Journal of the Nigerian Association of Mathematical Physics
Volume **19 (November, 2011), pp 249 – 254**

Flow of a Maxwell Fluid Through a Porous Medium Induced By a Constantly Accelerating Plate

Hassan A. R.[1]* and Fenuga O. J[2]

**[1*]Department of Mathematical Sciences,
Olabisi Onabanjo University, Ago – Iwoye, Ogun State.
[2]Department of Mathematics, University of Lagos, Lagos.**

Abstract

The unsteady flow of a Maxwell fluid through a porous medium induced by a constantly accelerating plate is studied. Exact solution is established by means of Adomian decomposition method. The similar solution for a Newtonian fluid is obtained as a limiting case of the Maxwell fluid solution.

Keywords: Maxwell Fluid, constantly accelerating plates, Porous medium, Adomian Decomposition Method.

1.0 Introduction

This paper revisits the work in [1] in which the exact solution corresponding to the flow of Maxwell fluid is established between two side walls induced by a constantly accelerating plate where the asymptotic behaviour of the relaxation time is studied. Due to the diversity of fluid in nature, a lot of models have been studied in [2, 3] to describe the behaviour of fluids in different circumstances. A porous medium is a solid with holes which is characterized or described in terms of properties that affect the flow as discussed in [3].

The use of Adomian decomposition method has been applied to a wide class of problems in the Sciences. Rich literature for the Adomian method can be found in [4]; the method has shown reliable results in supplying analytical approximations that converge very rapidly.

2.0 Governing Equations

Following [1], we examine the flow between two walls z = 0 and z = d

$$\mathrm{v} = \mathrm{v}(y, z, t) = (u(y, z, t), o, o) \tag{2.1}$$

In the absence of body forces, the balance of linear momentum reduces to

$$(1+\lambda\partial_t)\tau_1 = \mu\partial_y u \quad and \quad (1+\lambda\partial_t)\tau_2 = \mu\partial_z u \tag{2.2}$$

where u is the velocity,
τ is the shear stress exerted by the fluid (drag),
μ is the fluid viscosity

Simplifying from [1], we obtained

$$\frac{\partial u}{\partial t} + \lambda\frac{\partial^2 u}{\partial t^2} = \upsilon\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] - \frac{\partial P}{\partial x}\left(1+\lambda\frac{\partial u}{\partial t}\right) \tag{2.3}$$

Neglecting the z – co ordinate, allowing the pressure tends zero and adding the porous terms, equation (2.3) reduces to

$$\frac{\partial u}{\partial t} + \lambda\frac{\partial^2 u}{\partial t^2} = \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\upsilon u}{k} \tag{2.4}$$

where λ is the relaxation time
υ is the kinematic viscosity and
k is the permeability constant

with the initial and boundary conditions as follows

*Corresponding authors: Hassan A. R.; E-mail: anthonyhassan72@yahoo.co.uk, Tel: +2348054203428

$$u(y,0) = \frac{\partial u}{\partial t}(y,o) = 0$$
$$u(d,t) = 0, \tag{2.5}$$
$$u(0,t) = At$$

where A stands for the Acceleration.

3.0 Method of Solution

We now non – dimensionalise with $\bar{u} = \frac{u}{v_0}, \bar{t} = \frac{t}{t_0}$ and $\bar{y} = \frac{y}{d}$ (3.1)

this implies that

$$u = \bar{u}\,v_0\,,\ t = \bar{t}\,t_0 \text{ and } y = \bar{y}\,d \tag{3.2}$$

Substituting equation (3.2) into equation (2.4), we obtain

$$\lambda v_0 \frac{\partial^2 \bar{u}}{\partial t^2} + v_0 \frac{\partial \bar{u}}{\partial t} + = v_0 \upsilon \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{v_0 \upsilon \bar{u}}{k} \tag{3.3}$$

We divide through by υ_0 to get

$$\lambda \frac{\partial^2 \bar{u}}{\partial t^2} + \frac{\partial \bar{u}}{\partial t} = \upsilon \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\upsilon \bar{u}}{k} \tag{3.4}$$

But $\quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \bar{t}} * \frac{\partial \bar{t}}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \bar{t}}$

$$\therefore \ \frac{\partial \bar{u}}{\partial t} = \frac{1}{t_0}\frac{\partial \bar{u}}{\partial \bar{t}} \tag{3.5}$$

Also

$$\frac{\partial^2 \bar{u}}{\partial t^2} = \frac{1}{t_0^2}\frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \tag{3.6}$$

