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The Asymptotic Solution for the Steady Variable-Viscosity Free Convection Flow on a Porous Plate

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Abstract

Under an arbitrary time-dependent heating of an infinite vertical plate (or wall), the steady viscosity-dependent free convection flow of a viscous incompressible fluid is investigated. Using the asymptotic method of solution on the governing equations of motion and energy, the resulting Ordinary differential equations were solved numerically. And the results show that the fluid energy decreases as the distance moved by the plate increases.

1.0 Introduction

Transient convection is of fundamental interest in many industrial and environmental such as air conditioning systems, human comfort in buildings, atmospheric flows, motors, thermal regulation processes, and cooling of electronic devices. In view of these applications, [1] investigated transient free convection flow between two horizontal parallel plates. The results of a numerical study of the transient natural convection flow between two vertical parallel plates were presented by [2].

In this study, Joshi applied uniform heat flux on the walls. Singh [3] and Singh et al [4] studied the flow of behavior of a transient free convective flow of a viscous incompressible fluid between two vertical parallel plates on relative motion using Laplace transform technique.

The first numerical solution for developing natural convection flow in an isothermal channel was carried out by [5] using boundary-layer approximation. Aung [6], Aung et al [7], Miyatake and Fuzii [8], andMiyatake et al [9] presented their results for a steady free convective flow between vertical walls by applying different physical treatments for transport process.

Our interest in this paper is to show how the distance moved by the plate affects the energy distribution of the fluid, and our model is taken from the work of [10] and [11].

1. Mathematical formulation

The free-convection flow is two-dimensional and it is considered with the coordinate origin at an arbitrary point on an infinite, porous limiting vertical plate or wall. The x'-axis is along the plate and in the upward direction and the y'-axis normal towards it. The fluid is viscous and incompressible. The flow is induced either by the motion of the plate or by heating it or both.

The plate is at rest with a constant temperature T_∞, and it is suddenly moved with a constant velocity $u_\circ$. Its temperature is instantaneously increased (or decreased) by the quantity $\alpha(T'_w - T'_\infty)$ for $t \succ 0$, $T'_w \ (\neq T_\infty)$ a constant temperature of the plate.

On the physical grounds of the present problem, all the quantities are assumed to be functions of the space coordinate y' and t', so that the vector of the velocity is given by ($u', v', 0$).

Then the equation of continuity, on integration, gives $v' = cons\tan t = v'_\circ$ (say), where $v'_\circ$ is the normal velocity of suction or injection at the wall according as $v'_\circ \prec 0$ or $v'_\circ \succ 0$ respectively. $v'_\circ = 0$ represents the case of a non-permeable wall.

The corresponding equation of motion and energy for this case are respectively;

$$\rho\left[\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right] = \frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) + \rho g\beta'(T' - T'_\infty) \quad (1)$$

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$$\frac{\partial T'}{\partial t'}+v'_{\circ}\frac{\partial T'}{\partial y'}=\frac{k}{\ell Cp}\cdot\frac{\partial^2 T'}{\partial y'^2}+\frac{\mu}{\rho Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

where ρ denotes the fluid density, T' the temperature, g the acceleration due to gravity, β' the coefficient of volume expansion, k the thermal conductivity; and Cp the specific heat at constant pressure.

We assume that the viscosity $\mu=\mu_{\circ}\left(\frac{T'-T'_{\infty}}{T'_w-T'_{\infty}}\right)^n$, where n is a positive number.

We introduce the following non-dimensional variables and parameters;

$$t=\frac{t'u^2{}_{\circ}}{\upsilon},\qquad y=\frac{y'u_{\circ}}{\upsilon},\qquad v_{\circ}=\frac{v'_{\circ}}{u_{\circ}}, \tag{3}$$

Non-dimensional temperature, $\theta=\frac{T'-T'_{\infty}}{T'_w-T'_{\infty}}$ (4)

Prandtl number, $P=\frac{\mu Cp}{k}$ (5)

Grashof number, $G=\frac{\upsilon g\beta'(T'_w-T''_{\infty})}{u^3{}_{\circ}}$ (6)

The corresponding initial and boundary conditions of the system (1) and (2) after being non-dimensionalized are:

$$\theta(y,0)=0,\qquad \theta(0,t)=\alpha,\qquad \theta(\infty,0)=0 \tag{7}$$

$$u(y,0)=0,\qquad u(0,t)=0,\qquad u(\infty,0)=0 \tag{8}$$

The boundary conditions (7) and (8) are solved asymptotically as

$$\theta=\theta_{\circ}+a\theta_1+a^2\theta_2+a^3\theta_3+\ldots \quad\text{and}\quad u=u_{\circ}+au_1+a^2u_2+a^3u_3+\ldots \tag{9}$$

