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Turbulence Flow Characteristics of Suspended Sediments and its Transport Rate in Open Channel

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Abstract

This paper presents an attempt to describe velocity distribution of suspended sediment laden flow by using a theory based on Monin-Obukhov Length L. It will be shown that experimental results from open channel flow with suspended sediments are better accounted for by this theory. The method involves the coupling of energy production equation by Reynold stresses with the velocity distribution equation in flow with suspended sediments. These are inturn integrated to give the hydraulic resistance law for sediment laden flow. The law of velocity distribution in open channel flow with suspended sediments was derived introducing Monin-Obukhov Length L . The distribution equation agrees well with the observation of velocity profile in the experiments. The Von-Karman constant, has a value of 0.4 for flows with suspended sediments. The collapse of turbulence in open channel in the suspended sediments was predicted by the theory, and a good agreement with measured values was observed. These results were in agreement with the work of [1], [2] , [5] and, [23].

1.0 Introduction

Studies of the mechanics of suspended sediment laden flow have been made with varying degrees of success. Experimental observations were carried out by [2], [3], [6], [23] and [24]. From these investigations the following conclusions for open channel flows with suspended sediments have been obtained.

1. The friction factor for the flow decreases as an effect of suspended material prevails.
2. The von Karman universal constant, k, which has been given the value of about 0.4 in a pure water flow, may be smaller in flows with suspended sediment.
3. The mixing length and the scale of turbulence for the flow are reduced because of suspended sediments in the flow.
4. The velocity gradient du/dy becomes larger with increasing suspended sediment concentration.
5. Measured near the bottom of the channel appear to be large compared to values predicted by the velocity defect law.
6. The concentration of suspended sediments near the bottom is generally lower than a value computed from the generally accepted chemical equation.

Theoretical description of the phenomena mentioned in the foregoing have been attempted by several Japanese researchers, such as [19], [21], [20], and [4].

In most previous studies, the velocity distributions were examined so as to fit a straight line for points plotted on a semilogarithmic paper. Thus average slopes of the lines were used to evaluate over-all values of k. Some of the velocity distribution equations were shown by the forms of the velocity defect law. It is therefore necessary to give a value of maximum velocity, μ_{max}, to calculate the exact velocity distribution and a hydraulic resistance law of the flow.

In the present study, a new attempt to describe a velocity distribution of suspended, sediment laden flow was proposed by using a theory based on the Monin-Obukhow length. It will be shown that the experimental results for open channel flows with suspended sediments are better accounted for by this theory.

In the next section, a theoretical description of velocity distribution is presented and then a hydraulic resistance law for sediment laden flow will be established. The following sections deal with the distribution of sediment concentration and the transport rate of suspended materials, and include a reasonable definition of the reference concentration. Finally, some characteristics of turbulence for the flow with suspended sediments will be described.

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Velocity Distribution

An energy equation for a unit mass of water and sediment mixture is described by Krishnappen [14] as:

$$\tau \frac{du}{dy} = \frac{\tau}{g\rho' \upsilon'} + B\frac{\rho u_*^3}{kL} + \frac{\rho u_*^3}{ky} \qquad (1)$$

The left-hand side of equation 1 represents the energy production by the Reynolds stress, and the right-hand side expresses an energy distribution in the flow; the first term is the power per unit volume per unit time due to the pressure fluctuation, and it may be rewritten as equation (2) in steady state, which corresponds to the energy or power necessary to suspend sediment particles:

$$\frac{\tau}{gp'v'} = g\rho(\gamma-1)\frac{\tau}{c'v'} = g\rho(\gamma-1)w_o c \qquad (2)$$

The second term in equation is the rate of removal of energy through motions of suspended particles such as rotation, rectilinear motion relative to the fluid, collision, and the reduction of effective space to dissipate energy into heat. The last term indicates the rate of energy loss dissipated and diffused by a turbulent motion of liquid phase.

The velocity distribution for a flow with suspended sediments, therefore, will differ from the distribution in a pure water flow, depending on the ratio of energy described by the first and second terms to the third term on the right-hand side of equation (1). Both the first and second terms represent the effect of suspended sediments in the flow. Then it was assumed that they were proportional to each other, and the characteristic length, L, was defined by the ratio between them as

$$\frac{1}{L} = \frac{kg(\gamma-1)w_o c}{u^3_*} \qquad (3)$$

This assumption was verified by [9].

The characteristic length, L, defined by equation 3, is the ratio of a buoyancy flux, i.e., a velocity of energy transport by buoyancy, to a velocity of energy dissipation. It corresponds to the "Monin-Obukhov length" in meteorology, as introduced by [17] and [22].

In pure water flows, the first and second terms on the right-hand side of equation (1) should be zero. This shows that in flow with suspended sediments an additional energy described by the first and second terms in equation (1) is required, and that $\frac{du}{dy}$ should be larger than for the pure water flow. If two terms are combined by introducing a ratio, L, as described by equation (3) , then equation (1) leads to

$$\tau \frac{du}{dy} = \alpha \frac{\rho u_*^3}{kL} + \frac{\rho u_*^3}{ky} \qquad (4)$$

Or

$$\frac{du}{dy} = \frac{u_*}{ky}\left(1 + \alpha \frac{y}{L}\right) \qquad (5)$$

In equation (5), α is called the Monin-Obukhov coefficient, and α=5 was found from some observations of wind velocity over the ground by [26].

As shown in equation (3), L is a function of y, a distance from the boundary, since the concentration, c, depends on y. In meterological applications, however, L has been calculated at a reference point near the ground. In the present study, taking into account this method, a mean value of concentration $\overline{c}$ over the cross section was substituted for c in equation (3). Then the coefficient α was evaluated from many velocity distributions in the flows with suspended sediments as

$$\alpha = 7; \qquad \frac{1}{L} = \frac{kg(\gamma-1)w_o \overline{c}}{u_*^3} \qquad (6)$$

A log-linear law for the velocity distribution for the flow with suspended sediments is obtained by integrating equation (5) as

$$\frac{u}{u_*} = \frac{1}{k}\left(In\frac{u_* y}{\overline{v}} + \alpha\frac{y}{L}\right) + \left[B'\left(\frac{u_* k_s}{\overline{v}}, \frac{k_s}{L}\right) - \frac{1}{k}In\frac{u_* k_s}{\overline{v}}\right] \qquad (7)$$

in which $\overline{v}$ = a kinematic viscosity of sediment laden flow. A relationship between $\overline{v}$ and υ, the kinematic viscosity for pure water flow, was introduced by [3] as

$$\overline{v} = \upsilon(1+2.5c_o) \qquad (8)$$

The well known logarithmic distribution for velocity in a pure water flow is a particular form of equation (7) for L → ∞.

