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Free Convective Flow between Vertical Porous Plates with Variable Suction and Periodic Heating Input

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Abstract

Analytical solution for the flow of a viscous incompressible heat generation fluid between two periodically heated vertical porous plates with variable suction in free convective flow. The dimensionless governing equations are solved analytically using the two-term steady and non steady functions (temperature and velocity fields respectively) and the resulting second order ordinary differential equations solved. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The fluid is subjected to a constant pressure gradient and a uniform suction and injection through the plates which are kept at different but constant temperatures. The ordinary differential equations are then solved numerically using asymptotic expansion technique. The exact solutions are solved using undetermined coefficients. Graphical results for velocity and temperature profile of both phases based on the analytical solution are presented and discussed. The significant result from this study is that temperature is higher near the plate with injection while velocity is more enhanced near the plate with suction. Similarly, for large values of Strouhal number the flow behaves as if there is no suction/injection at the plates Wang(1988)

Keywords: free convection, periodic heating, variable suction, suction/injection, porous medium

1.0 Introduction

Heat transfer is the area that deals with the mechanisms responsible for transferring energy from one place to another when a temperature difference exists. In thermodynamics, heat is defined as energy in transient; however most of the thermodynamic processes are concerned with equilibrium or quasi equilibrium situations in [2]. [3] conducted the first comprehensive experimental work on free convection heat transfer. [4-5] presented laminar free-convective flow of a viscous incompressible fluid between two vertical walls. [6-8] investigated the combined effects of a steady free convective laminar flow and [9] investigated the boundary-layer approximation while [10-16] obtained analytical and numerical solutions for free convection flow along a porous plate with variable suction in porous medium. [17-18] studied Dufour and Soret effect with variable suction on unsteady MHD free convection flow along a porous plate while [19] investigated the periodic heat input of free convective fluid. Also [20-21] investigated flow in porous media and [22] presented a new numerical method for the solution of porous media equation and [23] investigated free convective flow between vertical porous plates.

2.0 Mathematical Formulation

The flow considered is a free convective flow of a viscous incompressible heat generating/absorbing fluid in a vertical channel with variable suction and periodic heating of the porous channel plates.

Periodic heating is introduced on both walls and due to temperature gradient between the walls and the fluid which resulted in buoyancy effect, natural convection automatically sets in. Due to the viscosity of the working fluid, velocity remains zero on both plates of the channel. Under these assumptions, incorporating the Boussinesq approximation within the boundary layer, the governing equations of continuity, momentum and energy [24, 25, 26, 27 and 28] respectively are given

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byEquation of continuity

$$\frac{\partial u}{\partial y} = 0 \qquad (1)$$

Momentum transport equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) \qquad (2)$$

Energy transport equation

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho Cp} \qquad (3)$$

where$u$ and v are velocity components along x- and y-directions, t is time variable and throughout we shall rely on the nomenclature as defined in appendix A and appendix B.

The channel walls are taken vertically, parallel to the x-axis at y = ± 1.It is assumed that the suction velocity at the plate is not a constant and takes the following exponential form

$$v = -v_0(1 + \varepsilon A^* e^{i\omega t}) \qquad (4)$$

and that it is socked off from the other plate y=-h at the same rate (see figure 1). The heat generation term in this problem is Q.

where $$Q = Q_0(T_0 - T) \qquad (5)$$

Substituting equation (4) into equation (2) and equation (3) yields

$$\frac{\partial u}{\partial t} - v_0(1 + \varepsilon A^* e^{i\omega t})\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_0) \qquad (6)$$

$$\frac{\partial T}{\partial t} - v_0(1 + \varepsilon A^* e^{i\omega t})\frac{\partial T}{\partial y} = \frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho Cp} \qquad (7)$$

The following expressions are used to split the velocity and temperature into steady and unsteady parts respectively.

$$u = \frac{g\beta h^2}{\nu}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}] + O(e^{i\omega t})^2 \qquad (8)$$

$$T = T_0 + (T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t} + O(e^{i\omega t})^2 \qquad (9)$$

substituting equation (5) and equation (9) into equation (7), the momentum equation above, yields

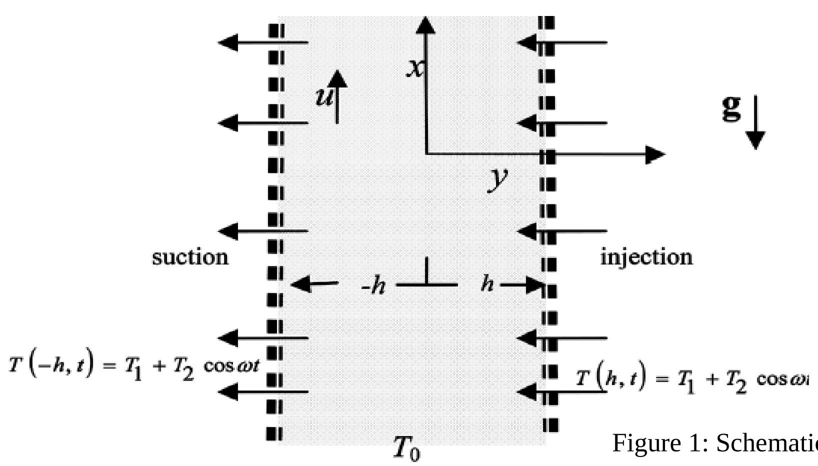

Figure 1: Schematic diagram of the problem

3.0 Method of Solution

Following [19], we use asymptotic expansion to obtain the following,

$$\frac{\partial}{\partial t}[T_0 + (T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t}] - v_0(1 + \varepsilon A^* e^{i\omega t})\frac{\partial}{\partial y}[T_0 + (T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t}] =$$

$$\frac{k}{\rho Cp}\frac{\partial^2}{\partial y^2}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]+\frac{Q_0(T_0-T)}{\rho Cp} \quad (10)$$

further substitution and collecting like terms, equation (10) reduces to

$$\frac{\partial}{\partial t}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]-v_0(1+\varepsilon A^* e^{i\omega t})\frac{\partial}{\partial y}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]=$$
$$\frac{k}{\rho Cp}\frac{\partial^2}{\partial y^2}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]+\frac{Q_0[T_0-\{T_0+[T_1-T_0]F(Y)+T_2G(Y)e^{i\omega t}\}]}{\rho Cp} \quad (11)$$

on opening the bracket in equation (11) and further simplification, equation (11) becomes

$$\frac{\partial}{\partial t}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]-v_0(1+\varepsilon A^* e^{i\omega t})\frac{\partial}{\partial y}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]=$$
$$\frac{k}{\rho Cp}\frac{\partial^2}{\partial y^2}[T_0+(T_1-T_0)F(Y)+T_2G(Y)e^{i\omega t}]+\frac{Q_0[-\{T_1-T_0\}F(Y)+T_2G(Y)e^{i\omega t}]}{\rho Cp} \quad (12)$$

