Journal of the Nigerian Association of Mathematical Physics
Volume **19 (November, 2011), pp 277 – 284**

A Mathematical Model and Simulation of Lime Shaft Kilns

R.O. Olayiwola
Department of Mathematics and Statistics,
Federal University of Technology, Minna, Nigeria.

Abstract

In this paper a mathematical model to predict the heat transfer in a lime kiln is presented. We assume the reaction is not well-stirred. We examine the properties of solution under certain conditions. The governing equations are solved analytically using high activation energy asymptotics. Results are presented and potential implications are discussed.

Keywords: lime shaft kiln, lime production, calcinations, simulation, shaft furnaces

1.0 Introduction

The primary function of the lime kiln is to convert $CaCO_3$ to CaO for reuse in the causticizing process. The process involves heat and mass transfer between the kiln, fuel, primary and secondary air, drying of lime mud, and calcining of $CaCO_3$.

Lime production is a global industry that contributes greatly to social and economic development throughout the world. Many beneficial industrial and consumer applications are made possible by the use of lime. A variety of business sectors including industrial manufacturing, utility suppliers, and environmental technologies, rely on the affordability, versatility and practicality of lime. The mining and distribution of lime stimulates commerce in other business sectors such as, transportation, shipping, storage, tooling suppliers, and heavy equipment suppliers.

A lime shaft kiln is basically a moving bed reactor with the upward-flow of hot gases passing counter-current to the downward-flow of a feed consisting of limestone particles undergoing calcination. A kiln basically has three operating sections: the preheating, the burning and the cooling zone. The preheating zone is that part of the kiln where the limestone is heated to its dissociation temperature. The burning zone is that part of the kiln in which reaction of the burden takes place. The cooling zone is that part of the kiln in which the lime emerging from the burning zone is cooled before discharge.

The most common fuels used in shaft kilns are coke, natural gas, weak gas and pulverized lignite [1]. The majority of shaft furnaces for limestone calcination operate with counter-current flows of burden materials and gases (Boynton [2], Terruzzi [3], Tabunshikov [4] and Monastirev and Aleksandrov [5]). The furnace incorporates three technological zones: preheating, calcination and cooling (from top to bottom).

Gordon et al [6] developed the multi-dimensional mathematical model to optimize the furnace design and the process parameters. The developed mathematical model belongs to the group of essentially non-linear models. According to them, it is not possible to develop an analytical solution of the problem. The finite element method was used to provide a solution. Olayiwola et al [7] developed a mathematical model of calcination process. The developed model took into account the Arrhenius heat generation and chemical reaction. They provided an analytical solution of the model and investigated the effects of activation energy and Frank-Kamenetskii parameters on the gas and material temperatures.

In this paper we extend the model in [7] to account for a situation where the reaction is not well-stirred. We examine the properties of solution under certain conditions. Using high activation energy asymptotics, we provide an analytical solution and investigate parameters involved.

2. Mathematical Formulation

Here, we assume that the reaction is not well-stirred, that is, change depends on both time and space variable. We make the additional assumption that the reaction is in a steady-state so that time derivatives are zero, $\frac{\partial}{\partial t} = 0$. Under these assumptions, we arrived at the following steady energy equation

For material:

Corresponding author: E-mail: olayiwolarasaq@yahoo.co.uk, *Tel*.: +234 805 254 8167; +234 806 774 3443

$$\frac{\lambda_m}{\rho_m(1-\varepsilon)C_m}\frac{d^2T_m}{dx^2}-\frac{\alpha_v}{\rho_m(1-\varepsilon)C_m}\left(T_g-T_m\right)+\frac{q_m}{\rho_m(1-\varepsilon)C_m}+\frac{QA}{\rho_m(1-\varepsilon)C_m}e^{-\frac{E}{RT_g}}=0 \quad (2.1)$$

For gas:

$$\frac{\lambda_g}{\rho_g\varepsilon C_g}\frac{d^2T_g}{dx^2}+\frac{\alpha_v}{\rho_g\varepsilon C_g}\left(T_g-T_m\right)+\frac{q_g}{\rho_g\varepsilon C_g}+\frac{QA}{\rho_g\varepsilon C_g}e^{-\frac{E}{RT_g}}=0 \quad (2.2)$$

The boundary conditions are

$$T_m(0)=T_0,\ T_m(L)=T_0,\ 0\le x\le L \quad (2.3)$$

$$T_g(0)=T_0,\ T_g(L)=T_0,\ 0\le x\le L \quad (2.4)$$

Here, as in [7], ρ is density, E is activation energy, R is gas constant, q is heat source, α is heat transfer coefficient, λ is thermal conductivity, C is heat capacity, ε is porosity, T is temperature, x is position, Q is heat of reaction, A is pre-exponential factor, g is gas, m is material.