In equation (7) B' is recognized to be a function of $u_*\ k_S\sqrt{v}$ and k_s/L . In a previous study [12], the writers attempted to determine whether or not B' in equation (7) depends on k_s/L. In this study, such pairs of velocity profiles were selected from flume data obtained by the writers, and by Einstein and Chien [2], for pure water and sediment laden flows with almost the same hydraulic conditions. All the flows chosen for the analysis were without any deposition of sediments on the bottom of the flume. Then, equation (9) was applied to a velocity profile of pure water flow and a roughness height, K_s, was evaluated.

$$\frac{u}{u_*} = \frac{1}{k}\ln\frac{u_* y}{v} + \left[B\left(\frac{u_* k_S}{v}\right) - \frac{1}{k} In\frac{u_* k_S}{v}\right] \qquad (9)$$

This value of k_s and equation (7) were applied to the velocity profile for sediment laden flow with the same hydraulic conditions, and B' in equation was evaluated. From an examination of many pairs of velocity profiles, it was concluded that B' was equivalent to B and was independent of L.

Equation (7) can, therefore, be reduced to a more convenient form for general applications.

$$\frac{u}{u_*} = \frac{1}{k}\left[\ln\left(\frac{u_* y}{\bar{v}}\right) + \varphi\frac{u_* y}{\bar{v}}\right] + \left[B\left(\frac{u_* y}{\bar{v}}\right) - \frac{1}{k} In\frac{u_* k_S}{\bar{v}}\right] \qquad (10)$$

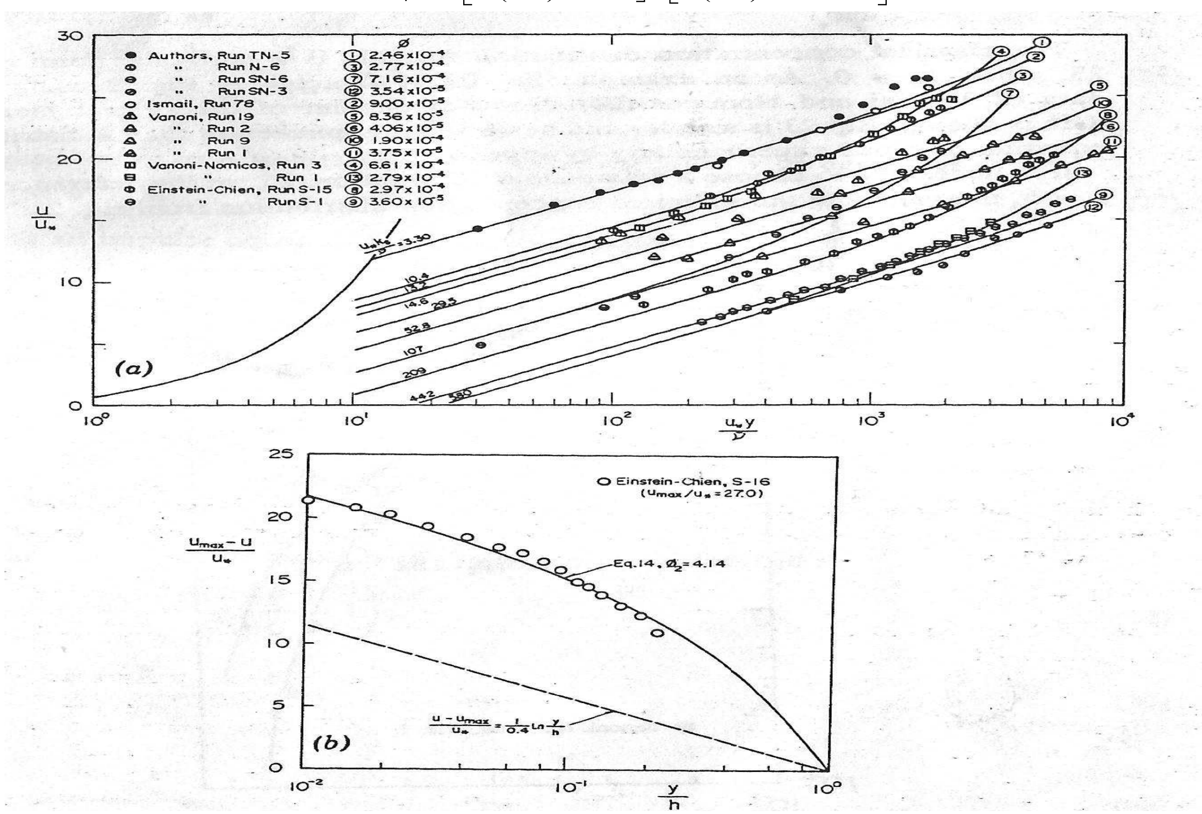

Fig. 1: (a) Velocity Distribution in Open Channel Flow with Suspended Sediments:
(b) Velocity Defect Law Plot

in which

$$\phi = \alpha\frac{\bar{v}}{u_* L} \qquad (11)$$

and k = 0.4 (12)

Equation (10) shows that the effect of suspended sediment on the velocity is predominant not near the bottom, but near the water surface as $u_* y/v$ increases. A new parameter, ϕ, defined by equation (11) increases almost proportionally to the concentration of suspended sediments.

The velocity distributions given by equation (10) were fitted well to the measured distributions, as shown in Fig. 1(a), and it is concluded that the theoretical distribution shows a good agreement to the flume data.

