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Comparison Between Laminar and Turbulent Flow in a Single Phase Horizontal Pipe Gas Flow

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Abstract

With the increase in consumption of natural gas in the world, it is important to understand how dry natural gas will flow in a horizontal pipe especially during production and transportation, and also during laminar and turbulent flow. Many attempts have been made to compare and distinguish both flows but only little is still known about these flows.

A novel approach reported in literature suggests the use of Weymouth and modified Panhandle equations as models for single phase horizontal flow. This paper presents the use of models to compare both laminar and turbulent flow in a single phase horizontal flow . the results obtained are then compared to existing Weymouth flow model.

Results show that laminar flow depends on viscous forces and is independent of gravity whereas turbulent flow does not depend on viscous forces but pipe roughness, gas gravity, pressure drop and gas flow rate.

Nomenclatures

W = Flow work Energy
$\rho = Density\ of\ the\ fluid$, Ibm /cuft
$dp = Change\ in\ Pressure$, psi
u_g = Average Velocity of the gas, ft/sec
$du = Change\ in\ velocity, ft/sec$
$dz = Elevation\ increase, ft$
$f_m = f = moody\ frictional\ factor$
$dL = change\ in\ length, ft$
$g = Acceleration\ due\ to\ gravity, Ft/sec^2$
$g_c = gravity$ Conversion factor, 32.2 Ibmft /Ibfs
D = Diameter of conduit, ft
L = Length of the horizontal pipe, ft
$q_g = volumetric\ flow\ rate\ of\ the\ gas, MM\ \ Ft^3/D$
R = Gas constant, 10.73 cuft psia / Ib mole R
$R_e = reynolds\ number$
P_1 = pressure at point 1, Psia
P_2 = pressure at point 2, Psia
Z = Gas compressibility
R_e = Reynolds number
e = Roughness
P_b = Base pressure, psia
T_b = Base temperature, Rankine
T = Well temperature, Rankine
Z= Fluid compressibility factor, fraction
G_g = Gas gravity, fraction

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1.0 Introduction

Production and pipeline engineers are interested in the way fluid flows in conduits. When a fluid flows through a conduit the internal roughness of the conduit can cause local eddy current within the fluid adding resistance to the flow of the fluid. Conduits having smooth walls such as glass, copper and polythene do have very low frictional resistance while conduit such as cast iron and steel create larger eddy currents which pose significant effect on the frictional resistance. The velocity profile in a conduit will show that the fluid at the middle of the conduit will have higher velocity than that towards the edge of the stream and therefore friction will occur between layers within the fluid. Fluid with high viscosity will naturally flow more slowly and will not support eddy current and therefore the internal roughness of the conduit will have no effect on the frictional resistance. This condition is called laminar flow but in turbulent flow when viscosity is relatively low, the fluid tends to flow faster and will definitely support eddy current, thus internal roughness of the conduit is a key factor. Both flows are illustrated in Fig. 1.

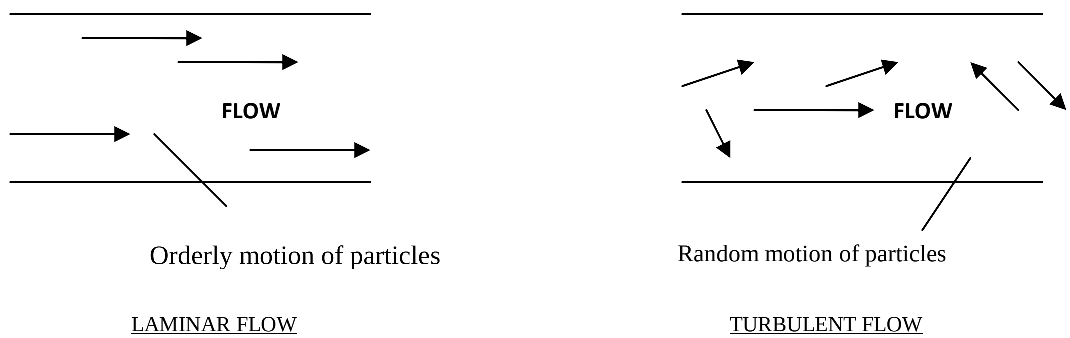

Fig. 1: Laminar and Turbulent Flow

In fluid flow the basic parameters are (1) Reynolds number, Re (2) conduit roughness, e, and (3) frictional factor. These parameters are used to differentiate between the flow regimes.

Flow Reynold number of value less than 2000 is termed laminar; if greater than 4000, it is termed turbulent and between 2000 and 4000 is termed transition flow. The frictional factor is negligible for laminar flow and all laminar but turbulent flow is greatly affected by the pipes roughness.

