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Time-Dependent Natural Convection Couette Flow of Heat Generating/Absorbing Fluid between Vertical Parallel Plates Filled With Porous Material

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Abstract

This paper, investigates the time-dependent behavior of natural convection Couette flow of a viscous and incompressible heat generating/absorbing fluid in a porous medium bounded by two infinite vertical parallel plates. The Brinkman-extended Darcy model is considered to simulate the momentum transfer within the porous medium. The flow is generated by the asymmetric heating and the impulsive motion of one of the infinite vertical parallel plates. Laplace transform techniques is used to obtain the analytical solutions for the temperature and the velocity profiles while the rate of heat transfer as well the skin friction are consequently derived. The numerical simulation conducted for some saturated liquids reveled that at $t \geq Pr$ the steady and unsteady state velocities (as well as the temperature of the fluid) coincide. It is also observed that the temperature as well as the velocity of the fluid can be improved by increasing the gap between the plates. For $Gr = 0$, indicating the absence of convection current, the results are comparable to the results obtained in Schlichting (1979) as $Da \to \infty$ and $\gamma = 1.0$.

Keywords: heat generating/absorbing fluid, natural convection Couette flow, porous material asymmetric heating and impulsive motion.

1.0 Introduction

The past two decades has witnessed several research works on the free convection flows and heat transfer on a vertical surface subject to various conditions depending on industrial, geophysical and technological problems, [2].

Heat transfer and fluid flow by time-dependent natural convection resulting from temperature difference between two vertical parallel plates has been examined by several researchers: Ingham *et al* [3] studied the time-dependent boundary layer flow triggered by the sudden imposition of a temperature difference between a wall and fluid-saturated porous medium. Nithiarasus *et al* [4] takes into account the linear and non-linear matrix drag components as well as the inertia and viscous forces within the fluid by examining the generalized non-Darcian porous medium of variable porosity for the natural convection flow. Cheng and Minkowycz [5] obtained the similarity solutions for free-convection flow in a porous medium adjacent to a vertical plate. The velocity distribution and heat transfer have been found to be greatly affected by the variation of permeability of the porous media by [7, 8]. The boundary and inertia effects on convective flow and heat transfer for constant porosity media has been investigated by [9]. Vafai [10, 11] examined the effect of flow channeling in the case of flat forced convection. Wooding [12] treated the problem of natural convection in a saturated porous medium at large Rayleigh or Peclet number.

Poulikakos [13] investigated the impact of flow inertia on natural convection in a vertical porous layer. Jha [14, 15] studied the natural convection through vertical porous stratum and transient free convective flow in a vertical channel with sink respectively. Chandrasekhara and Narayanan [16] studied the laminar convection under a pressure gradient through a vertical porous channel, the walls of which are heated or cooled. Singh [17] examined the natural convection in unsteady Couette motion.

In the literature, several studies have been undertaken to investigate the natural convection flow for difference physical situations and under different operating conditions [18, 19, 20]. The problem of unsteady natural convection of heat transfer due to the internal heat generating/absorbing fluids has recently gained considerable attention among researchers. This is because internal heat generation/absorption plays significant role in various physical phenomena such as in fire and

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combustion modeling by [21], post accident heat removal by [22] and production of metal waste from spent nuclear fuel by [24]. Recently, [23] studied the free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input.

One of the basic flows in fluid mechanics is the Couette flow where the fluid motion is triggered by the movement of the bounding surface (wall or plate). Such flow occurs in fluid machineries associated with moving parts and is therefore important for hydrodynamic lubrications, [25].

In the present work, it is intended to present analytical tools to study the time-dependent natural convection Couette flow of heat generating/absorbing fluid between vertical parallel plates filled with porous material. A step plate velocity and temperature change is applied to the system under consideration to stimulate the unsteadiness in the velocity and temperature variables.

2.0 MATHEMATICAL ANALYSIS

Consider the time-dependent natural convection Couette flow between two infinite vertical parallel plates filled with porous material. The x'-axis is taken along one of the parallel plates in a vertical upward direction while y'-axis is normal to it into the fluid. At $t' \leq 0$, both the fluid and the plates are at rest and at same temperature. At $t' > 0$, the temperature of the plate $y' = 0$ is raised or falls to T_w and also begins to move with an impulsive motion U_0 while the other plate $y' = h$ is kept at rest and at temperature T_h.