Thus, when the flows are of completely rough or smooth hydraulic conditions, the following velocity distributions may be derived from equation (10).

$$\frac{u}{u_*} = 8.5 + \frac{1}{k}\left(In\frac{y}{k_s} + \phi_1 \frac{y}{k_s} \right) for \frac{u_* k_s}{\bar{v}} > 70 \tag{13a}$$

$$\frac{u}{u_*} = 8.5 + \frac{1}{k}\left(In\frac{u_* y}{\bar{v}} + \phi_1 \frac{u_* y}{\bar{v}} \right) for \frac{u_* k_s}{\bar{v}} < 5 \tag{13b}$$

in which
$$\phi_1 = \alpha \frac{k_s}{L} \tag{13c}$$

A velocity distribution for the flow with suspended sediments in the form of the velocity defect law is obtained from equation (10) as

$$\frac{u - u_{max}}{u_*} = \frac{1}{k}\left[\ln\frac{y}{h} + \phi_1 \left(\frac{y}{h} - 1 \right) \right] \tag{14}$$

in which
$$\phi_1 = \alpha \frac{h}{L} \tag{15}$$

Fig. 1(b) shows an example of the velocity defect law of equation (14) in comparison with Einstein and Chien's flume data, and a good coincident of the theory is recognized. For the application of equation (14), the only parameter to be given is ϕ_2, which differentiates the velocity defect law for he velocity distribution for flows with suspended sediments from that for flows in pure water. The parameter, ϕ_2, can be obtained from flow data, using equations (15) and (16). In contrast, a velocity defect law obtained empirically by Ippen [5] is:

$$\frac{u - u_{max}}{u_*} = \frac{1}{k'} In\left[\frac{y}{h} - \Psi\, In\frac{y}{h} \right] \tag{16}$$

In equation (16), two parameters, k' and Ψ, must be known in order to calculate the velocity distribution.

The hydraulic resistance law for sediment laden flow will be derived in the next section using the velocity distribution formula obtained in this section.

Hydraulic Resistance Law for Sediment Laden Flow

The hydraulic resistance law for the flow with suspended sediments can be derived from the velocity distribution described in the preceding section.

The average velocity, u_m, of the flow can be obtained by integration of equations (13a) and (13b), giving:

$$\frac{u_m}{u_*} = 6.0 + \frac{1}{k} In\frac{h}{k_s} + \frac{\phi_2}{2k} \text{ for } \frac{u_* k_s}{\bar{v}} > 70 \tag{17a}$$

$$\frac{u_m}{u_{.*}} = 3.0 + \frac{1}{k} In\frac{u_* h}{\bar{v}_s} + \frac{\phi_2}{2k} \text{ for } \frac{u_* k_s}{\bar{v}} < 5 \tag{17b}$$

Assuming v = $\bar{v}$, a relationship between the Darcy-Weisbach friction factors for clear and sediment laden flows may be derived.

$$\sqrt{\frac{8}{f}} = \sqrt{\frac{8}{f_o}} + \frac{\phi_2}{2k} \tag{18}$$

in which f and f_o = friction factors for the sediment laden and pure water flows, respectively.

It was concluded from equation (18) that the friction factor for the sediment laden flow is less than that of the pure water flow. The relationship of equation (18) showed a satisfactory agreement with much experimental data [7].

On the contrary, it was observed by Itakura et al [7] that the friction factor for sediment laden flow was larger than friction factor for pure water f_o, and was a monotonically increasing function of the concentration of suspended sediments. Thus, the results reported by Montes are in conflict with the conclusion obtained in this study. It should be noted that his measurements were performed in a hydraulically smooth flume and for flow in transition from the smooth to rough hydraulic condition.

Distribution of Suspended Sediment Concentration

In this section, a theoretical approach for the distribution of suspended sediment concentration is explained.

Assuming a linear distribution of the shear stress, τ, the kinematic eddy viscosity, $\in_m$, i.e. momentum transfer coefficient, is described as:

$$\tau = \rho \in_m \frac{du}{d\rho}; \tau = \tau_o \left(1 - \frac{y}{h} \right); \in_m = \frac{u_*^2 \left(1 - \frac{y}{h} \right)}{\frac{du}{dy}} \tag{19}$$

If the dispersion coefficient of suspended sediment $\in_s$ is assumed to be equal to the kinematic eddy viscosity, equation (20) is obtained from equations (5) and (19). Thus

$$\in_S = ku_* y\left(1-\frac{y}{h}\right)\left(1+\alpha\frac{h}{L}\frac{y}{h}\right)^{-1} \tag{20}$$

Substituting equation (20) into the equation of equilibrium for suspended sediment given equation (21), in which w_o = the fall velocity of suspended materials, equation (22) can be derived:

$$\in_S \frac{dc}{dy} + w_O c = 0 \tag{21}$$

$$\frac{dc}{c} = -\frac{w_O}{ku_* y}\left(1+\alpha\frac{y}{L}\right)\left(1-\frac{y}{h}\right)^{-1} \text{dy} \tag{22}$$

Integrating equation (22) with a boundary condition of c = c_b at y = b, the distribution of suspended sediment concentration is determined as:

$$\frac{c}{c_b} = \left[\left(\frac{h-y}{h-b}\right)^{1+\phi_2}\left(\frac{b}{y}\right)\right]^z \tag{23}$$