term by term differentiation of equation (12) results in

$$i\omega T_2G(Y)e^{i\omega t}-v_0(1+\varepsilon A^* e^{i\omega t})[(T_1-T_0)F^{'}(Y)+T_2G^{'}(Y)e^{i\omega t}]=$$
$$\frac{k}{\rho Cp}[(T_1-T_0)F^{''}(Y)+T_2G^{''}(Y)e^{i\omega t}]+\frac{Q_0[-\{T_1-T_0]F(Y)-T_2G(Y)e^{i\omega t}\}]}{\rho Cp} \quad (13)$$

on expanding all the products of equation (13),

$$i\omega T_2G(Y)e^{i\omega t}-(v_0+v_0\varepsilon A^* e^{i\omega t})[(T_1-T_0)F^{'}(Y)+T_2G^{'}(Y)e^{i\omega t}]= \quad (14)$$
$$\frac{k}{\rho Cp}[(T_1-T_0)F^{''}(Y)+T_2G^{''}(Y)e^{i\omega t}]+\frac{Q_0[-\{T_1-T_0]F(Y)-T_2G(Y)e^{i\omega t}\}]}{\rho Cp}$$

on opening the bracket and expanding all the terms in equation (14),

$$i\omega T_2G(Y)e^{i\omega t}-v_0(T_1-T_0)F^{'}(Y)-v_0\varepsilon A^* e^{i\omega t}(T_1-T_0)F^{'}(Y)-v_0T_2G^{'}(Y)e^{i\omega t}-$$
$$v_0\varepsilon Ae^{i\omega t}T_2G^{'}(Y)e^{i\omega t}]=\frac{k}{\rho Cp}(T_1-T_0)F^{''}(Y)+\frac{k}{\rho Cp}T_2G^{''}(Y)e^{i\omega t}-\frac{Q_0[T_1-T_0]F(Y)}{\rho Cp} \quad (15)$$

now equating orders of the periodic functions of order zero, order one and neglecting other higher powers, gives (1). for order zero, we have

$$-v_0(T_1-T_0)F^{'}(Y)=\frac{k}{\rho Cp}(T_1-T_0)F^{''}(Y)-\frac{Q_0[T_1-T_0]F(Y)}{\rho Cp} \quad (16)$$

divide through equation(16) by (T_1-T_0) gives

$$-v_0F'(Y)=\frac{k}{\rho Cp}F''(Y)-\frac{Q_0F(Y)}{\rho Cp} \quad (17)$$

and then, multiply through equation (17) by $\frac{\rho Cp}{k}$ gives

$$-v_0F'(Y)\frac{\rho Cp}{k}=\frac{k}{\rho Cp}F''(Y)\frac{\rho Cp}{k}-\frac{Q_0F(Y)}{\rho Cp}\frac{\rho Cp}{k} \quad (18)$$

on simplification, equation (18) becomes

$$-v_0 F^{'}(Y)\frac{\rho Cp}{k} = F''(Y) - \frac{Q_0 F(Y)}{k} \tag{19}$$

recall, by definition, $\alpha = \dfrac{k}{\rho Cp}$, (20a)

therefore, it implies that $\dfrac{1}{\alpha} = \dfrac{\rho Cp}{k}$ (20b)

substituting equation (20a) and equation (20b) into equation (19), we have

$$-\frac{v_0}{\alpha}F'(Y) = F''(Y) - \frac{Q_0}{k}F(Y) \tag{21}$$

simple re-rearrangement of equation (21), gives

$$F''(Y) = -\frac{v_0}{\alpha}F'(Y) + \frac{Q_0}{k}F(Y) \tag{22a}$$

or better still in differential sign notation,

$$\frac{d^2F}{dy^2} = -\frac{v_0}{\alpha}\frac{dF}{dy} + \frac{Q_0}{k}F(Y) \tag{22b}$$

using the following non-dimensional quantities;

$$s = \frac{v_0 h}{v}, St = \frac{h^2\omega}{v}, \Pr = \frac{v\rho Cp}{k} = \frac{v}{\alpha}, \alpha = \frac{k}{\rho Cp} \, and \, \delta = \frac{Q_0}{\alpha\rho Cp} \tag{23}$$

when $Y = \dfrac{y}{h}$, (24)

by definition, $\dfrac{d}{dy} = \dfrac{d}{dY}.\dfrac{dY}{dy}$ (25)

substituting equation(24) into equation (25)

$$\frac{d}{dy} = \frac{d}{dY}.\frac{1}{h} = \frac{1}{h}.\frac{d}{dY}; \tag{26}$$

invoke equation (26) into equation (22b)

therefore, $\dfrac{dF}{dy} = \dfrac{dF}{dY}.\dfrac{1}{h} = \dfrac{1}{h}.\dfrac{dF}{dY}$ and $\dfrac{d^2F}{dy^2} = \dfrac{d^2F}{dY^2}.\dfrac{1}{h^2} = \dfrac{1}{h^2}.\dfrac{d^2F}{dY^2}$ (27a)

$$\frac{d^2F}{dy^2} = \frac{d}{dy}(\frac{dF}{dy}) = \frac{d}{dy}(\frac{dF}{dY}.\frac{dY}{dy}) = \frac{d}{dy}(\frac{1}{h}.\frac{dF}{dY}) = \frac{1}{h}.\frac{dF}{dY}(\frac{1}{h}.\frac{dF}{dY}) = \frac{1}{h^2}.\frac{d^2F}{dY^2} \tag{27b}$$

substituting equation (27a) and equation (27b) into equation (22b) above

$$\frac{1}{h^2}.\frac{d^2F}{dY^2} = -\frac{v_0}{\alpha h}.\frac{dF}{dY} + \frac{Q_0}{k}F(Y) \tag{28}$$

multiply through equation (28) by h^2

$$h^2.\frac{1}{h^2}\frac{d^2F}{dY^2} = h^2\frac{v_0}{\alpha h}\frac{dF}{dY} + \frac{h^2Q_0}{k}F(Y) \tag{29}$$

on further simplification of equation (29), it reduces to

$$\frac{d^2F}{dY^2} = -\frac{v_0 h}{\alpha}\frac{dF}{dY} + \frac{h^2Q_0}{k}F(Y) \tag{30a}$$

$$F''(Y) = -\frac{v_0 h}{\alpha}F'(Y) + \frac{h^2 Q_0}{k}F(Y) \qquad (30b)$$

and applying the non-dimensional quantities in equations (23) into equation (30b), it results into

$$s = \frac{v_0 h}{v}, \Pr = \frac{v\rho Cp}{k}, s\Pr = \frac{v_o h}{v}.\frac{v\rho Cp}{k} = v_o h.\frac{\rho Cp}{k} = \frac{v_o h}{\alpha}, \delta = \frac{Q_o h^2}{\alpha\rho Cp} = \frac{Q_o h^2}{k} \qquad (31)$$