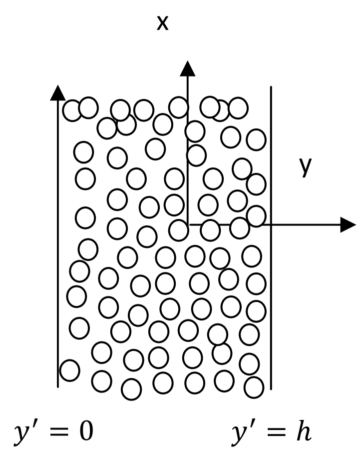

Fig.1.0. The physical configuration of the system

Considering the works conducted by [14.15, 17], the mathematical model for the time-dependent natural-convection Couette flow of heat generating/absorbing fluid between two infinite vertical parallel plates filled with porous material is governed by the following dimensionless set of second order partial differential equations:

$$\frac{\partial U'}{\partial t'} = \upsilon_{eff}\frac{\partial^2 U'}{\partial y'^2} + g\beta(T' - T_h) - \frac{U'\upsilon}{k} \tag{1.0}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_o(T_h - T')}{\rho C_p} \tag{2.0}$$

Under the following conditions

$$\begin{aligned} &t' \leq 0, \quad T' = T_h, && 0 \leq y' \leq h \\ &t' > 0; \quad U' = U_0,\ T' = T_w && y' = 0 \\ &\qquad U' = 0, T' = T_h && y' = h \end{aligned} \tag{3.0}$$

On applying the non dimensional quantities (4.0) below in equations (1.0) to (3.0)

$$U = \frac{U'}{U_0},\quad T = \frac{T' - T_h}{T_w - T_h},\quad y = \frac{y' U_0}{\upsilon}, \gamma = \frac{\upsilon_{eff}}{\upsilon}, t = \frac{t' U_0^2}{\upsilon}\quad H = \frac{hU_0}{\upsilon}, \Pr = \frac{\mu C_p}{k}, Da = \frac{KU_0^2}{\upsilon^2}$$

$$Gr = \frac{g\beta(T_w - T_h)\upsilon}{U_0^3} \text{ and } S = \frac{Q_o}{k}\left(\frac{\upsilon}{U_0}\right)^2 \tag{4.0}$$

we obtained the flow equations with the heated plate subjected to an impulsive motion:

$$\frac{\partial U}{\partial t} = \gamma\frac{\partial U^2}{\partial y^2} + GrT - \frac{U}{Da} \tag{5.0}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr}\frac{\partial T^2}{\partial y^2} - \frac{ST}{Pr} \tag{6.0}$$

The initial and boundary conditions in dimensionless form are:

$$
\begin{array}{llll}
t \leq 0; & U = T = 0 & \text{at } 0 \leq y \leq H & \\
t > 0; & U = 1, \quad T = 1 & \text{at } y = 0 & (7.0) \\
 & U = 0, \quad T = 0 & \text{at } y = H &
\end{array}
$$

3.0 SOLUTIONS

The analytical solutions of the equations (5.0) and (6.0) under the initial and boundary conditions (7.0) are given respectively by

$$U(y,t) = \frac{1}{2}\sum_{m}^{\infty}\Big[\big(f_4(a,t,\gamma,Da) - f_4(b,t,\gamma,Da)\big) + \frac{Gr}{c_1c_2}[(f_2(a,t,c_2,\gamma,Da) - f_2(b,t,c_2,\gamma,Da) + f_3(b,t,c_2,\gamma,Pr) - f_3(a,t,c_2,\gamma,Pr) + f_4(b,t,\gamma,Da) - f_4(a,t,\gamma,Da) + f_1(a,t,Pr,c,S) - f_1(b,t,Pr,c,S)]\Big] \tag{8.0}$$

and

$$T(y,t) = \frac{1}{2}\sum_{m}^{\infty}[f_1(a,t,Pr,c\,S) - f_1(b,t,Pr,c\,S)\ ] \tag{9.0}$$

Setting $erfc\ (\infty) = 0, \exp\ (-\infty) = 0$ and for small value of time (t), equations (8.0) and (9.0) above indicates that the velocity as well as the temperature fields near the plate $(y' = 0)$ are dominated by the expression;

$$\frac{1}{2}\big(f_4(y,t,\gamma,Da)\big) + \frac{Gr}{c_1c_2}(f_2(y,t,c_2,\gamma,Da) - f_3(y,t,c_2,\gamma,Pr) - f_4(y,t,\gamma,Da) + f_1(y,t,Pr,c\,s) \tag{10.0}$$

and

$$\frac{1}{2}f_1(y,t,Pr,c\,s) \tag{11.0}$$

The above expressions are solution for the velocity and the temperature fields for natural convection flow of heat generating/absorbing fluid past an impulsively started infinite vertical plate embedded in a porous media at constant temperature. This shows that for small time, the problem of flow formation in the presence of an impulsively started plate between two plates collapses to a flow at the plate $y' = 0$ only and is not influenced by the other plate $y' = h$.