in which $$z = \frac{w_O}{ku_*}; \text{k} = 0.4 \tag{24}$$

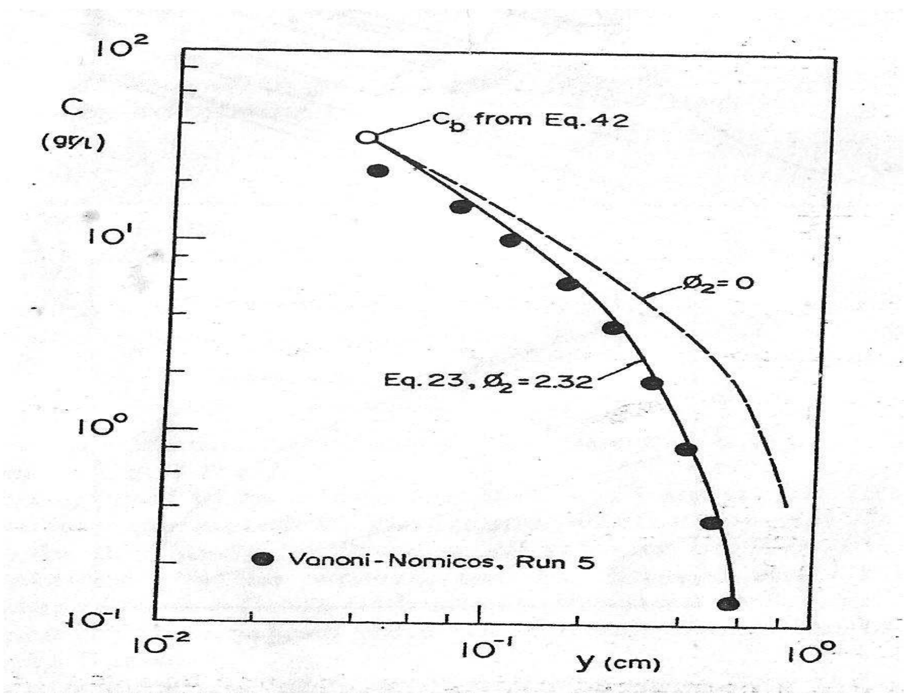

Fig. 2: Distribution of Suspended Sediments Concentration

The classical concentration distribution formula is a particular form of equation (23), when ϕ_2 = 0. As an example, equation (23) is depicted in Fig. 2 using some data of Vanoni and Nomicos [24]. Good agreement of theory and measured data is shown. Equation (23) is simpler and has a wider applicability than a distribution equation obtained experimentally by Ippen [5].

It is necessary to choose a reference concentration, c_a, at the reference level y =b, in evaluating the sediment concentration distribution from equation 23.

Based on the studies performed by [15], [10], and [25], a new attempt was made to evaluate c_b, introducing a modified velocity of sand grains in fluid.

It was assumed that the bed of the stream consists of uniform grains. A characteristic duration of time t_2, in which a grain at the bottom of the channel is replaced by a depositing grain, is defined by:

$$t_2 = k_2 \frac{d}{v_s} \tag{25}$$

in which d = the grain size of a uniform bed material; and v_s denotes the vertical component of relative velocity of the grain.

A pick-up rate, q_m, of the grains from unit area of the bed per unit time is described as

$$q_m = \frac{\pi d^2}{6} \frac{k_1}{\frac{\pi}{4} d^2 \frac{k_2 d}{v_2}} = K v_2 \tag{26}$$

Introducing an absolute velocity, v_o, of the grain, v_s is described as

$$v_s = v_o - w_o \tag{27}$$

and an impulse equation for an individual grain is

$$(\mathrm{F - G}) t_* = \left(p_S \frac{\pi d^3}{6} \right) v_o \tag{28}$$

in which

$$\mathrm{F} = \phi_s p d^2 \, u_*^2 \tag{29}$$

$$\mathrm{G} = (p_S - p) g \frac{\pi d^3}{6} \tag{30}$$

$$t_* = a_* \frac{d}{u_*} \tag{31}$$

Then the absolute velocity of the grain can be deduced as

$$v_o = t_* \frac{p_S - \rho}{p_S} g \left(\frac{F}{G} - 1 \right) \tag{32}$$

$$\frac{F}{G} = \frac{F}{\overline{F}} . \frac{\overline{F}}{G} = r . \frac{\phi_S}{\frac{\pi}{6}} \overline{\tau}_* \tag{33}$$

$$\overline{\tau}_* = \frac{u_*^2}{(\gamma - 1) g d} \tag{34}$$

In equation (33) r = a normalized hydrodynamic force and can be written as equation (35), in which r' denotes the fluctuating component of r.

$$r = \frac{F}{\overline{F}} = \frac{\overline{F} + F'}{\overline{F}} = 1 + r' \tag{35}$$

The grain on the bed will detach when F > G. Then a critical condition of the detachment can be written as

$$r = 1 + r' = \frac{F}{\overline{F}} > \frac{G}{F} = \frac{\frac{\pi}{6}}{\phi, \overline{\tau}_*} = a \tag{36}$$

$$r' = r - 1 > a - 1 \tag{37}$$

The fluctuating force component, r', was assume to have a Gaussian distribution with a mean of zero and a variance, a^2. An average velocity, $\overline{v}_*$, of grains in fluid is described as

$$\overline{v}_o = \frac{\int_{a-1}^{\infty} v_o \cdot \frac{1}{\sqrt{2\pi\sigma}} \exp\left(- \frac{r'^2}{2\sigma^2} \right) dr'}{\int_{a-1}^{\infty} v_o \cdot \frac{1}{\sqrt{2\pi\sigma}} \exp\left(- \frac{r'^2}{2\sigma^2} \right) dr'} \tag{38}$$

in which

$$v_o = t_* \frac{p_S - \rho}{p_S} g \left[(1 + r') \frac{\overline{\tau}_*}{B_* \eta_o} - 1 \right] \tag{39}$$

$$\eta_o = \sqrt{2\sigma}; \; \mathrm{B}_* = \frac{6}{\eta_o \phi_S} \tag{40}$$

A deposition rate, q_{sd}, of suspended grains per unit area of the bed per unit time is

$$q_{sd} = c_b w_o \tag{41}$$

Equating equation (26) and (41), the reference concentration, c_b, was obtained after

$$c_b = K\left(a_* \frac{p_s - \rho}{p_1} \frac{gd}{u_* w_o} \Omega - 1 \right) \qquad (42)$$

$$\Omega = \frac{\bar{\tau}_*}{B_*} \frac{\int_{a'}^{\infty} \xi \frac{1}{\sqrt{\pi}} \exp\left(-\xi^2\right) d\xi}{\int_{a'}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(-\xi\right) d\xi} + \frac{\bar{\tau}_*}{B_* \eta_o} - 1; \; a' = \frac{B_*}{\bar{\tau}_*} - \frac{1}{\eta_o} \qquad (43)$$

in which b = 0.05h.

Equation (42) contains four constants, K, a_*, B_* and η_o, which should be determined by experiments. According to Einstein's analysis (1), B_*, = 0.143 and η_o = 0.5 could be adopted for these constants in the present study.

A value of a_* = 0.14 was determined by an examination of the data reported by Kishi et al [11], which were of the observations of trajectories of slating artificial grains of several densities. Equation (38) is shown in fig. 3, compared to the experimental results using these constants.