$$F''(Y) = -s\Pr F'(Y) + \delta F(Y) \qquad (32)$$

Simple re-arrangement in equation (31) to the right hand side, gives

$$F''(Y) + s\Pr F'(Y) - \delta F(Y) = 0 \qquad (33)$$

(ii). secondly, for order 1, we have

$$i\omega T_2 G(Y) - v_o \varepsilon A^*(T_1 - T_0)F'(Y) - v_o T_2 G'(Y) = \frac{k}{\rho Cp}T_2 G''(Y) - \frac{Q_0}{\rho Cp}T_2 G(Y) \qquad (34)$$

multiply through equation (34) by $\frac{\rho Cp}{kT_2}$, it becomes

$$\frac{\rho Cp}{kT_2}i\omega T_2 G(Y) - \frac{\rho Cp}{kT_2}v_0\varepsilon A^*(T_1 - T_0)F'(Y) - \frac{\rho Cp}{kT_2}v_o T_2 G'(Y) = \frac{\rho Cp}{kT_2}\frac{k}{\rho Cp}T_2 G''(Y) - \frac{Q_0 T_2}{\rho Cp}G(Y)\frac{\rho Cp}{kT_2} \qquad (35)$$

and upon simplification, equation (35) becomes

$$\frac{\rho Cp}{k}i\omega G(Y) - \frac{\rho Cp}{kT_2}v_0\varepsilon A^*(T_1 - T_0)F'(Y) - \frac{\rho Cp}{k}v_o G'(Y) = G''(Y) - \frac{Q_0}{k}G(Y) \qquad (36)$$

and making $G''(Y)$ the subject of formula in equation (36) above

$$G''(Y) = -\frac{\rho Cp}{k}v_0 G'(Y) + \frac{\rho Cp}{k}i\omega G(Y) + \frac{Q_0}{k}G(Y) - \frac{\rho Cp}{kT_2}v_0\varepsilon A^*(T_1 - T_0)F'(Y) \qquad (37)$$

and applying the non-dimensional quantities in equations (23) into equation (37), and using

$$\frac{dF}{dy} = \frac{dF}{dY}.\frac{1}{h} = \frac{1}{h}.\frac{dF}{dY}; \frac{dG}{dy} = \frac{dG}{dY}.\frac{1}{h} = \frac{1}{h}.\frac{dG}{dY}; \frac{d^2G}{dy^2} = \frac{d^2G}{dY^2}.\frac{1}{h^2} = \frac{1}{h^2}.\frac{d^2G}{dY^2},$$

it reduces to

$$G''(Y) = -\frac{v_0}{\alpha}G'(Y) + \frac{i\omega}{\alpha}G(Y) + \frac{Q_0}{k}G(Y) - \frac{v_0}{\alpha}\varepsilon A^*(\frac{T_1 - T_0}{T_2})F'(Y) \qquad (38)$$

equation (38) further reduces to

$$\frac{1}{h^2}.\frac{d^2G}{dY^2} = -\frac{v_0}{\alpha h}\frac{dG}{dY} + \frac{i\omega}{\alpha}G(Y) + \frac{Q_0}{k}G(Y) - \frac{v_0}{\alpha}\varepsilon A^*(\frac{T_1 - T_0}{T_2})\frac{1}{h}\frac{dF}{dY} \qquad (39)$$

multiply through equation (40) by h^2, simplify it and it becomes

$$\frac{d^2G}{dY^2} = -\frac{v_0 h}{\alpha}\frac{dG}{dY} + \frac{i\omega h^2}{\alpha}G(Y) + \frac{Q_0 h^2}{k}G(Y) - \frac{v_0 h}{\alpha}\varepsilon A^*(\frac{T_1 - T_0}{T_2})\frac{1}{h}\frac{dF}{dY} \qquad (40)$$

and applying the non-dimensional quantities in equations (23) into equation(40), it results uniquely into

$$s = \frac{v_0 h}{v}, \Pr = \frac{v\rho Cp}{k}, s\Pr = \frac{v_o h}{v}.\frac{v\rho Cp}{k} = v_o h.\frac{\rho Cp}{k} = \frac{v_o h}{\alpha}, \delta = \frac{Q_o h^2}{\alpha\rho Cp} = \frac{Q_o h^2}{k},$$

$$St\Pr = \frac{h^2\omega}{v}\frac{v}{\alpha} = \frac{h^2\omega}{\alpha}$$

$$\frac{d^2G}{dY^2} = -s\Pr\frac{dG}{dY} + iSt\Pr G(Y) + \delta G(Y) - s\Pr\varepsilon A^*(\frac{T_1 - T_0}{T_2})\frac{dF}{dY} \qquad (41)$$

in equation (41) above, let $\gamma = \varepsilon A^*(\frac{T_1 - T_0}{T_2})$ (42)

then, equation (41) becomes

$$\frac{d^2G}{dY^2} = -s\Pr\frac{dG}{dY} + iSt\Pr G(Y) + \delta G(Y) - s\Pr\gamma\frac{dF}{dY} \qquad (43)$$

and (iii) other higher powers $O(e^{iwt})$ are neglected.

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Substituting equation (8) and equation (9) into equation (60), the momentum equation, upon correct substitution gives

$$\frac{\partial}{\partial t}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} - v_0(1 + \varepsilon A^* e^{i\omega t})\frac{\partial}{\partial y}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} = v\frac{\partial^2}{\partial y^2}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} + g\beta\{([T_0 + (T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t}] - T_0)\} \qquad (44)$$

equation (44) simplifies to

$$\frac{\partial}{\partial t}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} - v_0(1 + \varepsilon A^* e^{i\omega t})\frac{\partial}{\partial y}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} = v\frac{\partial^2}{\partial y^2}\{\frac{g\beta h^2}{v}[(T_1 - T_0)A(Y) + T_2 B(Y)e^{i\omega t}]\} + g\beta\{([(T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t}])\} \qquad (45)$$

term by term differentiation of equation of equation(45) gives

$$i\omega\frac{g\beta h^2}{v}T_2 B(Y)e^{i\omega t} - v_0(1 + \varepsilon A^* e^{i\omega t})\{\frac{g\beta h^2}{v}[(T_1 - T_0)A'(Y) + T_2 B'(Y)e^{i\omega t}]\} = v\{\frac{g\beta h^2}{v}[(T_1 - T_0)A''(Y) + T_2 B''(Y)e^{i\omega t}] + g\beta\{[(T_1 - T_0)F(Y) + T_2 G(Y)e^{i\omega t}]\} \qquad (46)$$

on expanding all the products of equation (46) asymptotically,

$$i\omega\frac{g\beta h^2}{v}T^2 B(Y)e^{i\omega t} - v_0\frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) - v_0\frac{g\beta h^2}{v}T_2 B'(Y)e^{i\omega t} - v_0\varepsilon A^* e^{i\omega t}\frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) - v_0\varepsilon A^* e^{i\omega t}T_2 B'(Y)e^{i\omega t} = v\frac{g\beta h^2}{v}(T_1 - T_0)A''(Y) + v\frac{g\beta h^2}{v}T_2 B''(Y)e^{i\omega t} + g\beta(T_1 - T_0)F(Y) + g\beta T_2 G(Y)e^{i\omega t} \qquad (47)$$

similarly, equating all the periodic functions of order, $(e^{i\omega t})^0$ from equation(47)