The frictional factor f_m, for a single phase laminar flow can be gotten analytically by the Hagen-Poiseuille equation[9], which is

$$f_m = \frac{64}{R_e} \tag{1}$$

Churchill Equation

Churchill (1977) has obtained an equation for the friction factor as the following form:

$$f = 8\left[\left(\frac{8}{R}\right)^{12} + (A+B)^{-3/2}\right]^{1/12} \tag{2}$$

where,

$$A = \left[-2log\left(\frac{e/D}{3.7}\right) + \left(\frac{7}{R_e}\right)^{0.9}\right]^{16} \tag{3}$$

$$B = \left(\frac{37530}{R_e}\right)^{16} \tag{4}$$

Chen Equation

Chen (1979) has also proposed an equation for friction factor of the form

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{e}{3.7065D} - \frac{5.0452}{R_e} log\left(\frac{e^{1.1098}}{2.8257D} + \frac{5.8506}{R_e^{0.8981}}\right)\right] \tag{5}$$

Round Equation

Round (1980) proposed an equation of the following form:

$$\frac{1}{\sqrt{f}} = -1.8log\left(0.27\frac{e}{D} + \frac{6.5}{R_e}\right) \tag{6}$$

Barr Equation

Barr (1981) Equation Is Of The Form:

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{e}{3.7D} + \frac{4.518log\left(\frac{R_e}{7}\right)}{R_e\left(1+\frac{R_e^{0.25}}{29}\left(\frac{e}{D}\right)^{0.7}\right)}\right] \quad (7)$$

Zigrang And Sylvester Equation

Zigrang and Sylvester (1982) have proposed the following equation:

$$\frac{1}{\sqrt{f}} = 2log\left[\frac{e}{3.7D} - \frac{5.02}{R_e}log\left(\frac{e}{3.7D} - \frac{5.02}{R_e}log\left(\frac{e}{3.7D} + \frac{13}{R_e}\right)\right)\right] \quad (8)$$

HAALAND

Haaland (1983) proposed frictional factor to be of the form:

$$f = \frac{1}{\left[-1.8log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]\right]^2} \quad (9)$$

Manadilli Equation

Manadilli (1997) proposed the following expressions valid for *Re* ranging from 5235 to 10^8 and for any value of *e*/D.

$$\frac{1}{\sqrt{f}} = -2log\left(\frac{e}{3.7D} + \frac{95}{R_e} + \frac{96.82}{R_e}\right) \quad (10)$$

ROMEO et al EQUATION

Romeo et al (2002) proposed an equation of the form:

$$\frac{1}{\sqrt{f}} = 2log\left(\frac{e}{3.7065D} - \frac{5.0272}{R_e}A\right) \quad (11)$$

$$A = log\left[\frac{e}{3.827D} - \frac{4.567}{R_e}log\left[\left(\frac{e/D}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815+R_e}\right)^{0.9345}\right]\right] \quad (12)$$

Nikuradse (1993)

Nikuradse equation is still one of the best equations for fully developed turbulent flow in rough pipe, and it is of the form:

$$f = \frac{1}{\left[1.74-2log\left(\frac{2e}{D}\right)\right]^2} \quad (13)$$

Von Karman (1939)

The Von Karman (1939) equation for moody frictional factor for rough pipes is given by

$$\frac{1}{\sqrt[2]{f_m}} = 2\log\left(\frac{3.7}{\varepsilon}\right) \quad (14)$$

where $\varepsilon = relative\ roughness$

$$\varepsilon = \frac{e(ft)}{D(ft)} \quad (15)$$

Swamee-Jain (1976)

The proposed frictional factor by Swamee-Jain (1976) is of the form:

$$f = \frac{0.25}{\left[log\left(\frac{e}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \quad (16)$$

Ohirhian

Ohirhian (2005) proposed one of the best frictional factor equation with staggering accuracy. And it is given by

$$f = \frac{1}{\left[-2log\left(\frac{e}{3.7D} + \frac{3.32}{Re^{0.0086logRe+0.81}}\right)\right]^2} \quad (17)$$

In this work, we adopted von Karman, Nikuradse, Swamee-Jain, Haaland and Ohirhian friction factor and considered their effect on pressure drop. We make the decision based on three factors:

(1) Required precision (2) Speed of computation required and (3) Available computational technology. The friction factor equations under consideration meet the above criteria.
The proposed model is compared with Weymouth flow model [1] stated as:

$$q = \frac{18.062T_b}{P_b}\left[\frac{(P_1^2 - P_2^2)D^{16/3}}{TZLG_g}\right]^{0.5}$$

where

$$f = \frac{0.032}{D^{\frac{1}{3}}}$$

Methodology

The motion of fluid is usually complex, and it is not always subjected to exact mathematical analysis. One of the important concept used in fluid flow is the energy equation associated with fluid flow over the length of a conduit and is given by (1)

$$\frac{144}{\rho_g}dp + \frac{u_g}{2g_c}du + \frac{g}{g_c}dz + \frac{f_m {u_g}^2}{2g_c D}dL + W = 0 \quad (18)$$

The assumptions made for the development of the methodology are, the flow is steady, temperature is assumed constant over the length of the conduit, kinetic energy is small and negligible, no mechanical work is done on or by the fluid and change in elevation is zero because flow is horizontal. Equation (18) becomes

$$\frac{144}{\rho_g}dp + \frac{f_m {u_g}^2}{2g_c D}dL = 0 \quad (19)$$

$$-\frac{144dp}{\rho_g} = \frac{f_m {u_g}^2 dL}{2g_c D} \quad (20)$$

where the density of gas is given by [1]

$$\rho_g = \frac{28.97PG_g}{ZRT} \quad (21)$$

For the velocity of the Gas [1]

$$u_g = \frac{0.4166q_g TZ}{PD^2} \quad (22)$$

Putting eqn. (21) and eqn (22) into eqn. (20)

$$-\frac{144dp}{\left(\frac{28.97PG_g}{ZRT}\right)} = \frac{\left(\frac{0.1736q_g^2 T^2 Z^2 f_m dL}{PD^2}\right)}{2Dg_c} \quad (23)$$

$$-Pdp = \frac{5.054\times10^{-5} G_g TZq_g^2 f_m}{D^5}dL \quad (24)$$

Integrating both sides,

$$\int_{p_1}^{p_2} -Pdp = \int_0^L \frac{5.054\times10^{-5} G_g TZq_g^2 f_m}{D^5}dL \quad (25)$$

$$\boldsymbol{P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZf_m L}{D^5}} \quad (26)$$

Recall, for laminar flow, the friction factor is given as:

$$f_m = \frac{64}{R_e} \quad (27)$$

But,

$$R_e = \frac{\rho uD}{\mu} \quad (28)$$

Therefore,

$$f_m = \frac{64\mu}{\rho uD} \quad (29)$$

Insert eqn. (21) and eqn. (22) into eqn (29) to get

$$f_m = \frac{56.9\mu D}{G_g q_g} \quad (30)$$

Inserting eqn. (30) into eqn. (26)

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{56.9\mu D}{G_g q_g} \quad (31)$$

$$P_1^2 - P_2^2 = \frac{0.00575TZ\mu Lq_g}{D^4} \quad (32)$$

where μ is in $Ib/Ftsec$
Equation (32) is for laminar flow. For turbulent flow, recall, von Karman frictional factor for rough pipes

$$\frac{1}{\sqrt[2]{f_m}} = 2\log\left(\frac{3.7}{\varepsilon}\right) \tag{33}$$

where

$$\varepsilon = \frac{e(ft)}{D(ft)} \tag{34}$$

Inserting eqn. (33) into eqn. (26)

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{1}{\left[2log\left(\frac{3.7}{\varepsilon}\right)\right]^2} \tag{35}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5\left[2log\left(\frac{3.7}{\varepsilon}\right)\right]^2} \tag{36}$$

Equation (36) is for turbulent flow. From Swamee-Jain

$$f = \frac{0.25}{\left[log\left(\frac{\varepsilon}{3.7D}+\frac{5.74}{Re^{0.9}}\right)\right]^2} \tag{37}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{0.25}{\left[log\left(\frac{\varepsilon}{3.7D}+\frac{5.74}{Re^{0.9}}\right)\right]^2} \tag{38}$$

$$P_1^2 - P_2^2 = \frac{2.528\times10^{-5} G_g q_g^2 TZL}{D^5\left[log\left(\frac{\varepsilon}{3.7D}+\frac{5.74}{Re^{0.9}}\right)\right]^2} \tag{39}$$

Equation (39) is for turbulent flow. From Haalands equation

$$f = \frac{1}{\left[-1.8log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right]\right]^2} \tag{40}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{1}{\left[-1.8log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right]\right]^2} \tag{41}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5\left[-1.8log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right]\right]^2} \tag{42}$$