Similarly,

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \bar{y}} * \frac{\partial \bar{y}}{\partial y} = \frac{1}{d}\frac{\partial}{\partial \bar{y}}$$

$$\therefore \ \frac{\partial \bar{u}}{\partial y} = \frac{1}{d}\frac{\partial \bar{u}}{\partial \bar{y}} \tag{3.7}$$

Also, $\quad \frac{\partial^2 \bar{u}}{\partial y^2} = \frac{1}{d^2}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$ (3.8)

Substitute equations (3.5), (3.6), (3.7) and (3.8) into equation (3.4) to obtain

$$\frac{\lambda}{t_0^2}\frac{\partial^2 \bar{u}}{\partial t^2} + \frac{1}{t_0}\frac{\partial \bar{u}}{\partial t} = \frac{\upsilon}{d^2}\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\upsilon \bar{u}}{k} \tag{3.9}$$

Rearranging and multiplying through by $\frac{d^2}{\nu}$

$$\frac{\lambda d^2}{\nu t_0^2}\frac{\partial^2 \bar{u}}{\partial t^2}+\frac{d^2}{\nu t_0}\frac{\partial \bar{u}}{\partial t}=\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{d^2 \bar{u}}{k} \tag{3.10}$$

Let $a=\frac{d^2}{\nu t_0}$, $k_0=\frac{\lambda}{t_0}$ and $Da=\frac{k}{d^2}$ where k_0 is the Maxwell parameter, Da is the Darcy's constant [3].
Hence, equation (3.10) reduces to

$$ak_0\frac{\partial^2 \bar{u}}{\partial t^2}+a\frac{\partial \bar{u}}{\partial t}=\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{\bar{u}}{Da} \tag{3.11}$$

$k_0\to 0$ implies Newtonian flow. Of interest is $k_0\neq 0$ and we let $a=1$. Also, we drop the bar over variables since there is no confusion.
That is,

$$k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\frac{u}{Da} \tag{3.12}$$

$$\frac{\partial^2 u}{\partial y^2}=k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da} \tag{3.13}$$

Using Adomian method, we integrate equation (3.13) as follows:

$$\left.\frac{\partial u}{\partial y}\right|_0^y=\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}dY \tag{3.14}$$

$$\frac{\partial u}{\partial y}(y)-\frac{\partial u}{\partial y}(0)=\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}dY \tag{3.15}$$

Let $$\frac{\partial u}{\partial y}(0)=a_0(t) \tag{3.16}$$

$$\frac{\partial u}{\partial y}=a_0(t)+\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}dY \tag{3.17}$$

Integrating equation (3.17) again

$$u|_0^y=\int_0^y\left[a_0(t)+\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}\right]dYdY \tag{3.18}$$

$$u(y)-u(0)=a_0(t)y+\int_0^y\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}dYdY \tag{3.19}$$

From initial boundary conditions (2.5),

$$\therefore u(y,t)=At+a_0(t)y+\int_0^y\int_0^y k_0\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-\frac{u}{Da}dYdY \tag{3.20}$$

We let $a_0(t)=t^2$ which implies that

$$u(y,t) = At + t^2 y + \int_0^y \int_0^y k_0 \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} - \frac{u}{Da} dYdY \qquad (3.21)$$

Using [4], we get Adomian polynomials as follows:

$$u_0 = At + t^2 y \qquad (3.22)$$

and

$$u_{n+1} = \int_0^y \int_0^y \left[k_0 \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} - \frac{u}{Da} \right] dYdY \qquad n \geq 0 \qquad (3.23)$$

So that

$$u_{n+1} = \int_0^y \int_0^y B_n dYdY \qquad (3.24)$$

where $$B_n = k_0 \frac{\partial^2 u_n}{\partial t^2} + \frac{\partial u_n}{\partial t} - \frac{u_n}{Da} \qquad (3.25)$$

Let $$u_n = u_0 + u_1 + u_2 + u_3 + \ldots \qquad (3.26)$$

$$\therefore B_n = k_0 \frac{\partial^2}{\partial t^2} [u_0 + u_1 + u_2 + u_3 + \ldots] + \frac{\partial}{\partial t} [u_0 + u_1 + u_2 + u_3 + \ldots] - \frac{1}{Da} [u_0 + u_1 + u_2 + u_3 + \ldots] \qquad (3.27)$$

Hence

$$B_0 = k_0 \frac{\partial^2 u_0}{\partial t^2} + \frac{\partial u_0}{\partial t} - \frac{u_0}{Da}$$

$$B_1 = k_0 \frac{\partial^2 u_1}{\partial t^2} + \frac{\partial u_1}{\partial t} - \frac{u_1}{Da} \qquad (3.28)$$

$$B_2 = k_0 \frac{\partial^2 u_2}{\partial t^2} + \frac{\partial u_2}{\partial t} - \frac{u_2}{Da}$$ and so on.