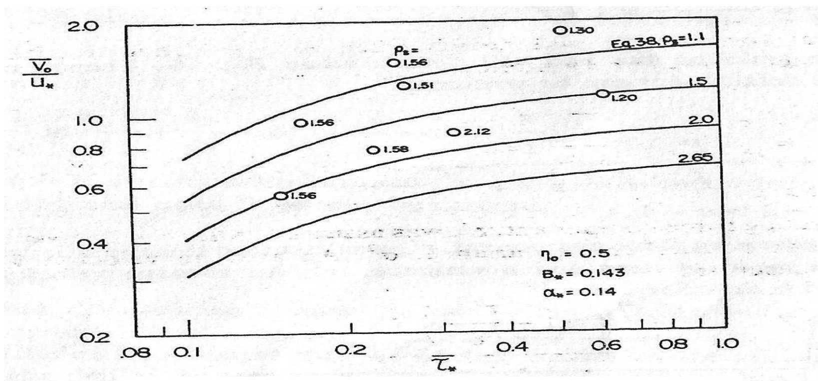

Fig. 3: Average Velocity of Grains in Fluid

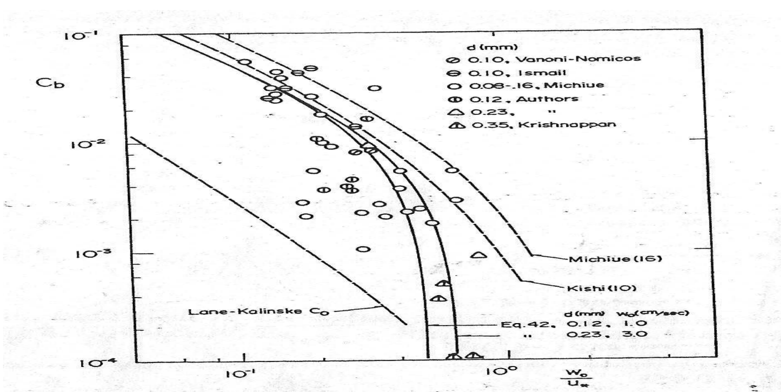

Fig. 4: Relationship between References Concentration C_b and w_0 / u_*

A value of K = 0.008 was determined, so as to fit many data of c_b, as shown in fig. 4. The numerical value of K = 0.008 means that 8% - 24% of detached grains go to the suspended load, according to a study of bed load sediment by Kishi et al [10]. This was confirmed by an examination of the data of saltating grains [13].

In conclusion, the reference concentration is calculated from equation (42), completely by the use of flow data. An example of the calculation is shown in Fig. (2) compared to the flume data. As shown in fig. 4, equation (42) well describes the fact that the suspended load suddenly increases when a certain value of w_o/u_* was attained in the alluvial steams.

When the distribution of sediment concentration is clarified, the transport rate of suspended sediment can easily be calculated, as shown in the next section.

Transport Rate of Suspended Sediments

The transport rate of suspended sediment per unit width of flow can be calculated by equation (44).

$$q_S = \int_b^h u \cdot c \, dy \qquad (44)$$

Introducing the velocity, u, of equation (10), and the sediment concentration, c, given by equation (23), into equation (44), the following equation is obtained.

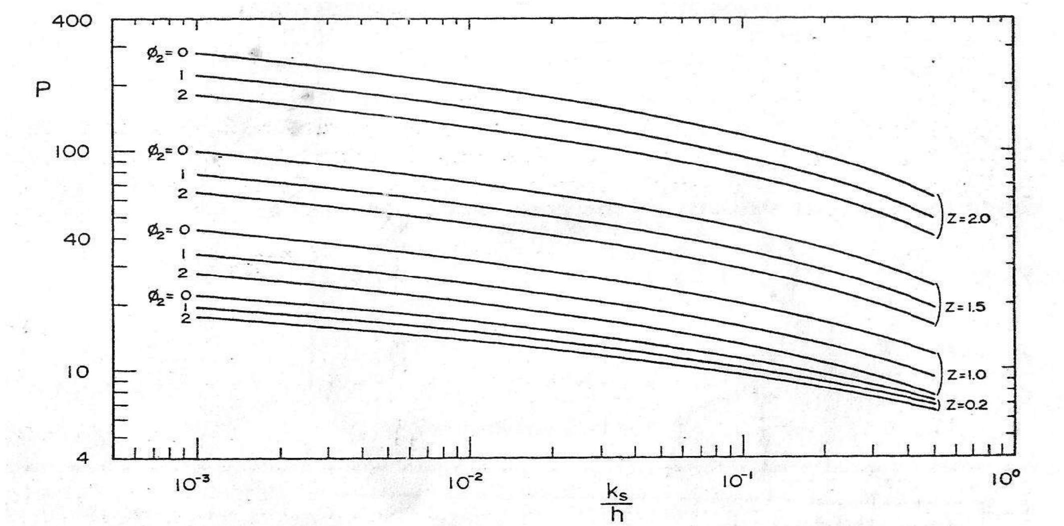

Fig. 5: Relationship between P and k_s / h

$$\frac{q_S}{u_* c_b} = \frac{\left(\frac{b}{h}\right)^z (h-b)}{\left[1-\frac{b}{h}\right]^{1+\phi_2}} \left\{ \frac{\phi_2}{k} I_3 + \frac{1}{k} I_2 + \left[B\left(\frac{u_* k_S}{\bar{v}}\right) - \frac{1}{k} In \frac{k_S}{h} \right] I_1 \right\} \qquad (45)$$

in which

$$I_1 = \int_{b/h}^1 \left[\frac{(1-\eta)^{1+\phi_2}}{\eta} \right]^z d\eta;$$

$$I_2 = \int_{b/h}^1 \left[\frac{(1-\eta)^{1+\phi_2}}{\eta} \right]^z \ln \eta d\eta;$$

$$I_3 = \int_{b/h}^1 \left[\frac{(1-\eta)^{1+\phi_2}}{\eta} \right]^z \eta d\eta; \text{ and } \eta = \frac{\mathrm{y}}{\mathrm{h}} \qquad (46)$$

when b/h is much smaller than unity, equation (47) can be derived from equation (45); thus

$$q_S = u_* c_b h \left(\frac{b}{h}\right)^z \cdot P \qquad (47)$$

$$\rho = \frac{\phi_2}{\mathrm{k}} I_3 + \frac{1}{k} I_2 + \left[B\left(\frac{u_* k_S}{\bar{v}}\right) - \frac{1}{k} \ln \frac{k_S}{h} \right] I_1 \qquad (48)$$

The suspended load formula given by Einstein is a particular form of equations (45 – 48) with $\phi_2 = 0$. The function P in equation (47) is shown in Fig. 5, the calculations for which were performed for B

$$\left(u_* k_s \sqrt{v}\right) = 8.5 \text{ and } b/h = 0.05.$$

Characteristics of Turbulence

In this section, a criterion between a complete suspension flow and a flow with bed deposit, based on an investigation of the characteristics of turbulence in open channel flow with suspended sediments, is presented.