Non – dimensionalization of (1) and (2) according to (3), (4), (5) and (6) gives;

$$\frac{\partial u}{\partial t}+v_{\circ}\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\theta^n\frac{\partial u}{\partial y}\right)+G\theta \tag{10}$$

$$\frac{\partial\theta}{\partial t}+v_{\circ}\frac{\partial\theta}{\partial y}=\frac{1}{P}\cdot\frac{\partial^2\theta}{\partial y^2}+a\theta^n\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

where $a=\frac{u^2}{Cp\left(T'_w-T'_{\infty}\right)}$

And for the steady case, $\frac{\partial u}{\partial t}=0$ and $\frac{\partial\theta}{\partial t}=0$, equations (10) and (11) become;

$$v_{\circ}\frac{du}{dy}=\frac{d}{dy}\left(\theta^n\frac{du}{dy}\right)+G\theta \tag{12}$$

$$v_{\circ}\frac{d\theta}{dy}=\frac{1}{P}\frac{d^2\theta}{dy^2}+a\theta^n\left(\frac{du}{dy}\right)^2 \tag{13}$$

We assume $0<a<<1$, and if we treat the coefficients of a° after expanding (12) and (13) asymptotically with $(\theta)^n=(\theta_{\circ}+a\theta_1+a^2\theta_2+a^3\theta_3+\ldots)^n$, we have;

$$v_{\circ}\frac{du_{\circ}}{dy}=\frac{d}{dy}\left(\theta_{\circ}{}^n\frac{du_{\circ}}{dy}\right)+G\theta_{\circ} \tag{14}$$

$$v_{\circ}\frac{d\theta_{\circ}}{dy}=\frac{1}{P_r}\frac{d^2\theta_{\circ}}{dy^2} \tag{15}$$

The corresponding asymptotic initial and boundary conditions for the energy $\theta_\circ$, and for the velocity $U_\circ$ are:

$$\theta_o(y, o), = 0,\ \theta_o(0, t) = a,\ \theta_o(\infty, t) = 0 \text{ and } U_\circ(y,0) = 0,\ U_\circ(\infty, t) = 0$$

Equation (15) has the auxiliary equation $m^2 - v_\circ pm = 0$, where $m = 0$ and $m = v_\circ p$

The solution for (15) is of the form $\theta_o(y) = A + B\,\ell^{v_o py}$ (16)

Using the boundary conditions for θ on (16) we have that:

$$\theta_o(y) = a\ell^{-\beta py} \tag{17}$$

where $v_\circ = -\beta$, $\beta \succ 0$ for finiteness

Putting equation (17) into equation (14), we have that:

$$-\beta\frac{d}{dy}u_o = \frac{d}{dy}\left(\alpha^n \ell^{-\beta npy}\frac{d}{dy}u_o\right) + \alpha\ell^{-\beta py}.G \tag{18}$$

And on integrating equation (18), we have

$$\frac{d}{dy}u_o + \frac{\beta}{\alpha^n}\ell^{\beta pny}.u_o = \frac{\alpha G}{\beta p\alpha^n}\ell^{(\beta np - \beta p)y} - \frac{k_1}{\alpha_n}\ell^{\beta pny} \tag{19}$$

Where k_1 is the constant of integration

Equation (19) can simply be written as

$$\frac{d}{dy}u_o + a\,.\,u_0\ell^{\gamma y} = b\ell^{dy} - c\ell^{fy} \tag{20}$$

where $a = \dfrac{\beta}{\alpha^n},\ b = \dfrac{\alpha G}{\beta p\alpha^n},\ c = \dfrac{k_1}{\alpha^n},\ d = \beta pn - \beta p \text{ and } f = \gamma = \beta pn$

If we obtain the coefficients of a, we will have;

$$v_o\frac{du_1}{dy} = \frac{d}{dy}\left(\theta_o^n\frac{du_1}{dy}\right) + G\theta_1 \tag{21}$$

$$v_o\frac{d\theta_1}{dy} = \frac{1}{P_r}\frac{d^2\theta_1}{dy^2} + \theta_o^n\left(\frac{du_o}{dy}\right)^2 \tag{22}$$

Satisfying

$$u_1(0) = 0,\ u_1(\infty) = 0,\quad \theta_1(0) = 0,\ \theta_1(\infty) = 0 \text{, where } \theta_0(y) = \alpha\,\ell^{-\beta py}$$

Unique solutions exist for (21) and (22) by the theorem below:

Theorem [1]:

Consider the initial value system;

$$\left.\begin{aligned} X_1^1 &= f_1(x_1, x_2, \ldots\ldots x_n, t),\ x_1(t_o) = x_{10} \\ X_2^1 &= f_2(x_1, x_2, \ldots\ldots x_n, t),\ x_2(t_o) = x_{20} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_n^1 &= f_n(x_1, x_2, \ldots\ldots x_n, t),\ x_n(t_o) = x_{n0} \end{aligned}\right\} \tag{23}$$

Let D denote the region in (n+1) – dimensional space, one dimension for t and one dimension for vector X.