$$-v_0\frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) = v\frac{g\beta h^2}{v}(T_1 - T_0)A''(Y) + g\beta(T_1 - T_0)F(Y) \qquad (48)$$

equation (48) simplify to

$$-v_0 \frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) = g\beta h^2 (T_1 - T_0)A''(Y) + g\beta(T_1 - T_0)F(Y) \qquad (49)$$

divide through equation (49) $(T_1 - T_0)g\beta h^2$; it results in

$$\frac{-v_0}{v}A'(Y) = A''(Y) + \frac{1}{h^2}F(Y) \qquad (50)$$

simple rearrangement, equation (50) becomes

$$A''(Y) = -\frac{v_0}{v}A'(Y) - \frac{1}{h^2}F(Y) \qquad (51a)$$

equation (51a) can also be written as

$$\frac{d^2A}{dy^2} = -\frac{v_0}{v}\frac{dA}{dy} - \frac{1}{h^2}F(y) \qquad (51b)$$

and applying the non-dimensional quantities in equations (23), $s = \frac{v_0 h}{v}$,

equation (51b) becomes

$$\frac{1}{h^2}\frac{d^2A}{dY^2} = -\frac{v_0}{v}\frac{1}{h}\frac{dA}{dY} - \frac{1}{h^2}F(y) \qquad (52)$$

multiply through equation (52) by h^2,

$$\frac{d^2A}{dY^2} = -\frac{v_0 h}{v}\frac{dA}{dY} - F(y) \qquad (53)$$

$$\frac{d^2A}{dY^2} = -s\frac{dA}{dY} - F(y)$$

$$A''(Y) = sA'(Y) - F(Y) \qquad (54)$$

Similarly, equating orders of periodic functions of order 1, from equation (47), we get

$$i\omega\frac{g\beta h^2}{v}T^2B(Y) - v_0\frac{g\beta h^2}{v}T_2B'(Y) - v_0\varepsilon A^*\frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) = \qquad (55)$$

$$v\frac{g\beta h^2}{v}T_2B''(Y) + g\beta T_2G(Y)$$

equation (55) decomposes to

$$i\omega\frac{g\beta h^2}{v}T_2B(Y) - v_0\frac{g\beta h^2}{v}T_2B'(Y) - v_0\varepsilon A^*\frac{g\beta h^2}{v}(T_1 - T_0)A'(Y) = \qquad (56)$$

$$g\beta h^2T_2B''(Y) + g\beta T_2G(Y)$$

divide through equation (56) by $T_2 g\beta h^2$

$$\frac{i\omega}{v}B(Y) - \frac{v_0}{v}B^{'}(Y) - \frac{v_0}{v}\varepsilon A^*(\frac{T_1 - T_0}{T_2})A'(Y) = B''(Y) + h^2G(Y) \qquad (57)$$

Making $B''(Y)$ the subject of formulae in equation (57) above,

$$B''(Y) = -\frac{v_0}{v}B'(Y) + \frac{i\omega}{v}B(Y) - \frac{v_0}{v}\varepsilon A^*(\frac{T_1 - T_0}{T_2})A'(Y) + h^2G(Y) \tag{58}$$

and applying the non-dimensional quantities in equations (23) into equation (58), and usi

$$\frac{dA}{dy} = \frac{dA}{dY}.\frac{1}{h} = \frac{1}{h}.\frac{dA}{dY};\frac{dB}{dy} = \frac{dB}{dY}.\frac{1}{h} = \frac{1}{h}.\frac{dB}{dY};\frac{d^2B}{dy^2} = \frac{d^2B}{dY^2}.\frac{1}{h^2} = \frac{1}{h^2}.\frac{d^2B}{dY^2},$$

equation (58) becomes

$$\frac{1}{h^2}\frac{d^2B}{dY^2} = -\frac{v_0}{v}\frac{1}{h}\frac{dB}{dY} + \frac{i\omega}{v}B(Y) - \frac{v_0}{v}\varepsilon A^*(\frac{T_1 - T_0}{T_2})\frac{1}{h}\frac{dA}{dY} + h^2G(Y) \tag{59}$$

multiply through equation (59) by h^2

$$\frac{d^2B}{dY^2} = -\frac{v_0 h}{v}\frac{dB}{dY} + \frac{i\omega h^2}{v}B(Y) - \frac{v_0 h}{v}\varepsilon A^*(\frac{T_1 - T_0}{T_2})\frac{dA}{dY} + G(Y) \tag{60}$$

from equation (42), $\gamma = \varepsilon A^*(\frac{T_1 - T_0}{T_2})$ and from equation (23), $s = \frac{v_0 h}{v}$, then equation (60) become

$$\frac{d^2B}{dY^2} = -s\frac{dB}{dY} + iStB(Y) - s\gamma\frac{dA}{dY} + G(Y)$$

$$B''(Y) = -sB'(Y) + iStB(Y) - s\gamma A'(Y) + G(Y) \tag{61}$$

In summary, equations (33), (43), (55) and (61) are ordinary differential equations obtained in dimensionless form:

$$F''(Y) + s\Pr F'(Y) - \delta F(Y) = 0 \tag{33}$$

$$\frac{d^2F}{dY^2} + s\Pr\frac{dF}{dY} - \delta F(Y) = 0 \tag{34}$$

$$G''(Y) = -s\Pr G'(Y) + iSt\Pr G(Y) + \delta G(Y) - s\Pr\gamma F'(Y) \tag{43}$$

$$\frac{d^2G}{dY^2} = -s\Pr\frac{dG}{dY} + iSt\Pr G(Y) + \delta G(Y) - s\Pr\gamma\frac{dF}{dY} \tag{44}$$

$$\frac{d^2A}{dY^2} = -s\frac{dA}{dY} - F(y) \tag{54}$$

$$A''(Y) = -sA'(Y) - F(Y) \tag{55}$$

$$\frac{d^2B}{dY^2} = -s\frac{dB}{dY} + iStB(Y) - s\gamma\frac{dA}{dY} + G(Y)$$

$$B''(Y) = -sB'(Y) + iStB(Y) - s\gamma A'(Y) + G(Y) \tag{61}$$

by simple rearrangement of the four coupled equation above,

$$F''(Y) + s\Pr F'(Y) - \delta F(Y) = 0 \tag{33}$$

$$G''(Y) = -s\Pr G'(Y) + iSt\Pr G(Y) + \delta G(Y) - s\Pr\gamma F'(Y)$$

$$A''(Y) = -sA'(Y) - F(Y)$$

$$B''(Y) = -sB'(Y) + iStB(Y) - s\gamma A'(Y) + G(Y)$$

Subject to the following boundary conditions

$$A(\pm1) = B(\pm1) = 0 \tag{62}$$

$$F(\pm1) = G(\pm1) = 1$$

4.0 Discussion of Results

In this work, free convective flow of an incompressible viscous fluid between vertical porous plates with variable suction and periodic heating input is studied in the presence of a uniform suction and injection considering variable properties. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature and the two plates are kept at two constant but different temperatures. The fluid is acted upon by a constant pressure gradient. The coupled set of the nonlinear equations of motion and the energy equation including the viscous dissipation term is solved numerically using the asymptotic expansion method to obtain the velocity and temperature distributions at any instant of time.