The kinematic eddy viscosity, which was defined by equation (19), can be written as follows, when ϕ_2 and $\eta = y/h$ are introduced.

$$\xi_m = \frac{u_{1*1}^2\left(1-\frac{y}{h}\right)}{\frac{u_*}{ky}\left(1+\alpha\frac{y}{L}\right)} \tag{49}$$

$$\xi_m = ku_*^2 h\eta(1-\eta)(1+\phi_2\eta)^{-1} \tag{50}$$

$$\xi_m = ku_*^2 h\eta(1-\eta)(1-\phi_2\eta)(1+\phi_2^2\eta^2) \tag{51}$$

A condition where the turbulence is collapsed or damped can be inferred from equation (51) as

$$\xi_m = 0 \text{ when } 1-\phi_2\eta = 0 \tag{52}$$

In equation (1), a ratio of the rate of removal of energy by suspending sediments to the turbulent energy production by shear defines the "flux Richardson number"
R_f. Introducing equations (2) and (5),
R_f is described as:

$$R_f = \frac{kg(\gamma-1)w_O'c}{u_*^3}y\left(1+\alpha\frac{y}{L}\right)^{-1} \tag{53}$$

As described by equation(53), the flux Richardson number is a function of position y, because it includes y, c, and L.
A reference flux Richardon number, R_{fh}, was defined by substituting y = h, corresponding to a depth, h, of an open channel flow, to obtain;

$$R_{fh} = \frac{h}{L}\left(1+\alpha\frac{y}{L}\right)^{-1} = \frac{\phi_2}{\alpha(1+\phi_2)} \tag{54}$$

A critical flux Richardson number, R'_{fc} is therefore deduced from equations (52) and (54).

$$R_{fc} = \frac{1}{\alpha(\eta+1)} \tag{55}$$

in which $R'_c = a$ function of η; and R'_{fc} =0.07 to 0.14 for η= 0.05 to 1.0

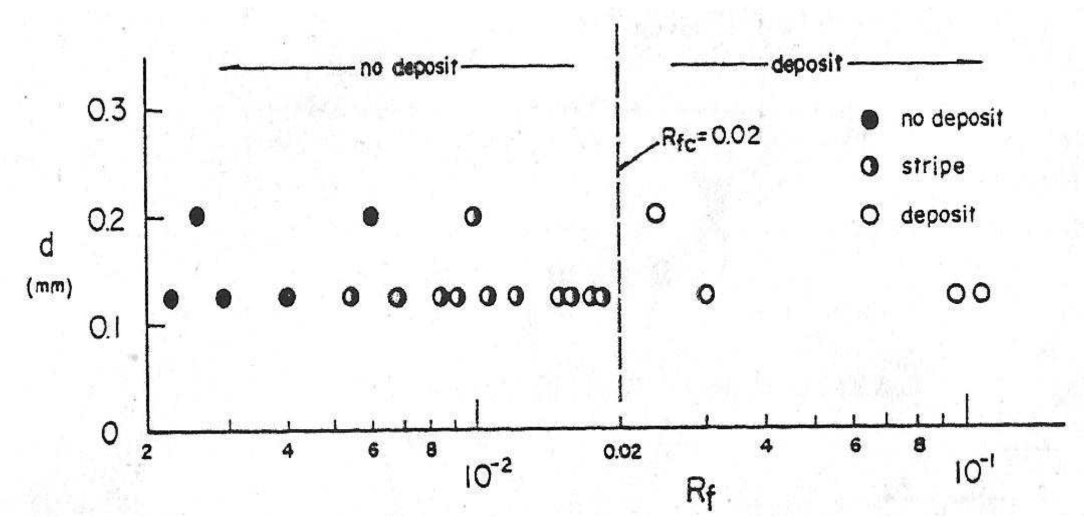

Fig. 6: Critical Flux Richardson Number

Experiments to determine the critical condition for the initiation of deposition of suspended materials were carried out by the writer in an open channel with rough bed of k_s = 0.33cm. A uniform pure water flow was first established and then sand grains were added to the flow. The results are shown in Fig. 6, and a critical flux Richardson number was obtained as

$$R_{fc} = 0.02 \tag{56}$$

This critical number was computed from equation (6) in which the characteristic length, L, was evaluated by using an average concentration, $\bar{c}$. In equation (54), it is reasonably assumed that αh/L << 1 and therefore R_f can be considered directly proportional to h/L, as well as c. In most of the concentration distribution profiles of suspended sediments in open channels, c = $\bar{c}$ and c = (4 ~ 5) $\bar{c}$ were observed at η=0.3 ~ 0.4 and η = 0.05~ 0.15, respectively. Thus, when R_{fc} = 0.02, a greater value of R'_{fc} is attained near the bed, and the deposition of suspended sediment initiates.