Simplifying we obtain

$$B_0 = \left[2k_0 + 2t - \frac{t^2}{Da} \right] y + A\left[1 - \frac{t}{Da} \right] \qquad (3.29)$$

Using equation (3.24),

$$u_1 = \int_0^y \int_0^y \left[2k_0 + 2t - \frac{t^2}{Da} \right] y + A\left[1 - \frac{t}{Da} \right] dYdY \qquad (3.30)$$

and integrating equation (3.30) twice

$$u_1 = \left[2(k_0 + t) - \frac{t^2}{Da} \right] \frac{y^3}{3!} + \frac{Ay^2}{2!} \left[1 - \frac{t}{Da} \right] \qquad (3.31)$$

Using equation (3.31), we obtain

$$B_1 = \left[\frac{-4}{Da}(k_0 + t) + 2 + \frac{t^2}{Da^2} \right] \frac{y^3}{3!} + \frac{Ay^2}{2!} \left[\frac{-2}{Da} + \frac{t}{Da^2} \right] \qquad (3.32)$$

And using equation (3.32), we arrive at

$$u_2 = \left[\frac{-4}{Da}(k_0 + t) + 2 + \frac{t^2}{Da^2}\right]\frac{y^5}{5!} + \frac{Ay^4}{4!}\left[\frac{-2}{Da} + \frac{t}{Da^2}\right] \tag{3.33}$$

Similarly, we obtain

$$B_2 = \left[\frac{6}{Da^2}(k_0 + t) - \frac{6}{Da} - \frac{t^2}{Da^3}\right]\frac{y^5}{5!} + \frac{Ay^4}{4!}\left[\frac{3}{Da^2} - \frac{t}{Da^3}\right] \tag{3.34}$$

and

$$u_3 = \left[\frac{6}{Da^2}(k_0 + t) - \frac{6}{Da} - \frac{t^2}{Da^3}\right]\frac{y^7}{7!} + \frac{Ay^6}{6!}\left[\frac{3}{Da^2} - \frac{t}{Da^3}\right] \tag{3.35}$$

Also, we use equation (3.35) to get

$$B_3 = \left[-\frac{8}{Da^3}(k_0 + t) + \frac{12}{Da^2} + \frac{t^2}{Da^4}\right]\frac{y^7}{7!} + \frac{Ay^6}{6!}\left[-\frac{4}{Da^3} + \frac{t}{Da^4}\right] \tag{3.36}$$

and

$$u_4 = \left[-\frac{8}{Da^3}(k_0 + t) + \frac{12}{Da^2} + \frac{t^2}{Da^4}\right]\frac{y^9}{9!} + \frac{Ay^8}{8!}\left[-\frac{4}{Da^3} + \frac{t}{Da^4}\right] \tag{3.37}$$

Generally,

$$u_n = \left[\frac{(\mathrm{n(n\ -1)}}{-Da^{n-2}} + \frac{2n}{-Da^{n-1}}(k_0 + t) + \frac{t^2}{-Da^n}\right]\frac{y^{2n+1}}{(2n+1)!} + \frac{Ay^{2n}}{2n!}\left[\frac{n}{-Da^{n-1}} + \frac{t}{-Da^n}\right] \tag{3.38}$$

Therefore,

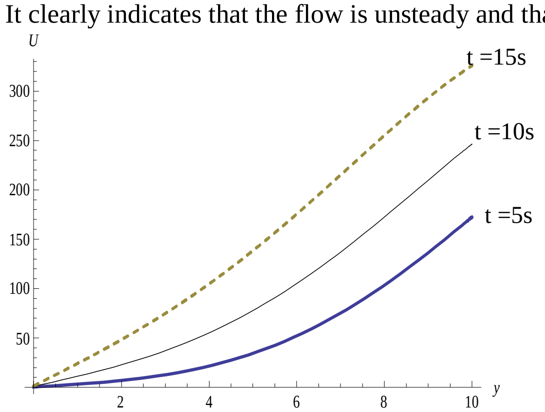
$$u(y,t) = \sum_{n=0}^{\infty}\left[\frac{(\mathrm{n(n\ -1)}}{-Da^{n-2}} + \frac{2n}{-Da^{n-1}}(k_0 + t) + \frac{t^2}{-Da^n}\right]\frac{y^{2n+1}}{(2n+1)!} + \frac{Ay^{2n}}{2n!}\left[\frac{n}{-Da^{n-1}} + \frac{t}{-Da^n}\right] \tag{3.39}$$