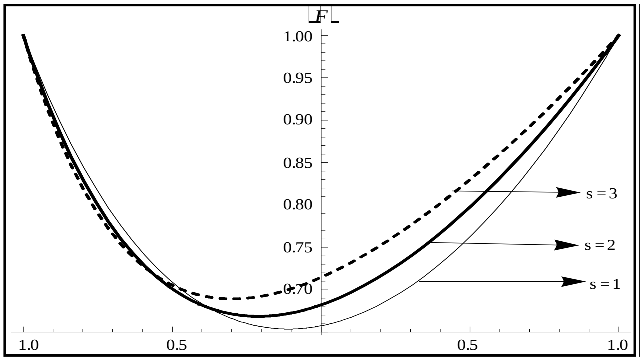


Figure 2

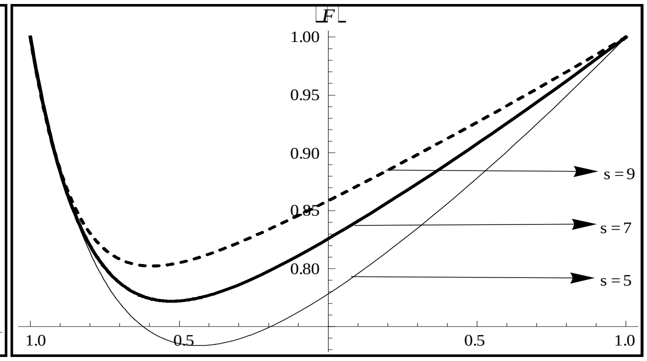


figure 3

Figure 2: Steady temperature profile for different suction (s). [δ=0.1,Pr=0.71 and St=5], Figure 3 Effect of suction/injection on temperature. [δ=0.1,Pr=0.71 and St=5]

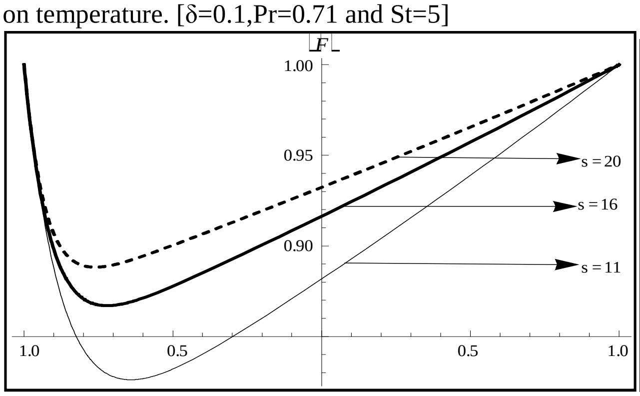


Figure 4

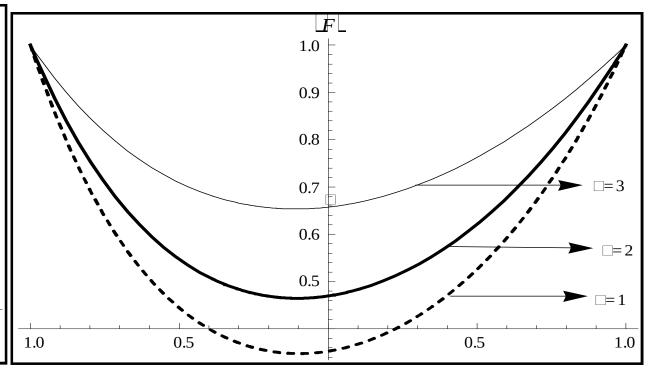


figure 5

Figure 4: Rate of heat transfer on different suction. [δ=0.1,Pr=0.71 and St=5], Figure 5: Influence of heat sink/source (δ) on temperature

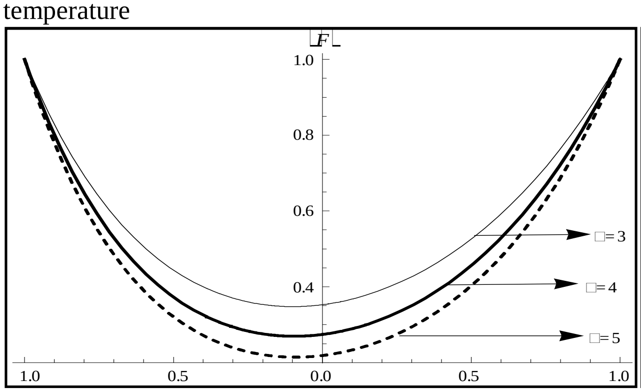


Figure 6

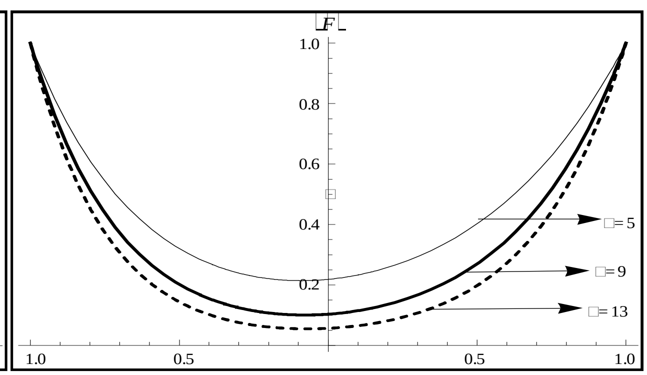


figure 7

Figure 6: Influence of heat sink/source (δ) on suction. Figure 7: Temperature profile for different heat sink/source (δ). F(y) is the steady temperature profile, the following δ = 0.1,P r = 0.71 and St = 5 were kept constant and stroughal (s) was varied in figure (2-4) above. In figure 2, s = {1, 2, 3} while in figure 3, s = {5, 7, 9}, and in figure 4, s = {10, 16, 20}:It was observed that suction tends towards the lower plate with lower plate with higher value but tends toward the upper plate with

small value ie when s = {1, 2, 3]. F(y) is the steady temperature profile, the following s = 1,Pr = 0.71 and St = 5 were kept constant and Delta, (δ), and the dimensionless heat generating parameter is varied in figure (5-7) above. In figure 5, (δ) = {3, 2, 1} while in figure 6, (δ) = {3, 4, 5}, and in figure 7, (δ) = {5, 9, 13}:It was observed that suction tends towards a minimum at the origin (ambient temperature T_o) and it increases symmetrically toward both the upper and lower plate. This is directly proportional to the frequency of heating of the channel walls Delta, (δ), and the variable suction parameter.