Equation (42) is for turbulent flow. From Nikuradse frictional equation,

$$f = \frac{1}{\left[1.74-2log\left(\frac{2e}{D}\right)\right]^2} \tag{43}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{1}{\left[1.74-2log\left(\frac{2e}{D}\right)\right]^2} \tag{44}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5\left[1.74-2log\left(\frac{2e}{D}\right)\right]^2} \tag{45}$$

Equation (45) is for turbulent flow. From Ohirhian frictional equation

$$f = \frac{1}{\left[-2log\left(\frac{\varepsilon}{3.7D}+\frac{3.32}{Re^{0.0086logRe+0.81}}\right)\right]^2} \tag{46}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5} \times \frac{1}{\left[-2log\left(\frac{\varepsilon}{3.7D}+\frac{3.32}{Re^{0.0086logRe+0.81}}\right)\right]^2} \tag{47}$$

$$P_1^2 - P_2^2 = \frac{1.011\times10^{-4} G_g q_g^2 TZL}{D^5\left[-2log\left(\frac{\varepsilon}{3.7D}+\frac{3.32}{Re^{0.0086logRe+0.81}}\right)\right]^2} \tag{48}$$

Equation (48) is for turbulent flow.

Results and Discussion

To accomplish our comparison, we assume the data shown Table 1.

Table 1: Data for Example Problem

Temperature	520 Rankine
Conduit diameter	2.5 inches
Conduit length	5280ft (1 mile)
Gas deviation factor	0.9
Roughness	0.0007 inches
Specific gravity of gas	0.8
Viscosity 1	0.0019cp
Viscosity 2	0.0200cp

Fig.2 shows how Δp varies with q, the developed model was solved by iteration. It is observed that in laminar flow, the rate of flow depends on the viscous force of the fluid. It was also observed that Δp varies inversely to D and q (gas) varies with D, which implies that pipe diameter is important in each of the flow regime. Slight variation in absolute viscosity yielded changes in corresponding parameters. This further proves that laminar flow depends basically on viscosity (viscous force).

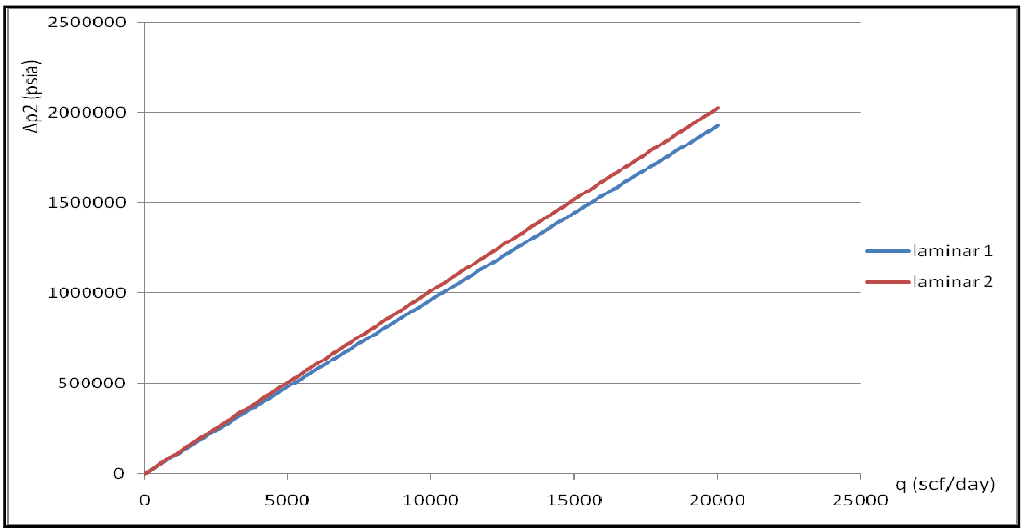

Figure 2: Δp^2 (psia) against q(Scf/d) for laminar flow.

From the **Fig. 3**, it's a curve is observed unlike laminar, which is a straight line, denoting quadratic relationship of pressure and rate. Comparing from graph, the models are relatively more accurate than Weymouth because Weymouth assumed frictional factor is not as accurate as those assumed for the developed model.
The model close to that of Weymouth are T4 and T5 with the use of Nikuradse friction factor and that of Ohirhian (2005). T4 should be used when the use of Reynolds number is not required and T5 when the use of Reynolds number is required.