As a summary, the critical condition for the initiation of deposition of suspended grains is represented as

$$R_{fc} = \frac{h}{L}\left(1+\alpha\frac{h}{L}\right)^{-1} = 0.02 \tag{57}$$

in which

$$\frac{1}{L} = \frac{kg(\gamma-1)w_o\bar{c}}{u_*^3} \tag{58}$$

Some quantities of the structure of turbulence in open channel flows with suspended sediments can be derived from the preceding analysis as follows:

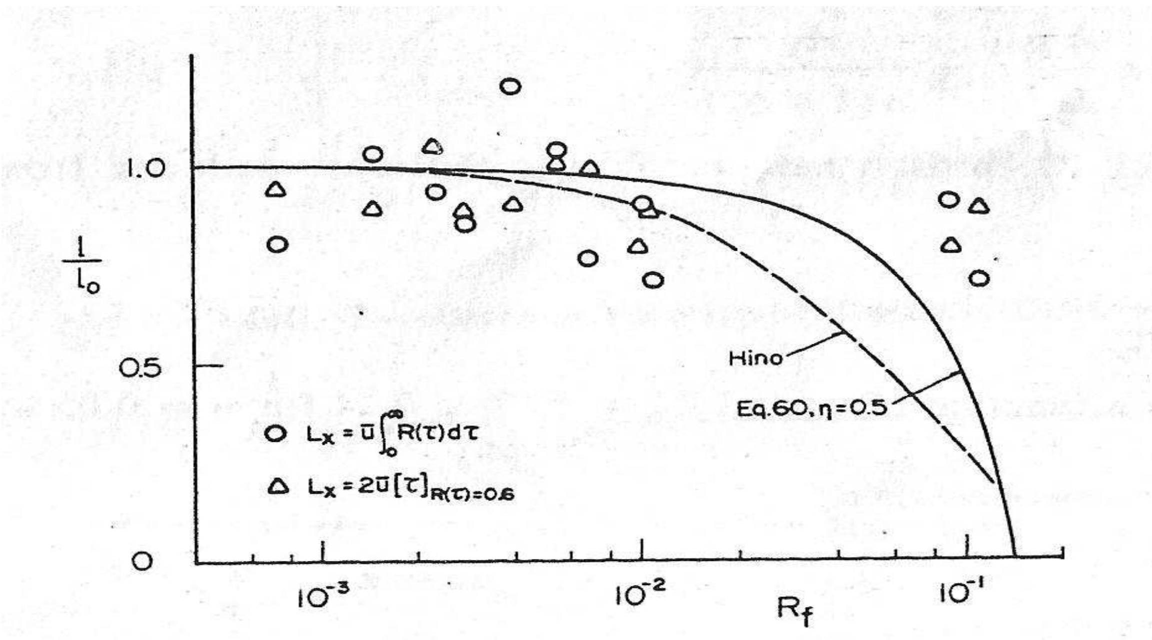

Fig. 7 Scale of Turbulence

According to the Karman-Prandtl mixing length theory

$$\frac{du}{dy} = \sqrt{\frac{\tau}{p}}\ \frac{1}{1} - \frac{u_*}{1}$$

$$1 - \frac{u_*}{\frac{du}{dy}} \tag{59}$$

The scale of turbulence can be estimated as

$$\frac{1}{l_o} = \frac{\left(\frac{u_*}{\frac{du}{dy}}\right)}{\left(\frac{u_*}{\frac{du}{dy}}\right)_o} = \left(1+\alpha\frac{y}{L}\right)^{-1} = \frac{1-\alpha R_f}{1-\alpha R_f(1-\eta)} \tag{60}$$

in which the subscript 0 denotes quantities for pure water flow. The lifetime of a turbulent eddy is

$$t - \frac{1}{\sqrt{u^{t2}}}$$

$$\frac{t}{t_o} = \frac{\left(\sqrt{\bar{u}'^2}\right)_o}{\sqrt{\bar{u}'^2}} \cdot \frac{1-\alpha R_f}{1-\alpha R_f(1-\eta)}$$

Figs 7, 8, and 9 show some examples of turbulence measurements by the writers using a hot-film anemometer. All the measurements were performed at $\eta = 0.5$. In order to resolve the highest frequency of 50Hz in the analysis, a sampling interval of 0.01 sec was used, and the period of sampling for all mean and root-mean-square velocity was 50 sec. From a consideration of the desired frequency definition and confidence limits, the degree of-freedom was determined to be 20, therefore, a lag of 500 was used in evaluating the autocorrelation function.