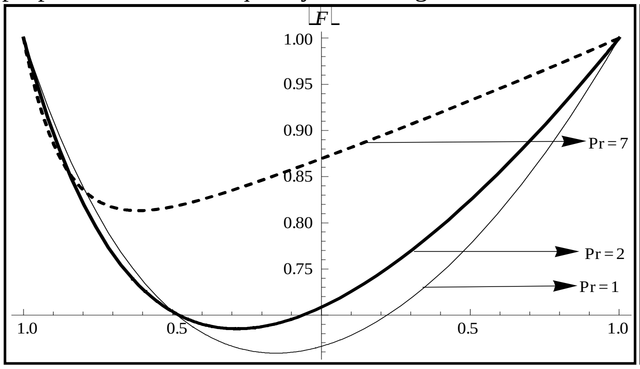


Figure 8

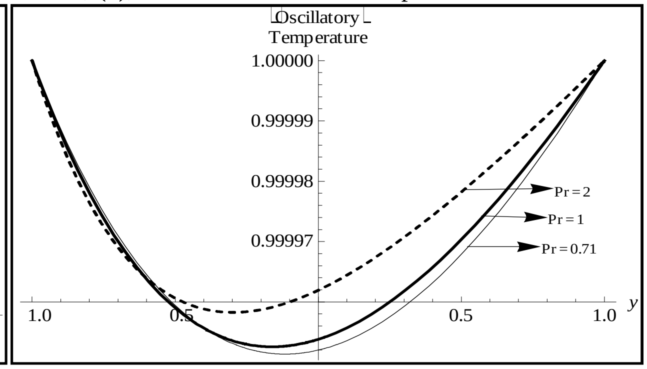


figure 9

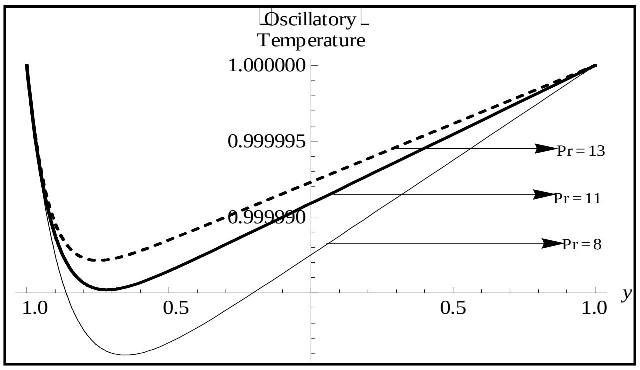


figure 10

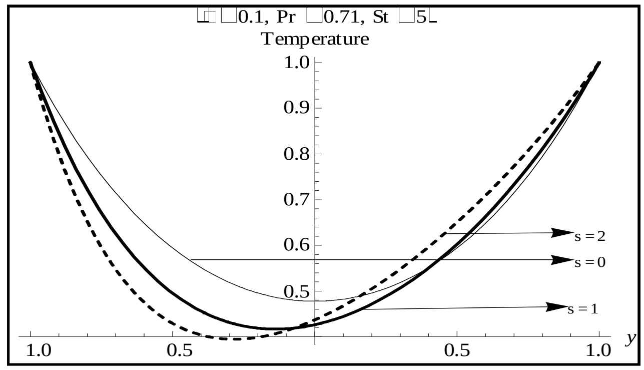


figure 11

Figure 8: Steady Temperature profile for different Prandtl, figure 9: Temperature profile for different Prandtl, Figure 10: Oscillatory Temperature profile for different Prandtl, Figures (8-10) is the steady oscillatory temperature profile of F(y) with the following s = 1, δ = 0.0001 and St = 5 kept constant and Delta, (δ) the dimensionless heat generating parameter is varied. In figure 8, (Pr) = {1, 2, 7} while in figure 9, (Pr) = {0.71, 1, 2}, and in figure 10, (Pr) = {8, 11, 13}: It was observed that the minimum is at the origin (ambient temperature T_o) when y = 0 and that it is increasing. Prandtl number, (Pr) which is inversely proportional to the thermal diffusivity of the working fluid, When (Pr) = {8, 11, 13}, the graphs tends towards saturation at the upper part of the left plate, Figure 10: Unsteady Temperature profile for different suction Figure 11: Temperature distribution for different suction.

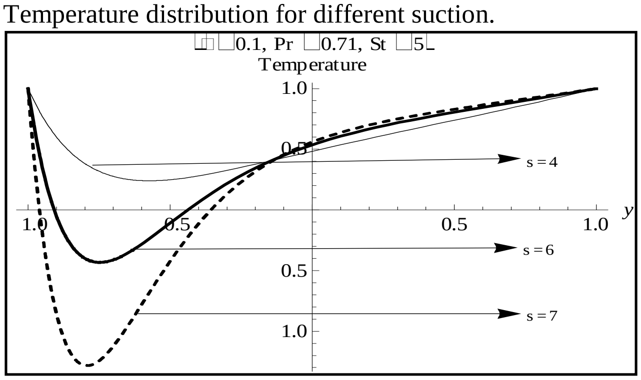


figure 12

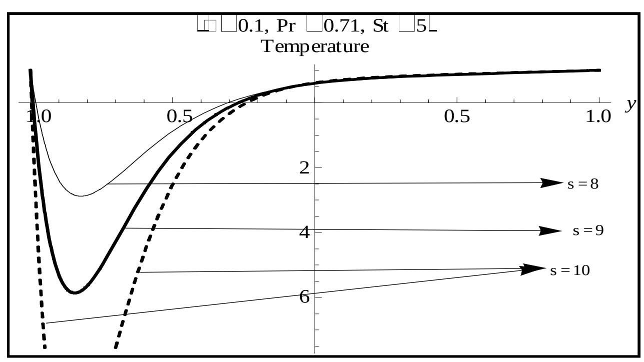
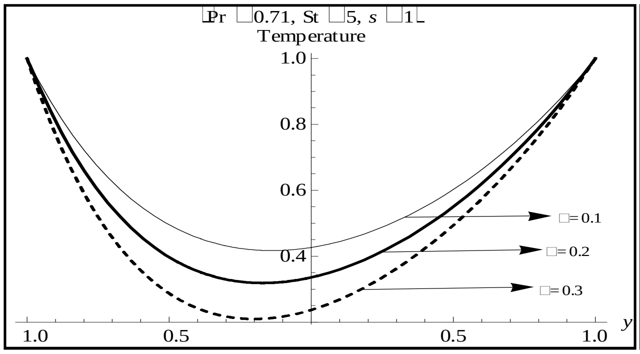


figure 13

Figure 12: Effect of suction/injection on Temperature, G(y) is unsteady temperature profile. The following δ = 0.1,Pr = 0.71 and St = 0.5 were kept constant and suction (s) was varied in figure (11 - 13) above. In figure 11, s = {0, 1, 2} while in figure 12, s = {4, 6, 7}, and in figure 13, s = {8, 9, 10}: It was observed that suction tends towards the lower plate with lower value but Suction and injection parameter, (s) simultaneously tends to the lower walls of the lower plate of figure 12 and figure 13. with total saturation at the upper side of the upper plate.



