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Determination of the Reliability of Product Quality Loss and Control Model for a Process Plant

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Abstract

Inconsistency in quality control is a major problem which plagues many indigenous firms in Nigeria. When product quality fluctuates, it negatively impacts on brand status, customer loyalty and confidence. The study was carried out to evaluate the best quality control practices for a brewing plant. Control charts were created and used for the analysis. These charts include the P-chart, C-chart, X-chart and R-chart. Product quality loss was determined to be 22. However, the X-chart was found to be the most suitable and effective. The values indicate that by using the X-chart model, about 96.4% of the suspected defective products would actually survive standard tests. This model has successfully eliminated the number of the proposed defective products to the barest minimum without altering the expected quality level of the final products.

1.0 Introduction

During mass production, product feature variations are an inevitable occurrence, and are a result of non standardization of production and materials handling processes. These variations may be obvious or subtle, leading to product inconsistency [11] and in some cases they could amount to marked defects, rendering the products useless. Quality control measures stand as a guide to filter the defective products from a large batch of products.

Quality control measures apply control charts for their selection procedures. The application of Control chart was invented in the Bell Labs by Walter Shewhart on May 24, 1924 (Juran,1997). Data is graphically represented by plotting process parameters against time under different scenarios. Control charts are intended to monitor process stability and variability [6, 12]. Control charts are one of the most important Statistical Quality Control (SQC) methods in quality control and improvement. They are proactive statistical tools intended to monitor processes and signal when they go out of control [5].These control charts help determine whether special cause variation is present, implying that action needs to be taken to either eliminate that cause if it has a detrimental effect on the process, or to make it standard operating procedure if that cause has a beneficial effect on the process. If no special cause variation is found to be present, Statistical Process Control (SPC) helps define the capability of the stable process to judge whether it is operating at an acceptable level.

Nembhard et al [9] said that control charts are used to keep a process in statistical control, where the output quality is at a target level. They said that [1] explains that a process can be disturbed by common causes and special causes. Common causes are associated with the usual steady state running of the process when it is in a state of control. Special causes may be thought of as problems that arise periodically in a somewhat unpredictable fashion. Control charts help find the special causes, eliminate them and return the process to its target level. The use of control charts has helped manufacturers to detect defective products and eliminating them early before they get into the market. Allowing these defective products into the market would definitely damage the brand's reputation

Quality control charts have also been used by other investigators to monitor the variations found in products. Sun and Matsui [13] studied the control chart design used in supply chain management systems. Luo and Wu [7] worked on the optimal np-control charts with variable sample sizes or variable sampling intervals. Ross [10] applied statistical control charts to a wide number of health care applications.

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Control charts typically display the limits that statistical variability can explain as normal, if the process is performing outside these limits, it is said to be out of control. Fu and Hu [4] said that the design of control charts in statistical quality control addresses the optimal selection of the design parameters. Control charts are represented as graphs, and they work by measuring a process value (statistic) sequentially, over a period of time. The question of whether a statistic such as the mean of a process, X is within upper and lower limits, determines whether a process is in control.

The graph includes a centre line, an upper control limit and lower control limit. In this study, the product quality loss would be determined, the control limits and the reliability of the quality control process would also be determined. The most appropriate control chart that can filter out the actual defective products in a most cost effective process would be determined. Control charts are used to monitor the products' defects, minimize cost and eventually maximize profit. The control chart that shows products with the minimum defects would be the one recommended.

2. Quality Evaluation Models

2.1 Quality Control Models

The quality control models used consist of control charts methods, reliability models to determine the potency of the method, and the quality loss caused by the defective products on the entire material used for production

Control Charts

The control charts models used for this study are itemized and expressed in Eqs (1 - 11)[11]

i P - Chart

The p-chart is used to distinguish between defective and non defective items and to state the number of defectives as a percentage of the whole. A p chart ensures that the proportion of defective items in the sample is determined to see if the proportion falls within the control limits on the chart. The control limits and average of the items are expressed in Eqs (1-3).

$$\bar{P} = \frac{\text{total no of defective products}}{\text{total no of sample observations}} \tag{1}$$

$\bar{P}$ defines the mean defective proportion of control limits

Upper control limit, $$UCL = \bar{\rho} + Z\sqrt{\frac{\bar{P}(1-\bar{\rho})}{n}} \tag{2}$$

and the lower control limit, LCL, is

$$LCL = \bar{\rho} - Z\sqrt{\frac{\bar{P}(1-\bar{\rho})}{n}} \tag{3}$$

Where n = No of Sample products
UCL = Upper Control Limit, LCL = Lower Control Limit
Z = quality control chart dimensionless parameter

ii. Range (-R-) Chart

The Range (R-) chart model reflects the amount of dispersion present in each sample. It is the difference between the smallest and largest values in the sample. The control limits and average range of items, $\bar{R}$ are expressed in Eqs (4-6).

