{e^{-0.7072r}}{0.7072r}.$$

As is well-known in nuclear matter calculations, the presence of the direct and exchange terms alone cannot reproduce the binding energy of nuclear matter, except a density dependence is included. We included the density dependence in the form similar to that of Ref. [2] in the form:

$$V(r,\rho) = V^{D(EX)}(r) f(\rho), \quad (8)$$

with

$$f(\rho) = A\left(1 + Be^{-\alpha\rho}\right), \quad (9)$$

where *A*, *B* and α are constants.

With this form of the density dependence, the binding energy of nuclear matter per nucleon can be written as [3]:

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} + \frac{f(\rho)\rho}{2}\left\{J_V + \int [j_B(k_F r)]^2 V^{EX}(r) d^3r\right\}, \quad (10)$$

where

$$J_V = \int V^D(r) d^3r,$$

and

$$j_B(x) = 3\frac{j_1(x)}{x},$$

while $j_k(x)$ is the k^{th} -order spherical Bessel function.

The compression modulus, k_∞ of the spin and isospin symmetric cold infinite nuclear matter (INM) is defined as

$$k_\infty = k_F^2 \frac{\partial^2 \in}{\partial k_F^2} = 9\rho^2 \left.\frac{\partial^2 \in}{\partial \rho^2}\right|_{\rho=\rho_0}, \tag{11}$$

where the Fermi momentum k_F for the spin and isospin symmetric INM is given by

$$k_F^3 = 1.5\pi^2\rho. \tag{12}$$

Here ρ is the nucleonic density while ρ_0 is the saturation density for the spin and isospin symmetric INM.

For the calculation of the compression modulus we have used as a first approximation the zero-range pseudo-potential, $J_{00}(\in)\delta(r)$ instead of the full exchange potential. In this case we define [11]:

$$\in = \frac{3\hbar^2 k_F^2}{10m} + \frac{f(\rho,\in)\rho J_D}{2} + J_{00}(\in), \tag{13}$$

where the zero-range pseudo-potential representing the single-nucleon exchange term is given by [12]:

$$J_{00}(\in) = -276\,(1 - 0.005\in)\,MeV\ \ fm^3. \tag{14}$$

Here the density-dependent part has been taken to be [13]

$$f(\rho,\in) = C\left[1 - \beta(\in)\rho^{2/3}\right], \tag{15}$$

which takes care of the higher order exchange and the Pauli blocking effects, C and $\beta(\in)$ are constants, $\beta(\in)$ depends on the energy where m is the nucleonic mass which is equal to 931.4943 Mev/c^2 and J_D represents the volume integral of the direct term supplemented by the zero-range pseudo-potential having the form

$$J_D(\in) = 4\pi\int V^D(r)r^2 dr\ . \tag{16}$$

The equilibrium density of nuclear matter is defined from the saturation condition

$$\frac{\partial \in}{\partial \rho} = 0\ . \tag{17}$$

The expressions for the parameters C and $\beta(\in)$ in eqn. (15) are respectively given by [11]

$$\beta(\in) = \left[\frac{(3-3p)}{9-5p}\right]\Big/\rho^{2/3}, \tag{18}$$

where

$$p = \frac{10\,m\in}{\left[\hbar^2(1.5\pi^2\rho)^{2/3}\right]},$$

and

$$C = -\frac{2\hbar^2 k_F^2}{\left[5mJ_v\rho\left(1 - 5\beta(\in)\rho^{2/3}/3\right)\right]}. \tag{19}$$

Finally, the Compression modulus, k_∞ can be evaluated as [11]:

$$k_\infty = \left[\frac{3\hbar^2 k_F^2}{5m} - 5JvC\,\beta(\in)\rho^{5/3}\right]_{\rho=\rho_0}. \tag{20}$$

4. Results.

In Fig. 1, the graph of the calculated values of the binding energy per nucleon for infinite nuclear matter is plotted with the requirement that the nuclear matter binding energy be reproduced at the correct nuclear matter density.

As can be seen from the graph, the binding energy per nucleon for infinite nuclear matter of *-16* MeV was reproduced at the nuclear density of ≈ 0.17 fm^{-3}.

In Table 1, we have presented the results of our calculations of k_∞ for different values of *C* and $\beta(\in)$. Since the values used for the saturation density ρ_0 by different groups do differ, a narrow range of its acceptable values ($\rho_0 = 0.170 - 0.150$ fm^{-3}) have been used. The values obtained for the compression modulus ranges from 301 to 307 MeV for the acceptable range of values of saturation densities used here.