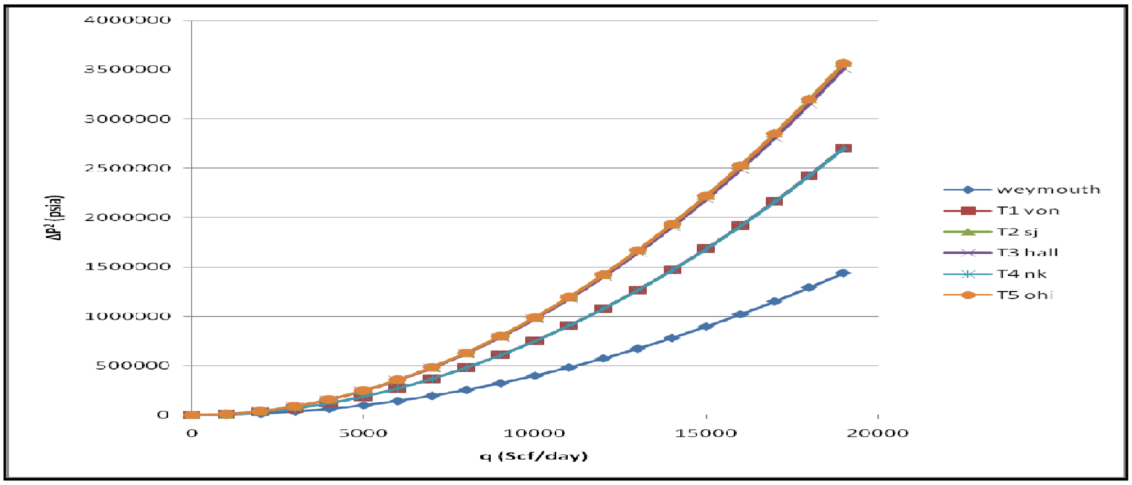

Fig. 3: Δp^2 (psia) against q(Scf/d) for turbulent flow.

From Fig. 4, the pressure drop in laminar is relatively small as compared to turbulent flow and also pressure increases in laminar flow as the viscosity increases.

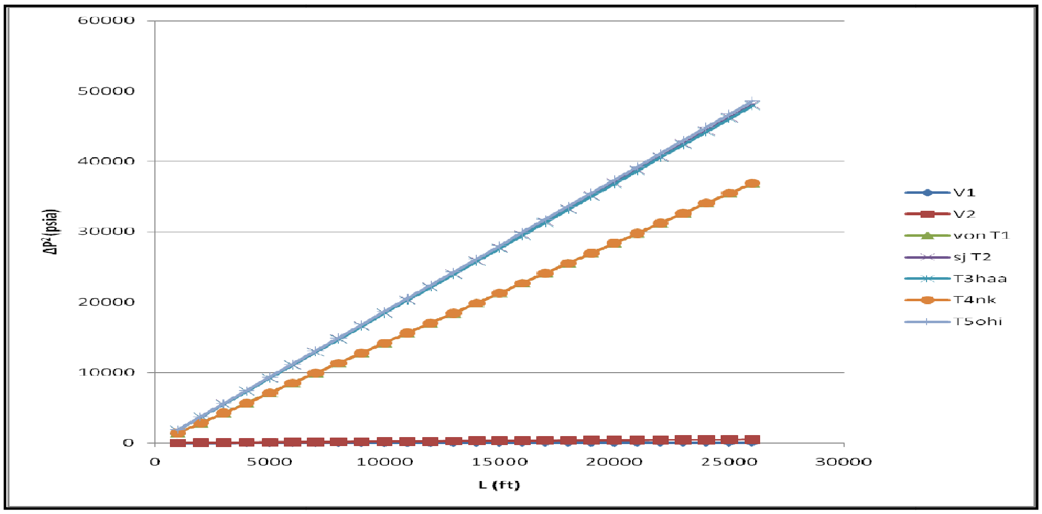

Fig. 4: Δp^2 (psia) against L (ft) for turbulent flow.

Conclusion

From the study, it was observed that laminar flow depends on viscous force (viscosity), as the viscosity increases, the pressure drop increases and gas flow rate decreases. Also in laminar flow, the flow is independent on the specific gravity of the gas. Here, the change in the square of the pressures varies with the gas flow rate and inversely to D^4. But it should be noted that natural gases do not flow laminarly. In turbulent flow, the viscous force is irrelevant and the flow is dependent on relative roughness and Reynolds number. When the flow is not completely turbulent it will depend more on the Reynolds number and when it is completely turbulent, it will depend more on the relative roughness. Also in turbulent flow, the specific gravity of the gas is important in that as the specific gravity increases, the pressure drop increases and the gas flow rate decreases. Here, the change in the square of the pressures varies with the square of the gas flow rate and inversely with D^5.

The diameter of the conduit affects the pressure drop more on the turbulent flow than in the laminar flow; the larger the diameter the larger the gas flow rate.

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