Mean range, $$\bar{R} = \frac{\sum R}{K} \tag{4}$$

Where $$K = n - 1 \tag{5}$$

$$UCL = D_4\bar{R} \tag{6}$$

$$LCL = D_3\bar{R}$$

Where D_3, D_4 are table values used for determining control limits for R-charts

iii. Mean $(\bar{X})$ Chart

The mean (X-) chart model shows how sample results relate to the process average or mean, $\bar{X}$. The control limits and average items are expressed in Eqs (7-9).

$$\overline{X} = \frac{X}{n} \quad (7)$$

$$UCL = \overline{X} + A_2\overline{R} \quad (8)$$

$$LCL = \overline{X} - A_2\overline{R} \quad (9)$$

A_1,A_2 are table values used for determining control limits for x-charts .

iv. C – Chart

The c-chart model is used when it is not possible to compute a proportion defective and the actual number of defects must be used. The control limits and average number of items, $\overline{C}$ are expressed in Eqs (10-11)

$$UCL = \overline{C} + Z\sqrt{\overline{C}} \quad (10)$$

$$LCL = \overline{C} + -\sqrt{\overline{C}} \quad (11)$$

Values obtained from Tables 1 and 2 were used for the computation, utilizing Eqs 1 – 11

2.2 Reliability Models

This model was proposed by Ebeling (2005) is used to evaluate the reliability of the quality control process adopted

$$F(t) = \int_0^t f(t^1)dt \quad (12)$$

$$R(t) = \int_t^\infty f(t^1)dt \quad (13)$$

Where F(t) is the probability of failure at time, t; f(t) is the probability density function and R(t) is the reliability with respect to time, t.

F(t) is defined as

$$F(t) = 1 - R(t) = Pr(T? t) \quad (14)$$

where $R(t) \geq 0$, R (0) = 1 and $\lim_{t\to\infty} R(t) = 0$. For a given value of t, R (t) is the probability that the time to failure is greater than or equal to t. R (t) is referred to as the reliability function. The probability density function, f (t) is defined by

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \quad (15)$$

Where t is the time to product failure

.This function describes the shape of the failure distribution.

The above reliability models are simplified and used for the calculation herein under.

Applying Eq (12) and substituting values in Table 1, F(t) becomes

$$F(t) = 1 - R(t) = 1 - \frac{1}{0.9072t + 1} = \frac{0.9072t}{0.9072t + 1}$$

applying Eq. (13), R (t) becomes

$$R(t) = \left.\frac{-1}{(0.9072t' + 1)}\right|_t^\infty = \frac{1}{0.9072t + 1}$$

Since t = 90720 mins (the value of t, is obtained from Table 1), $R(t) = 0.99998 \approx 1.0$.

and $$f(t) = \begin{cases} \dfrac{0.9072}{(0.9072t + 1)^2} & t \geq 0 \end{cases}$$

Applying the value of R(t) to Eq. 16

The reliability, R(system) of an n-out-of-m system is given by a binormial distribution on the assumption that each of the m units is independent and identical. The R (system) is then expressed as

$$R(\text{system}) = \sum_{i=n}^{m} \binom{m}{i} R^{i} (1-R)^{m-i} \tag{16}$$

Where

$$\binom{m}{i} = \frac{m!}{i!(m-i)!}$$

n is the number of products subject to failure, m is the number of failure parameters and i is the corresponding ith failed component.

Expanding Eq.(16) by inputing the values of m and i, leads to

$$R(system) = \binom{16}{8} R^{8}(1-R)^{8} + \binom{16}{9} R^{9}(1-R)^{7} + \binom{16}{10} R^{10}(1-R)^{6} + \binom{16}{11} R^{11}(1-R)^{5}$$
$$+ \binom{16}{12} R^{12}(1-R)^{4} + \binom{16}{13} R^{13}(1-R)^{3} + \binom{16}{14} R^{14}(1-R)^{2} + \binom{16}{15} R^{15}(1-R)^{5}$$
$$+ \binom{16}{16} R \tag{17a}$$

Where the number of products available is 16 and the observed defective sample products is 8 (see Table 1)

Since R(t) = R = 1.0, therefore the calculated $R(system) = 0.9998 \approx 1.0$

2.3 Determination of Product Quality Loss, Q

The Product quality loss theory was used by Dieter [2. This theory was analyzed with the aid of the quadratic loss function proposed by Taguchi (1990) and is expressed in Eq (17) as

$$L(y) = k(y-m)^{2} \tag{17b}$$

Where L (y) is the quality loss when the quality characteristic is y, m is the target value and k is a constant, the quality loss coefficient.