At relatively large time, the response of velocity and temperature to time becomes insignificant. Consequently, the flow becomes steady so that the velocity and temperature become time independent.

To derive the steady-state equations we put $\frac{\partial U}{\partial t} = 0$ and $\frac{\partial T}{\partial t} = 0$ in equations (5.0) and (6.0) respectively:-

$$\frac{d^2U}{dy^2} + \frac{GrT}{\gamma} - \frac{U}{\gamma Da} = 0 \tag{12.0}$$

$$\frac{d^2T}{dy^2} - ST = 0 \tag{13.0}$$

Solving equations (12.0) and (13.0) under the boundary conditions

$$
\begin{array}{lll}
U = 1, \quad T = 1 & \text{at } y = 0 & \\
U = 0, \quad T = 0 & \text{at } y = H & (14.0)
\end{array}
$$

we obtain the following steady state solutions:

$$U(y) = \frac{Da(S\gamma + Gr) - 1}{(\gamma SDa - 1)}\sum_{m}^{\infty}\left[\mathrm{e}^{-\frac{\mathrm{a}}{\sqrt{\gamma \mathrm{Da}}}} - \mathrm{e}^{-\frac{\mathrm{b}}{\sqrt{\gamma \mathrm{Da}}}}] \ - \frac{DaGr}{(\gamma SDa - 1)}\sum_{m=0}^{\infty}[e^{-a\sqrt{S}} - e^{-b\sqrt{S}}\right] \tag{15.0}$$

and

$$T(y) = \sum_{m}^{\infty} \exp\ (-a\sqrt{S}) - \exp\ (-b\sqrt{S})) \tag{16.0}$$

It is important to mention that equations (15.0) and (16.0) are free from Prandtl number Pr. Hence there is no role of Pr in the steady states solutions for velocity and temperature.

The skin-friction and the rate of heat transfer on the heated plate $y' = 0$ are obtained by using $\tau_0 = \frac{dU}{dy}\Big|_{y=0}$ and $Nu_0 = \frac{dT}{dy}\Big|_{y=0}$ respectively and are given below :-

$$\tau_0 = -\frac{1}{2}\sum_{m}^{\infty}\Big[m_6(f_4(c_3,t,\gamma,Da) + f_4(c_4,t,\gamma,Da)) - m_3(f_7(c_3,t,\gamma,Da) - f_7(c_4,t,\gamma,Da) + m_5(f_8(c_3,t,\gamma,Da) - f_8(c_4,t,\gamma,Da)) + \frac{Gr}{c_1c_2}(\ m_7(f_1(c_3,t,Pr,c,S) + f_1(c_4,t,Pr,c,S)) - m_4(f_6(c_3,t,Pr,c,0.0) + f_6(c_4,t,Pr,c,0.0)) - m_5(f_4(c_4,t,\gamma,Da) + f_4(c_3,t,\gamma,Da)) + m_3(f_5(c_3,t,\gamma,Da,0.0) + f_5(c_4,t,\gamma,Da,0.0)) + m_8[m_9(f_2(c_3,t,c_2,\gamma,Da) + f_2(c_3,t,c_2,\gamma,Da)) - m_3(f_5(c_3,t,\gamma,Da,c_2) + f_5(c_4,t,\gamma,Da,c_2)) - m_{10}(f_3(c_3,t,c_2,c,Pr) + f_3(c_4,t,c_2,c,Pr)) + m_4(f_6(c_3,t,Pr,c,c_2) + f_6(c_4,t,Pr,c,c_2))]\Big] \tag{18.0}$$

and

$$Nu_0 = \frac{1}{2}\sum_{0}^{\infty}[m_7(f_1(c_4,t,Pr,c,S) + f_1(c_5,t,Pr,c,S)) - m_4(f_6(c_4,t,Pr,c,0.0) + f_6(c_5,t,Pr,c,0.0)] \quad (19.0)$$