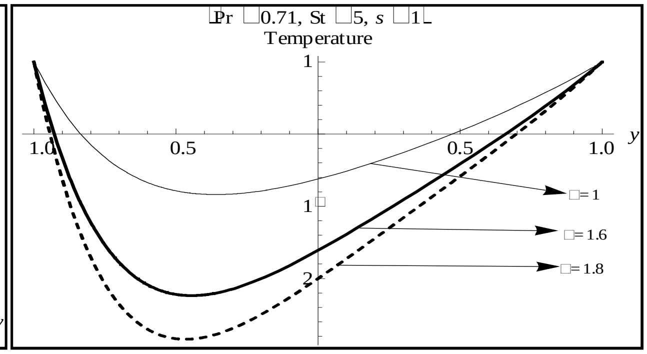
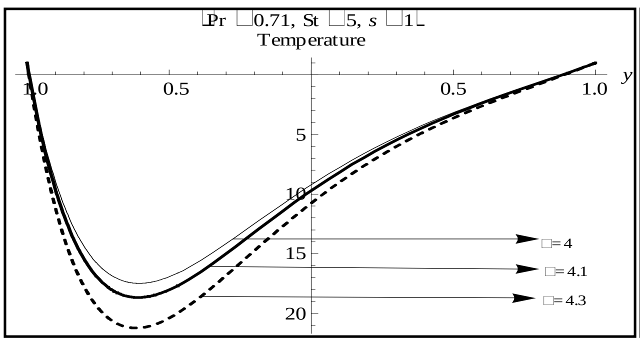


Figure 14 figure 15

Figure 14: Temperature distribution for different source/sink (δ), Figure 15: Influence of Temperature on heat sink/source (δ).

Influence of suction/injection over heat sink/sources (δ), Figures (14 - 16) below, represents G(y) is unsteady temperature profile of variance in heating of the channel walls Delta, (δ). The following Pr = 0.7, St = 5 and s = 1 were kept constant. In figure 14, δ = {0.1, 0.2, 0.3} while in figure 15, δ = {1, 1.6, 1.8}, and in figure 16, δ = {4, 4.1, 4.3}: It was observed that heating of the channel walls Delta, (δ) tends towards the lower plate with lower value. More symmetrical with lower value of (δ), Figure 17, effect of suction/injection on Prandtl number,(Pr) and Figure 18 below shows Unsteady Temperature profile for different Prandtl and figure 19, influence of Temperature on Prandtl (Pr)



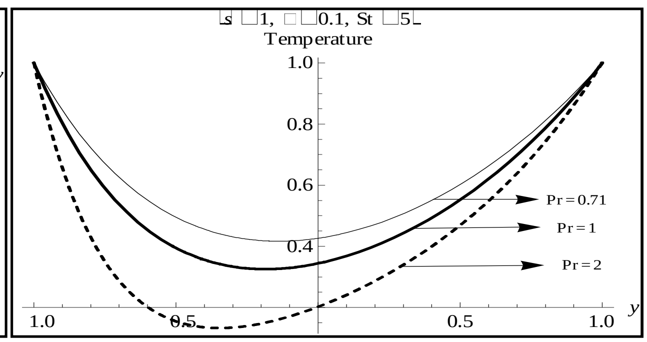
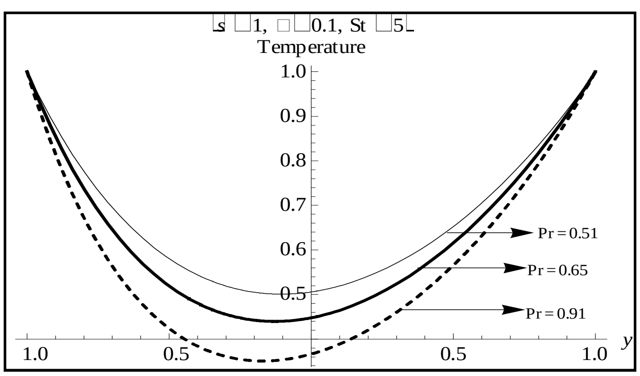


Figure 16 figure 17



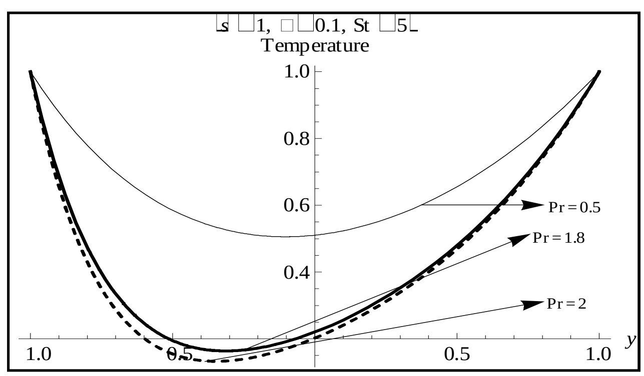


figure 18 figure 19

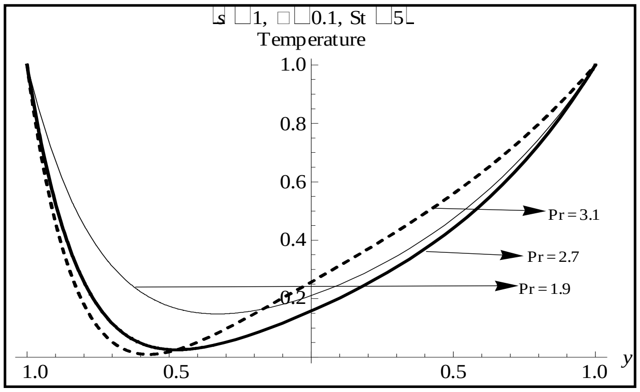


figure 20

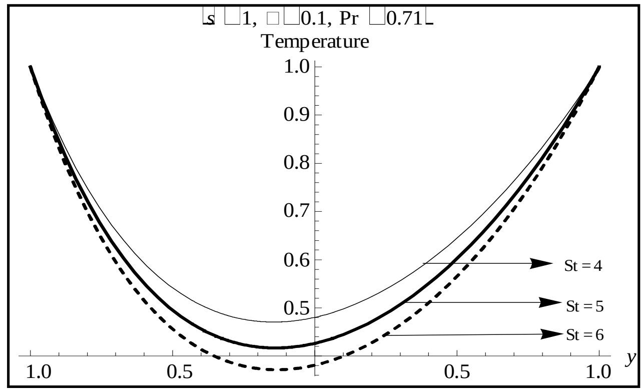


figure 21

Figure 20: High Prandtl number (Pr) and Temperature showing suction/injection (s), Figures (17 - 20) represents G(y), unsteady temperature profile of variance in heating of the channel walls delta, (δ). The following s = 1, (δ) = 0.1 and St = 5 were kept constant. In figure 17, Pr = {0.71, 1, 2 } while in figure 18, Pr = {0.51, 0.65, 0.91}, figure 19, Pr = {0.5, 1.8, 2} and in figure 20, Pr = {1.9, 2.7, 3.1}: It was observed that heating of the channel walls increase in prandtl number tends towards the lower plate with lower value. Figure 21: Unsteady Temperature profile for different Strouhal number. Figure 22 shows the influence of Strouhal number (St) on suction (s), and figure 23, influence of Strouhal number (St) and suction (s) on temperature. Figures (21 - 23) represents G(y) is unsteady temperature profile of variance in Strouhal number, (St). The following s = 1, δ = 0.1 and Pr = 0.71 were kept constant. In figure 21, St = {4, 5, 6 } while in figure 22, St = {2, 4, 6}.