If y_1, y_2, ---, y_n are measurements of y taken from n units, then the average quality loss, Q is given by

$$Q = \frac{1}{n}\left[L(y_1) + L(y_2) + \ldots + L(y_n)\right]$$
$$= \frac{1}{n}\left[(y_1 - m)^2 + (y_2 - m)^2 + \ldots + (y_m - m)^2\right] \tag{18}$$

Where

$$L(y) = \frac{A}{\Delta^2}(y-m)^2$$

L(y) is the quality loss when the quality characteristic is y

and $$K = \frac{A}{\Delta^2} \tag{19}$$

Δ is the difference between the highest number of effective product and the least number of defective product.
K is a constant, the quality loss coefficient

$$Q = K\left[(\mu - m)^2 + \frac{n-1}{n}\sigma^2\right] \tag{20}$$

$$\mu = \frac{1}{2}\sum_{i=1}^{n} y_i \tag{21}$$

and $$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \mu)^2 \tag{22}$$

When n is large, Q can be written as

$$Q = K\left[(\mu - m)^2 + \sigma^2\right] \quad (23)$$

Using values available in Table 1, Q is expressed as

$$Q = K\left[(55.625-16)^2 + \frac{8-1}{8}(238.7833)\right]$$

Where K = $\frac{29.6}{49^2} = 0.01233$

A is the cost of replacing the defective product

$$Q = 0.01233[1570.14 + 208.94] = 21.94$$

3: Discussion of Results

The statistical process control analysis was done using the values in Tables 1-2, for determining the control limits for the various charts.

Table 1: Quality Control Data of Defective Materials

Sample Name./ Monthly Record	Observations of Defective Samples in their various Brand Products								Total No. of Defective Products	Proportion Defective	Mean Defective	Range of Defective	Time to failure (mins)	Cost of replacing the product in Millions of naira	
Grains	4	5	5	5	4	6	7	4	40	0.045	5	3	7200	3.60	1
Sorgum	5	6	4	5	12	6	4	6	48	0.054	6	8	4320	0.80	2
malted Sorgum	6	3	4	9	6	3	3	2	36	0.054	4.5	7	4320	1.20	3
Barley Malt	6	4	4	2	6	4	3	4	33	0.037	4.125	4	5760	2.40	4
Maize	5	12	4	3	14	4	5	2	69	0.078	8.625	12	10,080	1.60	5
Wheat	4	3	5	4	16	8	3	4	47	0.053	5.875	13	7200	1.35	6
Hop	3	4	5	6	13	4	4	2	41	0.046	5.125	11	11520	1.25	7
Sugar	2	2	16	12	6	18	4	5	65	0.073	8.125	16	4320	2.45	8
Malt Extract	13	20	11	9	6	3	2	18	82	0.092	10.250	18	4320	2.30	9
Odefenous Malt	4	11	9	3	10	6	8	6	57	0.064	7.125	8	2880	1.55	10
Yeast Food	1	8	2	9	6	3	4	6	39	0.044	4.875	8	2880	2.40	11
Finings	6	13	8	10	14	12	6	4	73	0.082	9.125	10	4320	3.20	12
Biofoam	4	9	8	8	8	7	2	18	65	0.073	8.125	16	5760	1.10	13
Keslguhr powder	5	10	12	11	11	6	12	11	75	0.084	9.375	7	5760	0.90	14
Dal	12	6	3	6	6	10	4	6	55	0.062	6.875	9	4320	0.80	15
Vitamin C	14	12	4	7	7	11	2	8	65	0.073	8.125	12	5760	2.70	16
Total									890		111.250	162	90,720	29.6	

Table 2: Control Chart Parameters

Parameter	*Value*
A_2	0.37
C	59.3
D_3	0.14
D_4	1.86
$\bar{R}$	10.8
$\bar{X}$	7.42
$\sum \bar{X}$	111.25
Z	3.00

The quality control practices utilized to detect defective raw materials used for processing food drinks and beer by a renowned Brewing Plant is investigated in this study. Control charts were used for the quality control analysis.

Russell and Taylor III [11] said that charts are used to determine when the production process might be out of control. Four control chart models were used for the analysis. These are the p-chart, c-chart, X-chart and R-chart. These charts were used to determine and recommend an appropriate model to the organization geared towards minimizing defective products and cost of product refinement and at the same time maximizing organizational revenue. Further analysis was done by determining the product quality loss. This method was able to determine the actual loss of product due to defects.