While on the plate $y' = h$, they are respectively obtained by using

$\tau_H = \frac{dU}{dy}\Big|_{y=H}$ and $Nu_0 \frac{dT}{dy}\Big|_{y=H}$ thus:-

$$\tau_H = -\sum_{m}^{\infty}\Big[m_6 f_4(c_5,t,\gamma,Da) + \frac{Gr}{c_1 c_2}(m_7 f_1(c_3,t,Pr,c,S) - m_4 f_6(c_3,t,Pr,c,0.0) - m_5 f_4(c_4,t,\gamma,Da) + m_3 f_5(c_3,t,\gamma,Da,0.0) + m_8[m_9(f_2(c_3,t,c_2,\gamma,Da) - m_3 f_5(c_3,t,\gamma,Da,c_2) - m_{10} f_3(c_3,t,c_2,c,Pr) + m_4 f_6(c_4,t,Pr,c,c_2)]\Big] \quad (20.0)$$

and

$$Nu_H = \sum_{0}^{\infty}[m_7 f_1(c_6,t,Pr,c,S) - m_4 f_6(c_6,t,Pr,c,0.0)] \quad (21.0)$$

The list of the constants and functionals used are provided in appendix I and II respectively.

4.0 RESULTS AND DISCUSSIONS

The results of the numerical simulation are presented graphically and discussed in this section. The governing non-dimensional parameters are the gap between the plates (H), heat generating/absorbing parameter (S), Prandtl number (Pr), Grashof number (Gr), ratio of viscosity (γ) and Darcy number (Da). Their influence on the velocity, temperature, skin friction and the rate of heat transfer are analyzed.

Using the liquid-metal fluids, mercury (Hg) with $Pr = 0.044$, $\upsilon = 1.15\mathrm{e}^{-3}\mathrm{cm}^2\mathrm{s}^{-1}$ at 20°C and $\beta = \frac{1}{293}$, °C^{-1} as the working fluid, different values of Grashof number (Gr) for $H = 2.0cm$, $U_o = 0.1cms^{-1}$ and g = 980.0 cms^{-2} are determined at different temperatures (see Table 1).

From Fig 2.0, it is clear that increasing the gap between the plates increases the temperature of the fluid trapped between the plates. From Fig. 3.0, it is observed that as the distance (H) between the parallel plates is increased, the fluid velocity is increased. Furthermore, at *t*=0.003, the maximum velocity attained (which obviously increase with time) for all values of H is near the heated plate and is increased by increasing the gap H between the plates. This is as a result of the increase in the fluid temperature resulting from increase of the gap between the plates.

Within the different sections of the channel however, Fig. 4.0, reveals that the fluid temperature increases with time (t) and is higher near the moving plate. This is also the case with the fluid velocity profile (Fig. 5.0).

In Fig. 6.0 and Fig. 7.0 it is clear that the heat generating/absorbing parameter (S), exerts an inversely proportional influence the temperature of the system and on the fluid velocity. Increasing S at fixed $Q_{o,}U_o$ and υ, result to the decrease of the fluid's thermal conductivity and hence a decrease in the temperature. This decrease in temperature of the fluid leads to weak convection current in the system and therefore a decrease in velocity is obseved.

The relative magnitude of the fluid's thermal conductivity and the kinematic viscosity play a vital role on both the fluid flow and temperature profile. As the fluid's kinematic viscosity become greater than its thermal diffusivity $(\upsilon > \alpha)$, the Prandtl number Pr is greater than unity. Hence for larger values of Pr, the fluid thermal diffusivity is small thereby decreasing the temperature as well as the velocity of the fluid (Figs. 8.0 and 9.0). Also as $t \geq Pr$, the steady and the unsteady states temperatures coincide. The same trend is also seen in the velocity profiles Figs. 10.0 and 11.0).

Increasing the temperature difference between the plates increases the Grashof number (see table). Fig. 12.0 reveals that the variation of Gr is directly propotional to the fluid velocity. For $Gr = 0$, the fluid flow is triggered by the implusive motion of the plate $y' = 0$ only and the influence of temperature become immaterial. This flow is governed by the equation:

$$U(y,t) = \frac{1}{2}\sum_{m=0}^{\infty}[(f_4(a,t,\gamma,Da) - f_4(b,t,\gamma,Da)] \quad (22.0)$$

Equation (22.0) represents the velocity profile for unsteady Couette flow between vertical parallel plates filled with porous media in the absence of the convection current $(Gr = 0)$.

From Fig 13.0, it is evident that increasing the coefficient of the ratio of viscosity (γ) increases the frictional force within the system which reduces the velocity of the fluid within the channel. It is also noticed that close to the boundary plates, the effect of the variation of γ is negligible. This physically implies that fluid motion is small near the plates. Fig. 14.0 shows that increasing Da, increases the permeability of the porous matrix and therefore increases the fluid motion.