3. Method of Solution

Let

$$\theta=\frac{E}{RT_0^2}\left(T_g-T_0\right),\ \phi=\frac{E}{RT_0^2}\left(T_m-T_0\right),\quad x'=\frac{x}{L},\quad \in=\frac{RT_0}{E}$$

Then (2.1) and (2.2) become

$$\lambda_1\frac{d^2\phi}{dx^2}-\alpha_1(\theta-\phi)+\beta_1+\delta_1\exp\left(\frac{\theta}{1+\in\theta}\right)=0 \quad (3.1)$$

$$\lambda_2\frac{d^2\theta}{dx^2}+\alpha_2(\theta-\phi)+\beta_2+\delta_2\exp\left(\frac{\theta}{1+\in\theta}\right)=0 \quad (3.2)$$

together with the boundary conditions

$$\phi(0)=\phi(1)=0 \quad (3.3)$$

$$\theta(0)=\theta(1)=0, \quad (3.4)$$

where

$$\alpha_1=\frac{\alpha_v}{\rho_m(1-\varepsilon)C_m},\ \beta_1=\frac{q_m}{\in T_0\rho_m(1-\varepsilon)C_m},\ \alpha_2=\frac{\alpha_v}{\rho_g\varepsilon C_g},\ \beta_2=\frac{q_g}{\in T_0\rho_g\varepsilon C_g},$$

$\delta_1=\dfrac{QA\exp\left(-\frac{E}{RT_0}\right)}{\in T_0\rho_m(1-\varepsilon)C_m}$ is the Frank-Kamenetskii parameter for material

$\delta_2=\dfrac{QA\exp\left(-\frac{E}{RT_0}\right)}{\in T_0\rho_g\varepsilon C_g}$ is the Frank-Kamenetskii parameter for gas

$\lambda_1=\dfrac{\lambda_m}{\rho_m(1-\varepsilon)C_mL^2}$ is the scaled material thermal conductivity for material

$\lambda_2=\dfrac{\lambda_g}{\rho_g\varepsilon C_gL^2}$ is the scaled gas thermal conductivity for gas

3.1 Properties of Solution

Theorem 3.1

Let $\alpha_1=\alpha_2=0$ and $\lambda_1=\lambda_2=1$ in (3.1) and (3.2). Then $\theta(x)$ and $\phi(x)$ are symmetric about $x=\frac{1}{2}$.

Proof: Let $\alpha_1=\alpha_2=0$ and $\lambda_1=\lambda_2=1$ in (3.1) and (3.2). We obtain

$$\frac{d^2\phi(x)}{dx^2} + \delta_1 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right) + \beta_1 = 0, \quad \phi(0) = 0, \quad \phi(1) = 0$$

$$\frac{d^2\theta(x)}{dx^2} + \delta_2 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right) + \beta_2 = 0, \quad \theta(0) = 0, \quad \theta(1) = 0$$

Let $z = 2x - 1$
Then

$$\frac{d^2}{dx^2} = 4\frac{d^2}{dz^2}$$

So the problem becomes

$$\frac{d^2\phi(z)}{dz^2} + \frac{\delta_1}{4}\exp\left(\frac{\theta(z)}{1+\in\theta(z)}\right) + \frac{\beta_1}{4} = 0, \quad \phi(-1) = 0, \quad \phi(1) = 0$$

$$\frac{d^2\theta(z)}{dz^2} + \frac{\delta_2}{4}\exp\left(\frac{\theta(z)}{1+\in\theta(z)}\right) + \frac{\beta_2}{4} = 0, \quad \theta(-1) = 0, \quad \theta(1) = 0$$

It suffices to show that $\theta(-z) = \theta(z)$ and $\phi(-z) = \phi(z)$.
Replace z by $-z$. We obtain

$$\frac{d^2\phi(-z)}{d(-z)^2} + \frac{\delta_1}{4}\exp\left(\frac{\theta(-z)}{1+\in\theta(-z)}\right) + \frac{\beta_1}{4} = 0$$

$$\frac{d^2\theta(-z)}{d(-z)^2} + \frac{\delta_2}{4}\exp\left(\frac{\theta(-z)}{1+\in\theta(-z)}\right) + \frac{\beta_2}{4} = 0$$

Hence θ and ϕ are symmetric about $z = 0$ i.e. θ and ϕ are symmetric about $x = \frac{1}{2}$. This completes the proof.