For the p-chart model, an upper control limit (UCL) of 0.071, a lower control limit (LCL) of 0.044 and mean defective proportion of 0.059 were obtained using Eqs (1-3). From Fig.(1), it was observed that the production process of processing products denoted by sample numbers 5,8,9,12,13,14 and 16 was out of control whereas, only sample number 4 under the LCL indicates that there were few defects. This suggests that there must be something wrong with the specified sample numbers denoting the raw materials shown in Table 1. The method of materials inspection with regard to the p-chart model should be investigated.

The c-chart model showed that the UCL is 82.4, LCL is 36.2 and the average number of the products is 59.3 using Eqs (10-11). However, the production process was found to be within the control limits as shown in Fig. (2). This indicates that there was significant improvement over the p-chart model but for sample numbers 3 and 4, which appear below the LCL, indicate there are more defective products when compared with the p-chart model, detected by the use of the c-chart model.

The mean (x-) chart model used in this study showed that the UCL is 11.42, LCL is 3.42, whereas, the average number of products is 7.42 respectively using Eqs (7-9). From Fig. 3 the chart shows that all the sample numbers fall within the control limits. This indicates that the production process was in control. However, the pattern of the graph arrangement shown by sample numbers 5,8,9,12,13,14 and 16 which are above the average value of 7.42 is random. These sample numbers are likely to have the tendency of getting out of control, if appropriate measures are not taken to monitor their sampling process.

The R-chart model was also used to monitor the production process of these raw materials. The UCL, LCL and average number of products are 20.09, 1.51 and 10.8 respectively using Eqs (4-6). From Fig. 4 depicting the R-chart, all the sample numbers are well fitted within the production control limits but the pattern of the graph arrangements show that the chart is non random, this calls for further investigation by management. There is the possibility that products depicting sample numbers 5,6,7,8,9,13 and 16 may cause the production process to get out of control, if appropriate control measures are not put in place.

The X-chart and R-chart are appropriate models to be used by the firm under study but from the analysis of the control charts, it is clear that the X-chart is more suitable and should be recommended to the firm. From Fig.(3), it was observed that the graph pattern indicates that the graph is random which suits the criteria specified by the control chart model. Although Fig.(4) depicts a random graph, it is much less random in appearance than Fig.(3) when compared with a standard graph specified to be random, and other graphs specified to be less random [11], which indicates that although all the sample numbers are within the control limits there is however the tendency for the presence of defective products amongst the entire batch products studied. In Figs (3-4) products depicting sample numbers 5, 8, 9, 13 and 16 are likely to become defective earlier than the other products. These products must be monitored closely as they possess the potential of causing the production process to get out of control. Early monitoring is considered to potentially reduce the number of suspected defective products.