Theoretical estimate by INM model [11] claims a well defined and stable value of k_∞ =288 ± 20 MeV. The determination of k_∞ based on the production of hard photons in heavy-ion collisions leads to the experimental estimate, k_∞ = 290 ± 50 MeV [14]. The results of the present calculations are in excellent agreement with the results of these researchers.

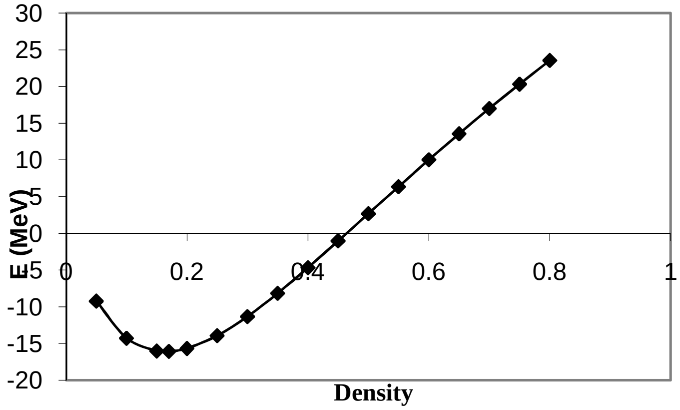

Fig. 1, the graph of the calculated values of the binding energy per nucleon for infinite nuclear matter against densities.

Table 1. Compression Modulus at different saturation densities. The results of the present calculations for k_∞ are given. These should be compared with those in ref. [11]

$\rho_\infty\,(fm^{-3})$	$\beta(\in)\,(fm^2)$	C [11]	C present	k_∞ (MeV) [11]	k_∞ (MeV) present
0.170	1.551	(1.98)	1.93	(309.6)	307.5
0.165	1.586	(2.02)	1.96	(308.2)	305.5
0.160	1.624	(2.07)	2.01	(306.9)	304.7
0.155	1.664	(2.11)	2.06	(305.5)	303.7
0.150	1.705	(2.16)	2.10	(304.0)	301.4

5. Conclusion.

We have derived a density-dependent potential from variational method which reproduces the binding energy per nucleon of infinite nuclear matter at the correct nuclear matter density of 0.17 fm^{-3}. We have next calculated the compression modulus of the spin and isospin symmetric cold INM using the potential. The resulting values of k_∞ are found to lie between the range 301 – 307 MeV, and are in excellent agreement with the theoretical results quoted in Table 1 and the experimental value of $k_\infty = 290 \pm 50$ MeV quoted in the text. In the next paper, we hope to use the full exchange term instead of the zero-range pseudo potential used as a first approximation in this paper.

References

[1] G. Bertsch, J. Borysowicz, H. McManus, and W. G Love, Nucl. Phys. A**284**, 399 (1977).
[2] Dao T. khoa and G. R. Satchler, Nul. Phys. **A668**, 3 (2000).
[3] M. Rashdan and T. A. Abdel-Karim ,Physca Scripta **65**, 133 (2002).
[4] A. Hosaka, K. I. Kubo, and H. Toki, Nucl Phys. **A444**, 76 (1985)
[5] J. Fiase, K.R.S. Devan and A. Hosaka, Phys. Rev. C **66** 014004 (2002).
[6] J. Fiase, A. Hamoudi, J. M. Irvine, and F. Yazici, J. Phys. G **14**, 27 (1988).
[7] J. Fiase, Phys. Rev C **63**, 037303 (2001).
[8] R. V. Reid, Ann. Phys. (N. Y.) **50**, 411 (1968).
[9] R. D Lawson, Theory of the Nuclear shell Model, Clarendon Press. Oxford (1980), p97.
[10] Dao T. Khoa and W. Von Oertzen, Phys Lett. **B 342**, 6 (1995)
[11] D. N. Basu (2004). J. Phys. G: Nucl. Part. Phys. **30** B7-B11
[12] Kobos A. M, Brown B. A, Lindsay R and Satchler G. R. (1984). Nucl. Phys. A **425** 205
[13] Dao T. Khoa and W. Von Oertzen, Physic Letters **B 3 04**, 8 (1993).
[14] Schutz Y et al (1996). Nucl. Phys. A **599** 97c