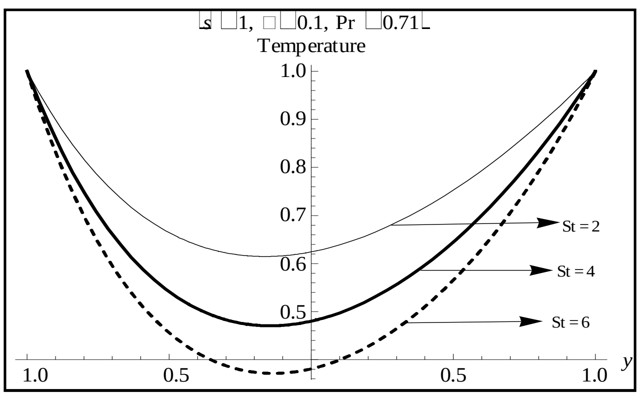


figure 22

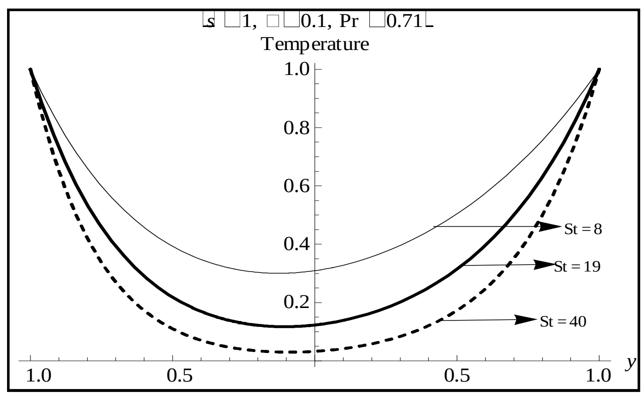


figure 23

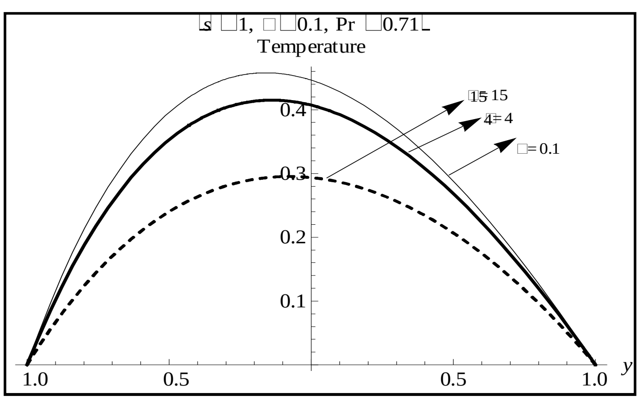


figure 24

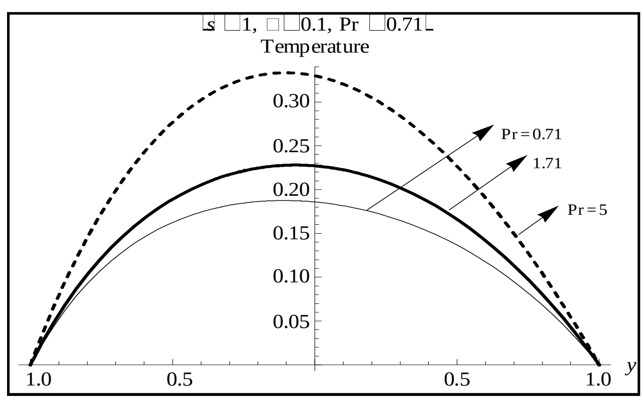


figure 25

It was observed that heating of the channel walls increase in prandtl number. Strouhal number, (St) is directly proportional to the frequency of heating of the channel walls Delta, (δ), and the heat generating parameter Gamma, and γ, the variable suction parameter. The graph is symmetrical about y = 0, Figure 24: Steady Temperature profile for different suction

Figure 25: Steady Temperature profile for different suction, A(y) is steady velocity profile and B(y) is unsteady velocity profile. Steady implies independent of time, i.e. t = 0.The basic parameter that governs the flow in this channel includes. Suction and injection parameter, (s) simultaneously applied each to opposite walls of the channel at the same rate. Prandtl number, (Pr) which is inversely proportional to the thermal diffusivity of the working fluid.It was observed that Strouhal number, (St) is directly proportional to the frequency of heating of the channel walls Delta, (δ), and the graph is symmetrical about y = 0.

4.0 Conclusion

The problem of heat flow in a channel with periodic suction has been investigated earlier byJha and Ajigbade.[19]. They considered constant suction; this work has now been extended to variable suction, with more readings considered. It can be concluded that heat source and suction affects the boundary layer thickness, temperature and velocity profiles and rate of heat transfer at the plate surface in such flows. In addition, exact solutions are obtained in the present work, which are very important as they serve as accuracy checks for experimental and analytical methods. Hence, the result of the study is expected to contribute to the literature in this field and enhance the understanding of free convection in vertical channels. And this will serve as a benchmark for researchers interested in this field for future study

APPENDIX A:
Nomenclatures:

A	steady velocity profile
B	unsteady velocity profile
F	steady temperature profile
G	unsteady temperature profile
g	gravitational force
ρ	density of the fluid
μ	viscosity of the fluid
Cp	specific heat at constant pressure
h	half channel width
Nu	Nusselt Number
Pr	Prandtl number
Q	heat generation term
Q_o	dimensional heat generation coefficient
s	dimensionless suction/injection parameter
St	Strouhal number
t	time
T	temperature of the fluid
T_o	ambient temperature
εA	any real positive constant
T_1	steady temperature on wall
T_2	unsteady temperature amplitude on wall
V_o	initial velocity of suction
y	horizontal coordinate
x	coordinate in the direction of the flow
y	coordinate across the flow
Y	dimensionless channel width
k	thermal conductivity
α	thermal diffusivity
β	coefficient of thermal expansion
δ	dimensionless heat generating parameter
□	kinematic viscosity
τ	skin friction
ω	frequency
ε	small parameter

APPENDIX B:
THE GENERAL VECTOR OPERATIONS

∇ : known as "nabla" or "del" is the vector differential operator .
For Cartesian coordinate

Del Operator: $$\nabla = (\frac{\partial}{\partial x} i, \frac{\partial}{\partial y} j, \frac{\partial}{\partial z} k)$$

∇^2 : known as the Laplacian operator

Laplacian Operator: $$\nabla^2 = \nabla.\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$$

Gradient: $$\nabla p = (\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z})$$

Vector Gradient: $$\nabla u = (\nabla u, \nabla v, \nabla w)$$

Divergence: $$\nabla . u = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Directional Derivative: $$u.\nabla = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

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