Theorem 3.2

Let $\alpha_1 = \alpha_2 = 0$ and $\lambda_1 = \lambda_2 = 1$ in (3.1) and (3.2). Then $\theta'\left(\frac{1}{2}\right) = 0$ and $\phi'\left(\frac{1}{2}\right) = 0$.

Proof: Let $\alpha_1 = \alpha_2 = 0$ and $\lambda_1 = \lambda_2 = 1$ in (3.1) and (3.2). We obtain

$$\frac{d^2\phi(x)}{dx^2} + \delta_1 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right) + \beta_1 = 0, \quad \phi(0) = 0, \quad \phi(1) = 0$$

$$\frac{d^2\theta(x)}{dx^2} + \delta_2 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right) + \beta_2 = 0, \quad \theta(0) = 0, \quad \theta(1) = 0$$

Since $\theta(x)$ and $\phi(x)$ are symmetric about $x = \frac{1}{2}$. Then $\theta'\left(\frac{1}{2}\right) = 0$ and $\phi'\left(\frac{1}{2}\right) = 0$. This completes the proof.

Theorem 3.3

Let $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$ and $\lambda_1 = \lambda_2 = 1$ in (3.1) and (3.2). Then $\theta'(x) > 0$ and $\phi'(x) > 0$ for $x \in \left(0, \frac{1}{2}\right)$.

Proof: Let $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$ and $\lambda_1 = \lambda_2 = 1$ in (3.1) and (3.2). We obtain

$$\frac{d^2\phi(x)}{dx^2} = -\delta_1 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right), \quad \phi(0) = 0, \quad \phi(1) = 0$$

$$\frac{d^2\theta(x)}{dx^2} = -\delta_2 \exp\left(\frac{\theta(x)}{1+\in\theta(x)}\right), \quad \theta(0) = 0, \quad \theta(1) = 0$$

Using Ayeni [8], we obtain

$$\theta(x)=\delta_2\int_0^{\frac{1}{2}}k(x,t)e^{\frac{\theta(t)}{1+\in\theta(t)}}dt$$

$$\phi(x)=\delta_1\int_0^{\frac{1}{2}}k(x,t)e^{\frac{\theta(t)}{1+\in\theta(t)}}dt,$$

where

$$k(x,t)=\begin{cases}x, & 0\le x\le t\\ t, & t\le x\le \frac{1}{2}\end{cases}$$

So

$$\theta'(x)=\delta_2\left[x\,e^{\frac{\theta(x)}{1+\in\theta(x)}}+\int_x^{\frac{1}{2}}e^{\frac{\theta(t)}{1+\in\theta(t)}}dt-x\,e^{\frac{\theta(x)}{1+\in\theta(x)}}\right]$$

$$=\delta_2\int_x^{\frac{1}{2}}e^{\frac{\theta(t)}{1+\in\theta(t)}}dt$$

and

$$\phi'(x)=\delta_1\left[x\,e^{\frac{\theta(x)}{1+\in\theta(x)}}+\int_x^{\frac{1}{2}}e^{\frac{\theta(t)}{1+\in\theta(t)}}dt-x\,e^{\frac{\theta(x)}{1+\in\theta(x)}}\right]$$

$$=\delta_1\int_x^{\frac{1}{2}}e^{\frac{\theta(t)}{1+\in\theta(t)}}dt$$

Hence, $\theta(x)$ and $\phi(x)$ are strictly monotonically increasing for $x\in\left(0,\frac{1}{2}\right)$. This completes the proof.