The skin-friction (τ) as well as the rate of heat transfer (*Nu*) are also considered for different values of H , Da, Pr and S. From Fig 15.0, it is observed that the skin-friction decreases with Pr but increases with time (t) at the heated plate $y' = 0$ while the effect of Pr $and\ t$ is almost insignificant on the other plate $y' = h$.

On the other hand, the rate of heat transfer (*Nu*), from the moving plate into the flowing fluid increases with Pr and decreases with time (Fig.16.0).

From Fig. 17.0, it is observed that the skin-friction decreases by increasing S and increases with time on the heated plate while the rate of heat transfer (*Nu*) is decreased by increasing time and shows no any change with the variation of S on the heated plate (Fig. 18.0). Fig.19.0 shows the influence of $Da\ \ on\ \tau$. It is clear that increasing Da and or time, increases the skin-friction on the moving plate. On the other hand, Fig. 20.0 shows that increase in H results to an increase in the skin-friction on the heated plate while a decrease in τ is noticed on the stationary plate ($y' = h$).

Lastly, it is evident from Fig. 21.0, that the rate of heat transfer decreases for some time and become constant at both plates by increasing the gap H between the plates and time (t).

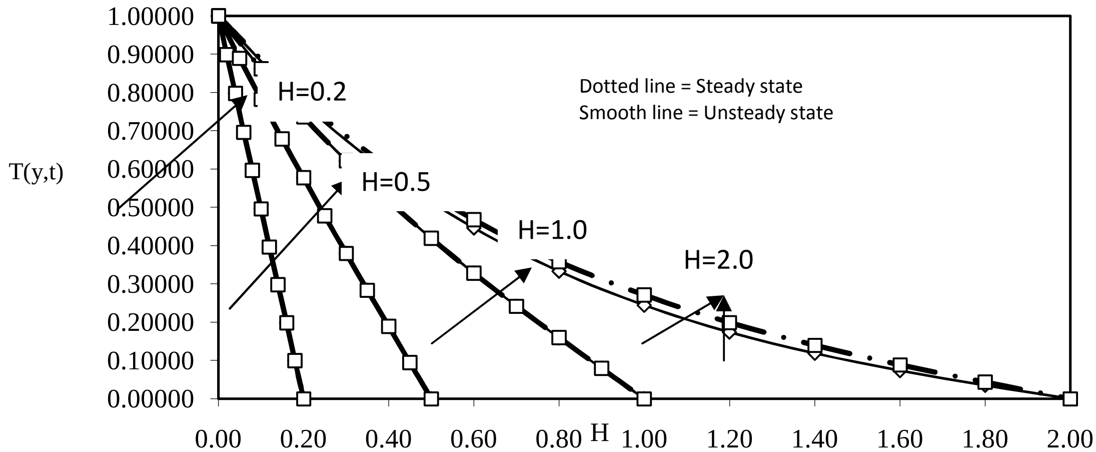

Fig. 2.0 Temperature profile with different H, $t = 0.003$, $Pr = 0.044$ and $S=1.5$

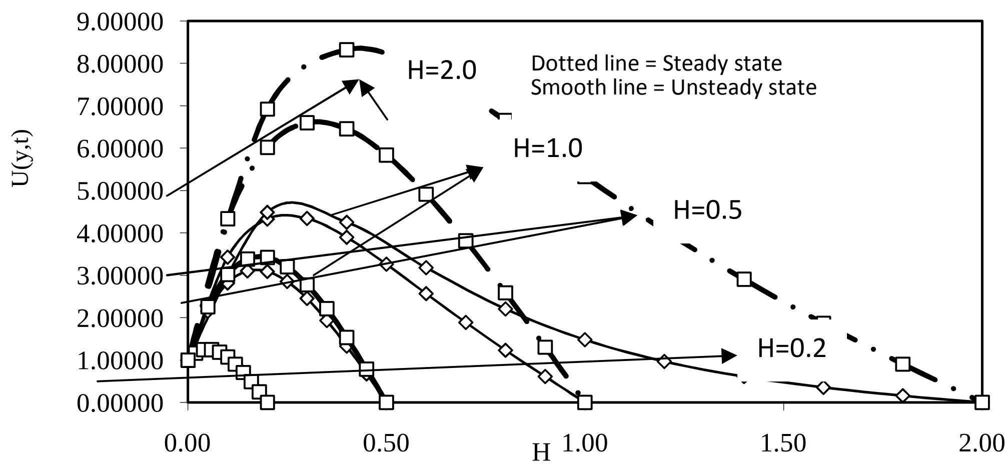

Fig. 3.0 Velocity profile with different H, $t=0.003$, $Pr = 0.044$, $Da = 0.05$, $\gamma = 1.5$ and $S=1.5$