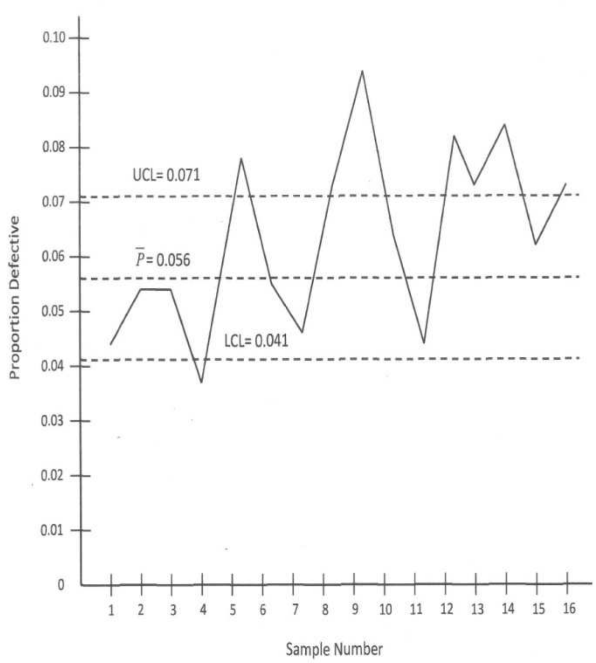

Fig. 1: The P Chart Graph

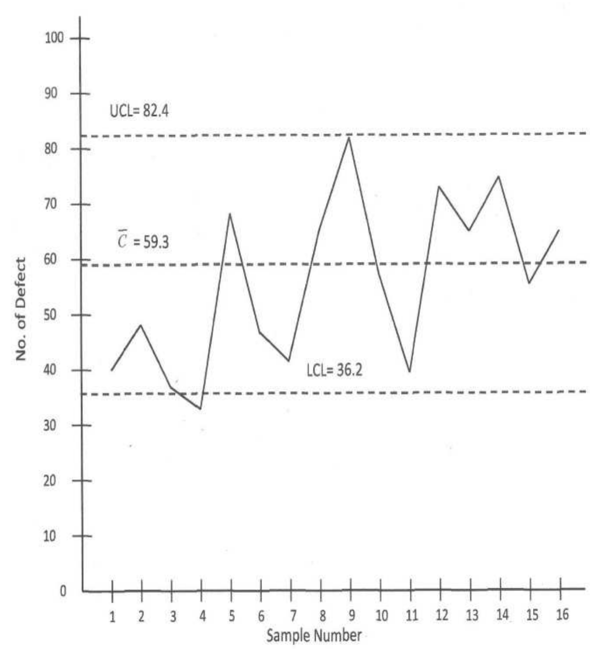

Fig 2: The C Chart Graph

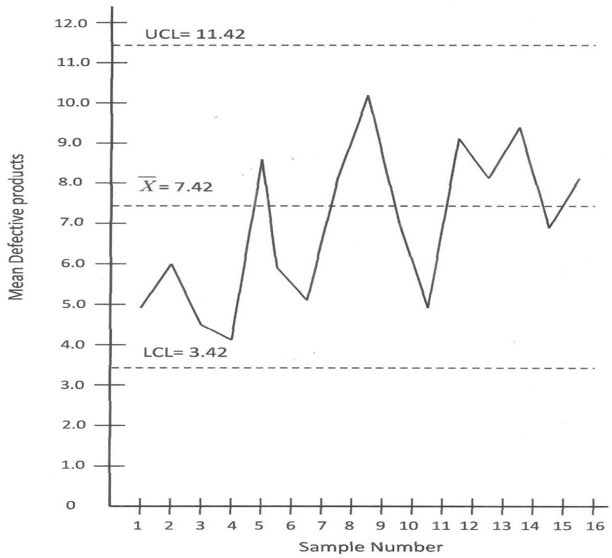

Fig. 3: X Chart Graph

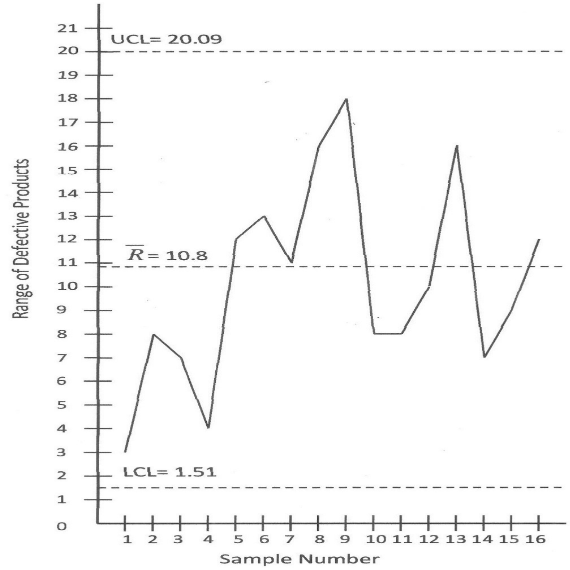

Fig 4: R Chart Graph

The quality loss study was carried out. Dieter [2] said that the quality loss theory indicates that some loss is inevitable from the time a product is produced to when it is shipped to the customer, so that the smaller the loss the more desirable the product. Dieter [2] wrote that, it is important to quantify the loss so that alternative designs and manufacturing processes can be compared. From the analysis, the total loss incurred by the organization due to the failure of the products (raw materials) to deliver the expected performance is 21.94; approximately 22 products were completely defectively out of a total number of 890 suspected defective products. This shows that from using the X-chart model about 858 products survived. This indicates that 96.4% of the entire suspected defective products were saved.

From the reliability analysis, the X-chart control model has been proven to be a very reliable process, with a reliability factor of 1.0 for detecting and minimizing defective products in a very cost effective manner.

CONCLUSION

In this study, four control chart models, such as the p-chart, c-chart, X-chart and R-chart were used to monitor product defects evaluation processes. After the rigorous evaluation processes were carried out by using these four control models, the X-chart model was found to be the most suitable and effective because this process filtered out the actual defective products which includes some products that would ordinarily have been ignored by the product sampling inspectors, and further saved some products that are of good or acceptable quality but may have been mistaken to be defective. The product quality loss and the reliability of the sampling process were studied and from the study it was found that the total loss of quality of the products is minimal and the production process adopted to control or monitor the entire sampling process system is found to be very reliable. This indicates that the system is potent.

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