3.2 Analytical Solution

Here, we consider (3.1) and (3.2) when $\in\to 0$ and assume $\lambda_1=\lambda_2=1$, $\delta_1=\delta_2=\delta$. Subtracting (3.1) from (3.2), we obtain

$$\frac{d^2u}{dx^2}+\alpha u+\beta=0 \qquad (3.5)$$

$$u(0)=u(1)=0,$$

where

$$u=\theta-\phi,\quad \alpha=\alpha_1+\alpha_2,\quad \beta=\beta_2-\beta_1$$

The solution of (3.5) is obtained as

$$u(x)=-\frac{\beta\left(\cos\sqrt{\alpha}-1\right)\sin\sqrt{\alpha}x}{\alpha\sin\sqrt{\alpha}}+\frac{\beta\cos\sqrt{\alpha}x}{\alpha}-\frac{\beta}{\alpha} \qquad (3.6)$$

Ayeni [9] has shown that $\exp(\theta)$ can be approximated as $1+(e-2)\theta+\theta^2$. In this paper we are going to take an approximation of the form

$$\exp(\theta)\approx 1+(e-2)\theta \qquad (3.7)$$

Then (3.1) and (3.2) can be written as

$$\frac{d^2\theta}{dx^2}+\alpha_2u+\beta_2+\delta\left(1+(e-2)\theta\right)=0 \qquad (3.8)$$

$$\frac{d^2\phi}{dx^2}-\alpha_1u+\beta_1+\delta\left(1+(e-2)\theta\right)=0 \qquad (3.9)$$

We obtain the solution of (3.8) and (3.9) as

$$\theta(x)=\frac{\left(\frac{\beta\alpha_2\delta(e-2)}{\alpha}+(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)\right)}{\delta(e-2)(\delta(e-2)-\alpha)}\cos\left(\sqrt{\delta(e-2)}\,x\right)-$$

$$\frac{\beta\alpha_2\cos\left(\sqrt{\alpha}\,x\right)}{\alpha(\delta(e-2)-\alpha)}-\frac{\beta\alpha_2\left(1-\cos\left(\sqrt{\alpha}\right)\right)\sin\left(\sqrt{\alpha}\,x\right)}{\alpha\sin\left(\sqrt{\alpha}\right)(\delta(e-2)-\alpha)}-\frac{\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}}{\delta(e-2)}+$$

$$\frac{\left(\begin{array}{l}\left(\begin{array}{l}(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)+\frac{\beta\alpha_2\delta(e-2)\left(1-\cos\left(\sqrt{\alpha}\right)\right)}{\alpha}+\\ \frac{\beta\alpha_2\delta(e-2)\cos\left(\sqrt{\alpha}\right)}{\alpha}-\frac{\beta\alpha_2\delta(e-2)}{\alpha}-(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)\\ \cos\left(\sqrt{\delta(e-2)}\right)\end{array}\right)\\ \sin\left(\sqrt{\delta(e-2)}\,x\right)\end{array}\right)}{\delta(e-2)(\delta(e-2)-\alpha)\sin\left(\sqrt{\delta(e-2)}\right)}\qquad(3.10)$$

$$\phi(x)=\alpha_1\left(-\frac{\beta\left(1-\cos\left(\sqrt{\alpha}\right)\right)\sin\left(\sqrt{\alpha}\,x\right)}{\alpha^2\sin\left(\sqrt{\alpha}\right)}-\frac{\beta\cos\left(\sqrt{\alpha}\,x\right)}{\alpha^2}-\frac{1}{2}\frac{\beta x^2}{\alpha}\right)-\frac{1}{2}(\delta+\beta_1)x^2-$$

$$\delta(e-2)\left(-\frac{\left(\begin{array}{l}\left(\begin{array}{l}(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)+\frac{\beta\alpha_2\delta(e-2)\left(1-\cos\left(\sqrt{\alpha}\right)\right)}{\alpha}+\\ \frac{\beta\alpha_2\delta(e-2)\cos\left(\sqrt{\alpha}\right)}{\alpha}-\frac{\beta\alpha_2\delta(e-2)}{\alpha}-(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)\\ \cos\left(\sqrt{\delta(e-2)}\right)\end{array}\right)\\ \sin\left(\sqrt{\delta(e-2)}\,x\right)\end{array}\right)}{\delta^2(e-2)^2(\delta(e-2)-\alpha)\sin\left(\sqrt{\delta(e-2)}\right)}\right)$$