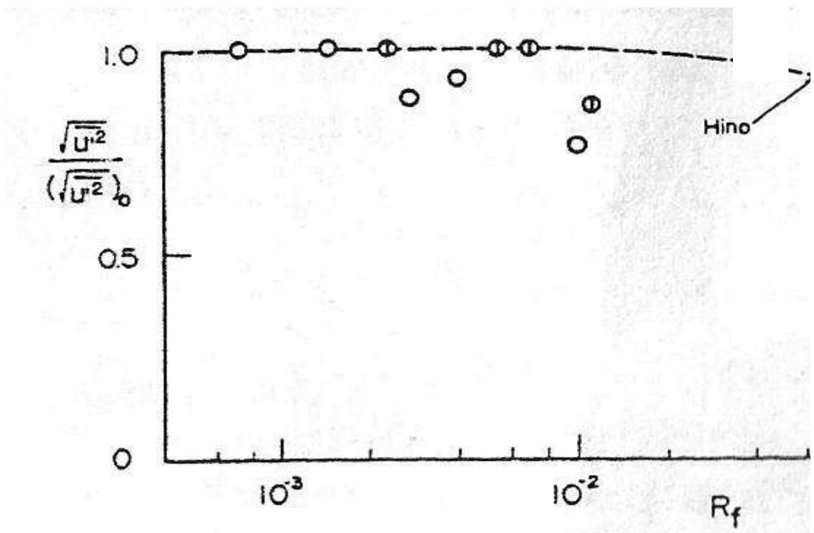

Fig. 8: Intensity of Turbulence

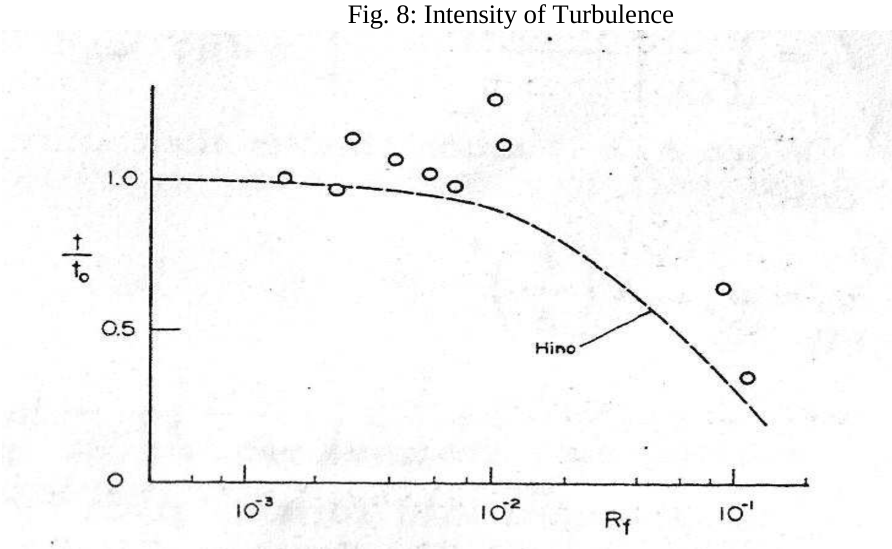

Fig. 9: Lifetime of Turbulent Eddies

Fig. 7 shows the variation of the scale of turbulence, represented by the turbulent macroscale and the semi-integral scale, with R_f. The measurements are compared to equation (60) and to Hino's theory. The intensity of turbulence, compared to Hino's theory, is shown in fig. 8. Fig. 9 shows the variation of the lifetime of turbulent eddies, in which the plotted points were calculated from equation (61) using the observed relative intensity of turbulence shown in fig. (8), and again Hino's theory is depicted for a comparison.

In all three figures, the collapse of turbulence can be observed around $R_f = 0.002$, as predicted by the theory.

Conclusions

A theoretical analysis of suspended sediment laden flow in an open channel was performed, and comparisons with much experimental data resulted in the following conclusions:

1. The law of the velocity distribution in open channel flow with suspended sediments was derived introducing Monin-Obukhov length L. The distribution equation agrees well with the observations of velocity profiles in experiments. The von Karman constant, has a value of 0.4 for flows with suspended sediment, as demonstrated here.
2. The hydraulic resistance law for the sediment laden flow was derived theoretically. The friction factor for the flow with suspended sediments in hydraulically rough channels was found to be smaller than that for pure water flow, and a good agreement of the theory with the experimental results was observed.
3. The distribution equation for the suspended sediment concentration and the transport rate formula for the suspended sediment load were derived. These describe well the concentration profiles measured in experiments, especially near the bottom of the channel where the classical equation is usually overestimated. The classical equations were shown to be particular forms of the new equation.
4. The equation for the reference concentration was derived and was fitted nicely to explain the observations. The reference concentration can be completely calculated by the flow data alone, without using concentration data.
5. The critical condition for the deposition of suspended particles in open channel flow was predicted in the theory by introducing the velocity flux and Richardson number in the experiments.
6. The collapse of turbulence in open channel flow with suspended sediments was predicted by the theory, and a good agreement with the measurements was observed.

Appendix II - Notation

The following symbols are used in this paper:

a = $\pi/(6\phi_s\bar{\tau}_*)$ = Parameter in equation (36)

$a' = B_*/\bar{\tau}_* - 1/\eta_o$ = parameter in equation (43).

B = constant in equation (1);

$B_*(u_*\frac{k_s}{v}$ = parameter in velocity distributions;

B_* = $\frac{\pi}{\left((6\eta_o\phi_s\right)} = 0.143$;

b = 0.05h, reference level;

c = volumetric concentration of suspended sediment at y;

c' = fluctuating component of c;

$\bar{c}$ = areal average of c;

c_b = reference concentration at y = b

d = diameter of suspended sediment and bed material;

F = hydrodynamic force (equation (29)

f = friction factor for sediment laden flow;

f_o = friction factor for pure water flow;

G = gravitational force (equation (30)

g = acceleration due to gravity;

h = depth of flow;

I_1, I_2, I_3 = parameters in equation (46)

K = 0.008, coefficient in (equation (42);

k_1 = coefficient in (equation (26);

k_2 = coefficient in (equation (25);

k_s = roughness height;

L = Monin-Obukhov length (equation (6);

I = scale of turbulence in sediment laden flow;

l_o = scale of turbulence in pure water flow;

P = parameter (equation (48);

q_s = transport rate of suspended sediment (L^2/T);

q_{ad} = rate of deposition (equation (41)

q_m = rate of detouchment (equation (26)

R_f	=	flux Richardson number;
R_{fc}	=	0.02 = critical flux Richardson number (equation (57)
r	=	normalizaed hydrodynamic force;
r'	=	fluctuating component of r;
t	=	life time of turbulent eddy in sediment laden flow;
t_o	=	life time of turbulent eddy in pure water flow;
t_*	=	characteristic time (equation (31)
u	=	point velocity in x direction at y;
u'	=	fluctuating component of u;
u_*	=	shear velocity
u_m	=	areal average velocity;
u_{max}	=	maximum value of u;
v'	=	fluctuating component of point velocity in y direction;
v_o	=	absolute velocity of grain;
v_s	=	relative velocity of grain;
w_o	=	fall velocity of suspended sediment;
x	=	coordinate in direction of flow;
y	=	coordinate perpendicular to x and to the bottom;
z	=	$w_o/(ku_*)$ in equation (24)
a	=	7.0 = Monin-Obukhov coefficient;
a_*	=	0.14 = coefficient in equations (31) and (42);
γ	=	p_S/ρ;
ε_m	=	kinematic eddy viscosity of fluid and sediment mixture;
ε_s	=	dispersion coefficient of suspended sediments;
η	=	y/h, relative height;
η_o	=	$\sqrt{2\sigma}$ =0.5 (equation 40)
k	=	0.4 = von Karman universal constant;
k'	=	constant analogous to k;
v	=	kinematic viscosity
$\bar{v}$	=	v for flows with suspended sediments;
ξ	=	$\frac{r''}{\sqrt{2\sigma}}$ = dummy variable in equation (43)
p	=	specific density of fluid and sediment mixture;
p'	=	fluctuating component of p;
p_s	=	specific density of sediment particle’
a^2	=	variance of hydrodynamic force;
τ	=	shear stress;
τ_*	=	$\frac{u_*^2}{[(\gamma-1)gd]}$ = dimensionless shear stress;
ϕ	=	$\frac{a\bar{v}}{u_* \text{L},}$ equation (29);
ϕ_1	=	$\frac{ak_s}{u_* \text{L},}$ equation (15);
ϕ_s	=	coefficient in equation (29)
Ψ	=	parameter in equation (16)
Ω	=	parameter in equations (42 and 43).

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