If the partial derivatives $\dfrac{\partial f_i}{\partial x_j}$, $i,\ j = 1, 2, \ldots, n$ are continuous and bounded on D. Then there is a constant δ > 0 such that there exist a unique continuous vector solution $X(t)$ of the system of equations (23).

Equation (17) is a simple function whose points are plotted in Figure1 for $\alpha = 1$ and $\beta\rho = \frac{1}{2}$.

Equation (19) which is a linear first order ordinary differential equation is numerically solved for $a = 1, b = 1, c = \frac{1}{2}, G = 1$ and $\beta\rho = \frac{1}{2}$ for $n = 2, n = 3$ and $n = 4$ using the forward different method with $u_\circ = 1$, and the result is shown in Figure 2.

2. Conclusion

The energy, $\theta_\circ$ decreases as the distance moved by the plates horizontally increases. The energy decreases creating a kink before attaining a constant value. The velocity of the fluid decreases as n decreases.

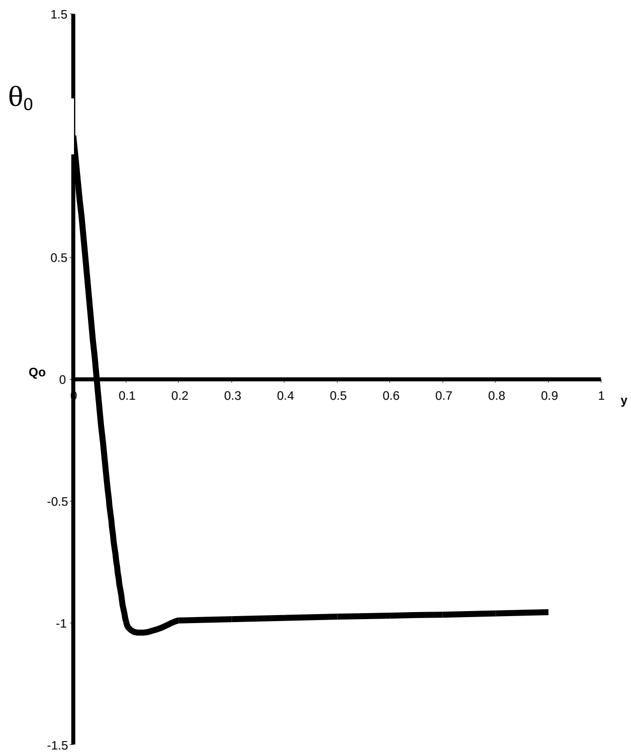

Figure 1: The graph of the energy (θ_0) against the distance (y) moved by the fluid

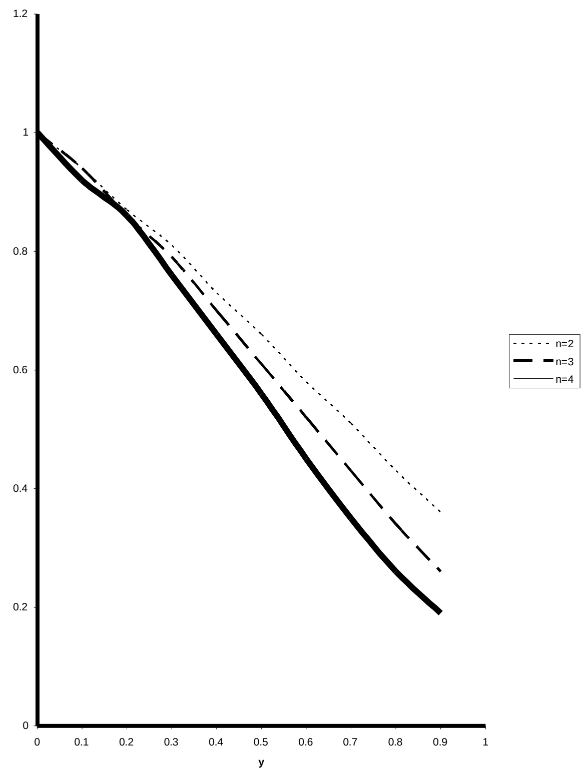

Figure 1: The graph of the energy (θ_0) against the distance (y) moved by the fluid

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