$$-\delta(e-2)\left(\begin{array}{l}-\frac{\left(\frac{\beta\alpha_2\delta(e-2)}{\alpha}+(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)\right)}{\delta^2(e-2)^2(\delta(e-2)-\alpha)}\cos\left(\sqrt{\delta(e-2)}\,x\right)+\\ \frac{\beta\alpha_2\cos\left(\sqrt{\alpha}\,x\right)}{\alpha^2(\delta(e-2)-\alpha)}+\frac{\beta\alpha_2\left(1-\cos\left(\sqrt{\alpha}\right)\right)\sin\left(\sqrt{\alpha}\,x\right)}{\alpha^2\sin\left(\sqrt{\alpha}\right)(\delta(e-2)-\alpha)}-\frac{1}{2}\frac{\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)x^2}{\delta(e-2)}\end{array}\right)$$

$$
+\frac{\beta\alpha_1}{\alpha^2}+\frac{\beta\alpha_2\delta(e-2)}{\alpha^2(\delta(e-2)-\alpha)}-\frac{\frac{\beta\alpha_2\delta(e-2)}{\alpha}+(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)}{\delta(e-2)(\delta(e-2)-\alpha)}+
$$

$$
\left(\delta(e-2)\left(\begin{array}{l}
-\frac{\left(\frac{\beta\alpha_2\delta(e-2)}{\alpha}+(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)\right)}{\delta^2(e-2)^2(\delta(e-2)-\alpha)}\cos\left(\sqrt{\delta(e-2)}\right)- \\
\frac{\left(\left(\begin{array}{l}(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)+\frac{\beta\alpha_2\delta(e-2)\left(1-\cos\left(\sqrt{\alpha}\right)\right)}{\alpha}+ \\ \frac{\beta\alpha_2\delta(e-2)\cos\left(\sqrt{\alpha}\right)}{\alpha}-\frac{\beta\alpha_2\delta(e-2)}{\alpha}-(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right) \\ \cos\left(\sqrt{\delta(e-2)}\right)\end{array}\right)\right)}{\delta^2(e-2)^2(\delta(e-2)-\alpha)}+ \\
\frac{\beta\alpha_2\cos\left(\sqrt{\alpha}\right)}{\alpha^2(\delta(e-2)-\alpha)}+\frac{\beta\alpha_2\left(1-\cos\left(\sqrt{\alpha}\right)\right)}{\alpha^2(\delta(e-2)-\alpha)}-\frac{1}{2}\frac{\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)}{\delta(e-2)}
\end{array}\right)\right)x+
$$

$$
\left(\begin{array}{l}
\frac{1}{2}\delta+\frac{1}{2}\beta_1-\alpha_1\left(-\frac{\beta\cos\left(\sqrt{\alpha}\right)}{\alpha^2}-\frac{\beta\left(1-\cos\left(\sqrt{\alpha}\right)\right)}{\alpha^2}-\frac{1}{2}\frac{\beta}{\alpha}\right)-\frac{\beta\alpha_1}{\alpha^2}-\frac{\beta\alpha_2\delta(e-2)}{\alpha^2(\delta(e-2)-\alpha)} \\
+\frac{\frac{\beta\alpha_2\delta(e-2)}{\alpha}+(\delta(e-2)-\alpha)\left(\delta+\beta_2+\frac{\beta\alpha_2}{\alpha}\right)}{\delta(e-2)(\delta(e-2)-\alpha)}
\end{array}\right)x \quad (3.11)
$$

4. Results and Discussion

We have shown, under certain conditions, that (i) $\theta(x)$ and $\phi(x)$ are symmetric about $x=\frac{1}{2}$, (ii) $\theta'\left(\frac{1}{2}\right)=0$ and $\phi'\left(\frac{1}{2}\right)=0$ and (iii) $\theta(x)$ and $\phi(x)$ are strictly monotonically increasing for $x\in\left(0,\frac{1}{2}\right)$.

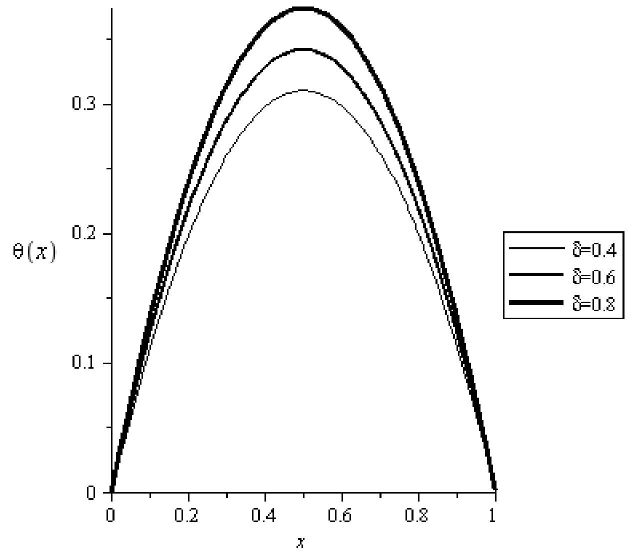

Figure 1: Plots of $\theta(x)$ against x for equation (3.8) at various values of δ when α=25, β=1, $\alpha_1 = 24$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 2$, $e = 2.718$