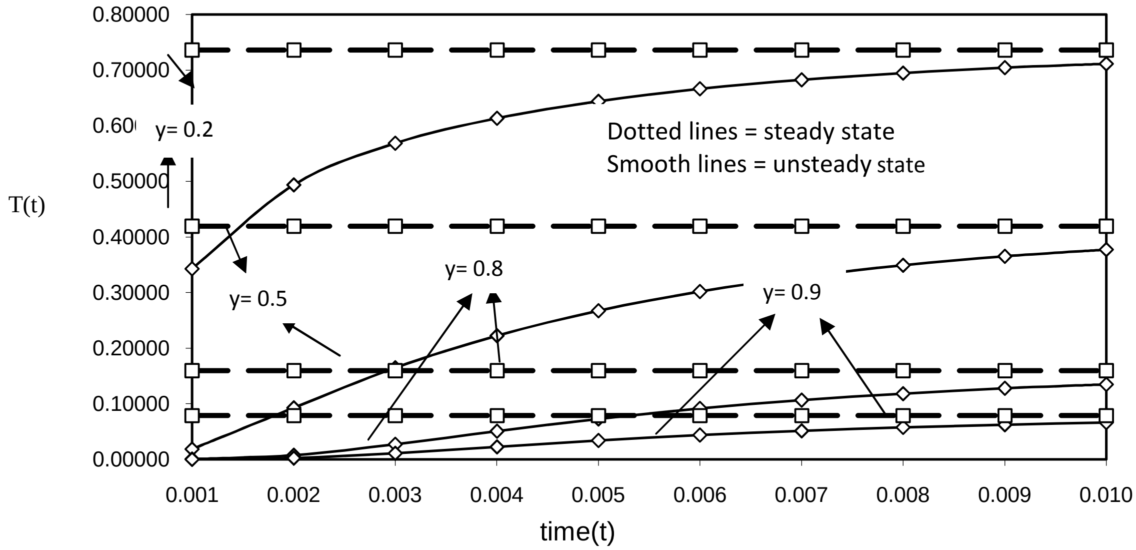

Fig. 4.0 Temperature profiles at different fluid sections with time (t),*Pr = 0.044,S=1.5 and H= 1.0*