Testing the result with [5], it shows that equation (3.39) can be written in a better form as follows:

$$u(y,t) = \frac{1}{4}\left[\begin{array}{c} 4At\,Cos\left[\frac{y}{\sqrt{Da}}\right] - 3\,y\,Da^2\,Cos\left[\frac{y}{\sqrt{Da}}\right] - 4\,y\,t\,Da\,Cos\left[\frac{y}{\sqrt{Da}}\right] + 3\,Da^{\frac{5}{2}}\,Sin\left[\frac{y}{\sqrt{Da}}\right] + \\ 4\,Da^{\frac{3}{2}}t\,Sin\left[\frac{y}{\sqrt{Da}}\right] + 4\sqrt{Da}\,t^2\,Sin\left[\frac{y}{\sqrt{Da}}\right] + 2\,A\,y\sqrt{Da}\,Sin\left[\frac{y}{\sqrt{Da}}\right] - \\ Da^{\frac{3}{2}}y^2\,Sin\left[\frac{y}{\sqrt{Da}}\right] - 4\,y\,k_0\,Da\,Cos\left[\frac{y}{\sqrt{Da}}\right] - 4\,k_0\,Da^{\frac{3}{2}}t\,Sin\left[\frac{y}{\sqrt{Da}}\right] \end{array}\right] \tag{3.40}$$

4.0 Discussion of Results

Figure 4.1 shows the graph of velocity against y for t equals 5s, 10s and 15s respectively when the relaxation time λ is small. It clearly indicates that the flow is unsteady and that the velocity increases as time also increases.



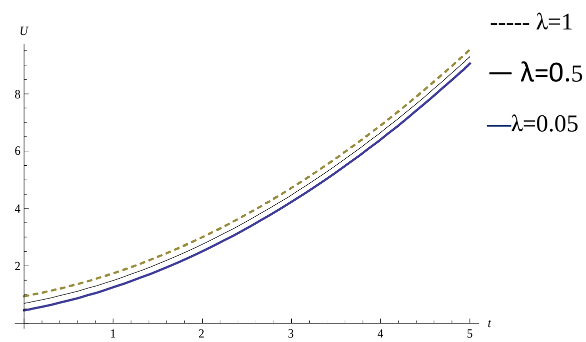


Figure 4.1: Graph of u against y for t = 5s, 10s and 15s.

Figure 4.2: Graph of u against t for λ = 0.01, 0.5 and 1

Figure 4.2 shows the graph of velocity against t for relaxation time (λ) equals 0.01s, 0.5s and 1s respectively which indicates that as relaxation time increases; each velocity increases and is at minimum when at rest.

5.0 Conclusion

We have studied the flow of a Maxwell fluid through a porous medium induced by a constantly accelerating plate and the result show that when there is porous medium like in sand, the relaxation time behaviour changes as time increases. Also it is clearly seen that velocity increases in both cases as time increases which satisfies the initial and boundary conditions. Thus, it implies that the porous medium has effect over the flow.

We have used Adomian decomposition method to find the nature of the Maxwell flow. It shows that our result exists and unique.

6.0 References

[1]**. Hassan A. R., Ayeni R.O.,(2010**); *On the flow of Maxwell Fluid between two walls induced by a constantly accelerating plate*, J of NAMP, Volume 17,pp 129 – 132.

[2]. **Akhtar W., Corina, F., Victor, T. and Constantine, F. (2008)**; *Flow of Maxwell fluid between two sides walls induced by a constantly accelerating plates,* Int. J, z. angew, Maths Physics DOI. 1007/S00033008 – 7129 – 8,Birkhauser Verlag, Basel.

[3]. **Robert A. Greenkorn, (1999**); *Flow in a Porous Media* in Fundamentals of Fluid Mechanics edited by Joseph A. Schetz and Allen E. Fuhs pp 818 – 823 John Wiley & Sons

[4]**. Abdul-Majid Wazwaz, (2000);** *A new algorithm for calculating Adomian polynomials for non linear operators,* Journal of Applied Mathematics and computation 111, pp 53 – 69.

[5]**. Wolfram, Stephen, (1959)** –*Mathematica 7*. — 7th ed.p. cm. Includes index. ISBN 1–57955–022–3(hard bound). I. Title.QA76.95.W65 2003 510 .285 5369—dc21 XX–XXXXX CIP