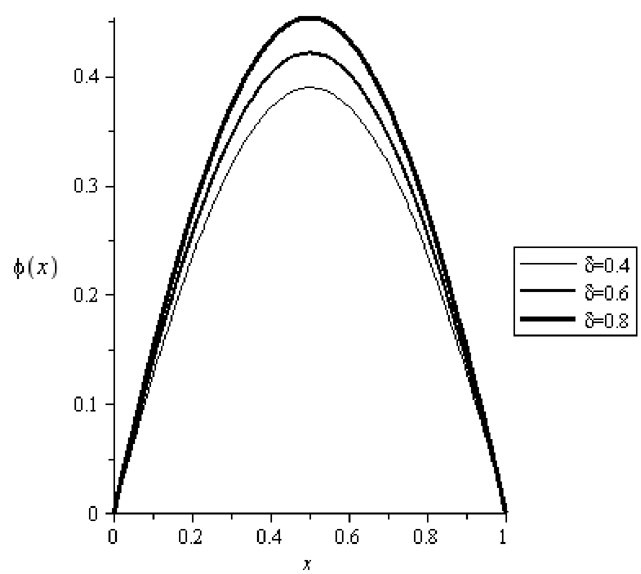

Figure 2: Plots of $\phi(x)$ against x for equation (3.9) at various values of δ when α=25, β=1, $\alpha_1 = 24$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 2$, $e = 2.718$

Figures 1 and 2 display the graphs of $\theta(x)$ and $\phi(x)$ versus x for various values of δ. It is easy to see that $\theta(x)$ and $\phi(x)$ increase as δ increases. Here, $\theta(x)$ and $\phi(x)$ are the gas temperature and material temperature respectively.

From the practical point of view, all these temperatures are favorable for formation of high quality quick lime since maintaining a high temperature of calcinations increases the furnace productivity.

5. Conclusion

In this paper a mathematical model to predict the heat transfer in a lime kiln has been presented. The model accounted for a situation where the reaction is not well-stirred. The governing equations have been solved analytically using high activation energy asymptotics. The presented analysis has shown that the Frank-Kamenetskii parameter has significant effects on the temperature field of the system.

In [7], the reaction was considered to be well-stirred, so the temperatures are function of time. The results of the simulation showed that these temperatures are proportional to time. But, in this study, we considered the two point boundary values and the results are in parabolic form.

References

[1] Höltje, W. (1989) " Alternatives to coke as fuel for conventional shaft kilns," Zement- Kalk-Gips 1 (1989), pp. 21-26

[2] Boynton R.S. (1980) "Chemistry and Technology of Lime and Limestone,'' John Wiley and Sons, Inc, New York, NY, USA.

[3] Terruzzi D. (1994) "Lime Shaft Kilns Using Two Way Pressure System,'' ZKG International, No. 6, p. 322-326.

[4] Tabunshikov N.P. (1974) "Lime Production,'' Chemistry Publishing House, Moscow.

[5] Monastirev A.V. and Aleksandrov A.V. (1979) "Furnace for Lime Production,'' Metallurgy, Moscow.

[6] Gordon Y.M., Blank M.E., Madison V.V. and Abovian P.R. (2003) "New Technology and Shaft Furnace for High Quality Metallurgical Lime Production," Proc. of Asia Steel International Conference, Jamshedpur, India, 9 -12 April 2003.

[7] Olayiwola R.O., Cole A.T., Hakimi D. and Ayeni R.O. (2009) "Mathematical Modeling of the Calcination Process," J. Nig. Assoc. of Mathl. Phys. 14 (1): 381 -388.

[8] Ayeni R.O. (1978) "Thermal Runaway," Ph.D Thesis Cornell University USA.

[9] Ayeni R.O. (1982) "On the Explosion of Chain-thermal Reactions," J. Austral. Math. Soc. (Series B), 24, 194-202.