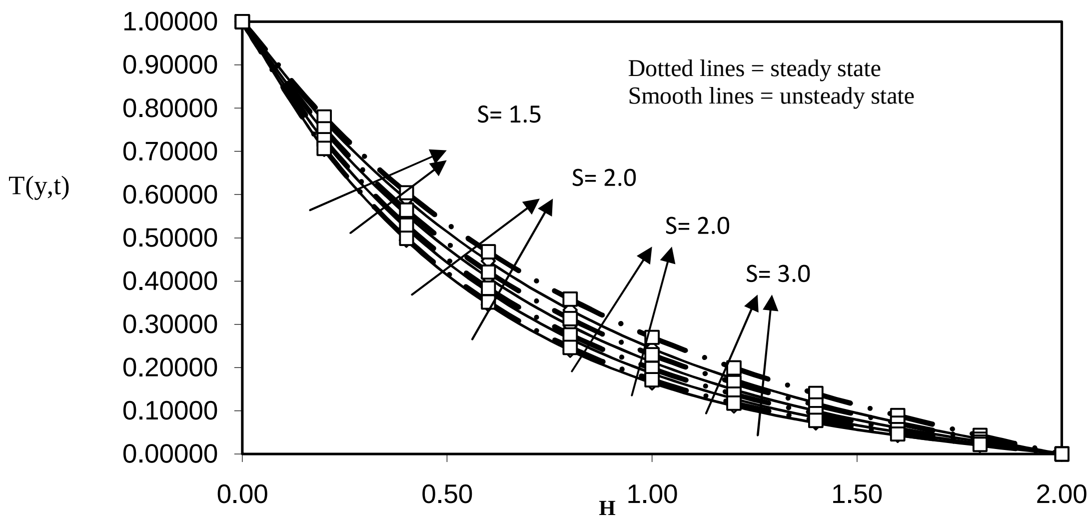

Fig. 6.0 Temperature profile with different S and t=0.003, H=2.0,Pr=0.044

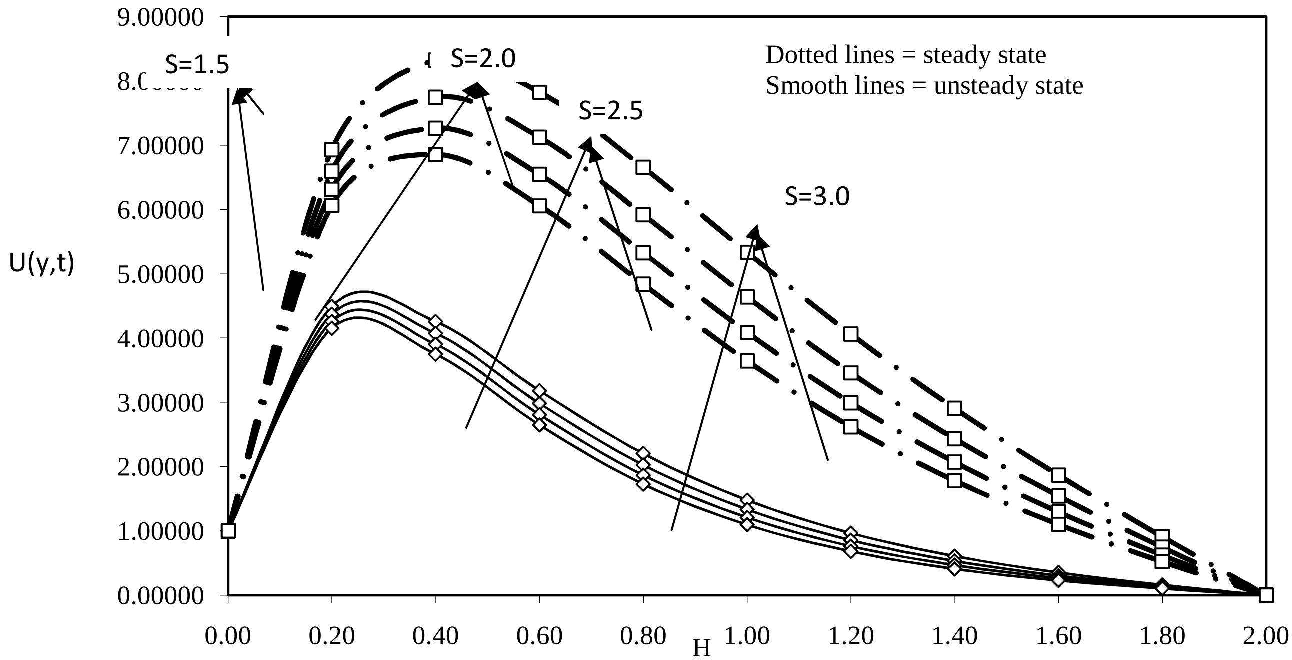

Fig.7.0 Velocity profile at different *S*, *t=0.03, Pr = 0.044, Da = 0.05, γ= 1.5 Gr = 385 and H = 2.0*

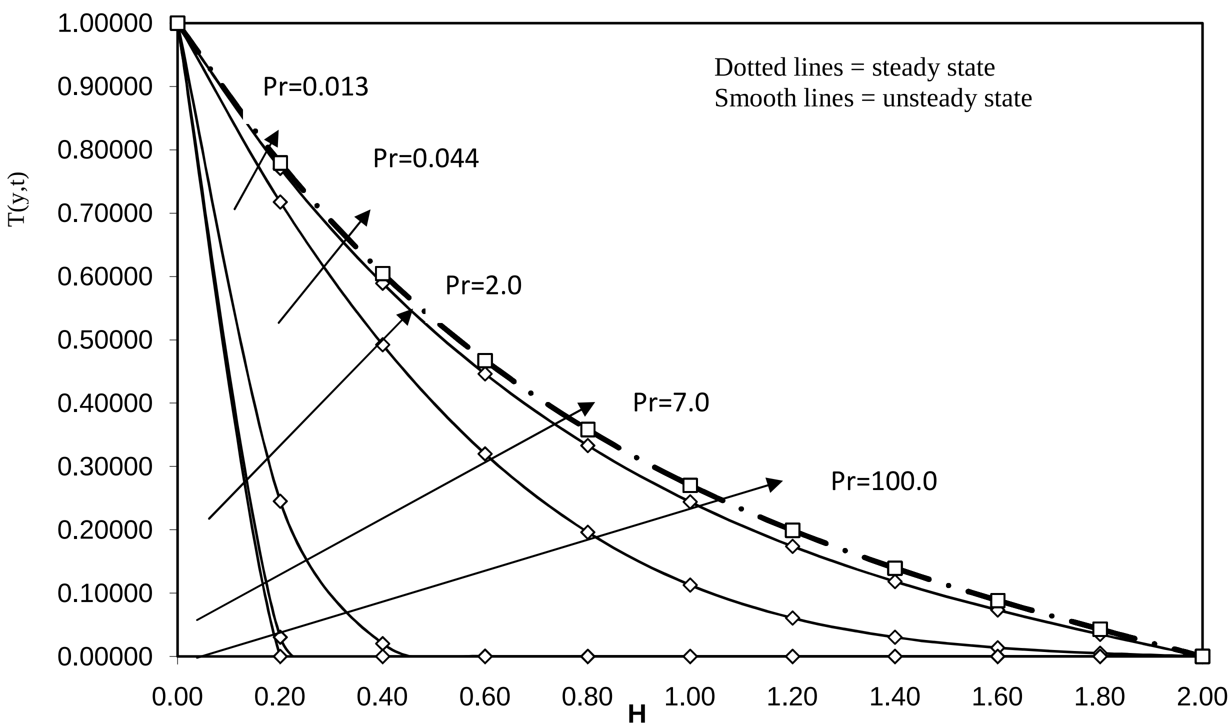

Fig. 8.0 Temperature profile with different Pr , t=0.003,S=1.5 and H = 2.0

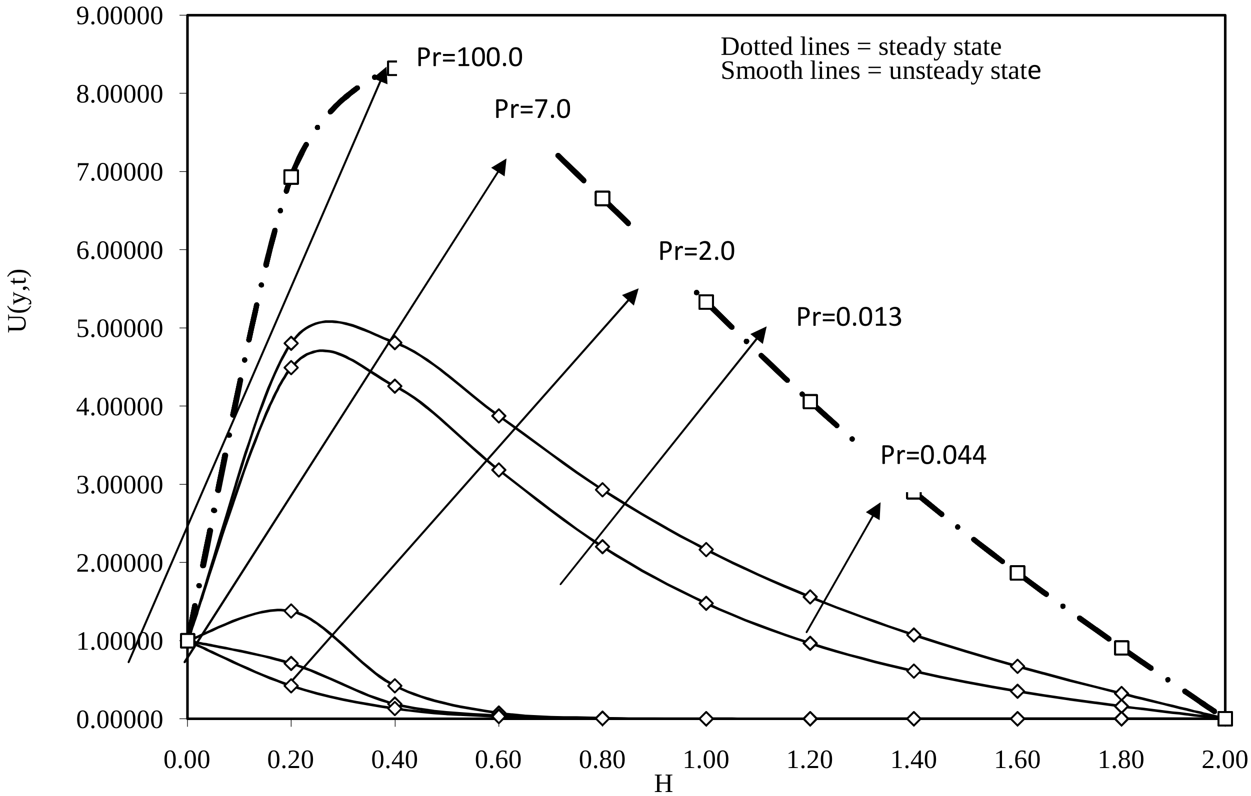

Fig. 9.0 Velocity profile with different Pr and t=0.003,S=1.5,γ=1.5,Da=0.05, Gr=385.0

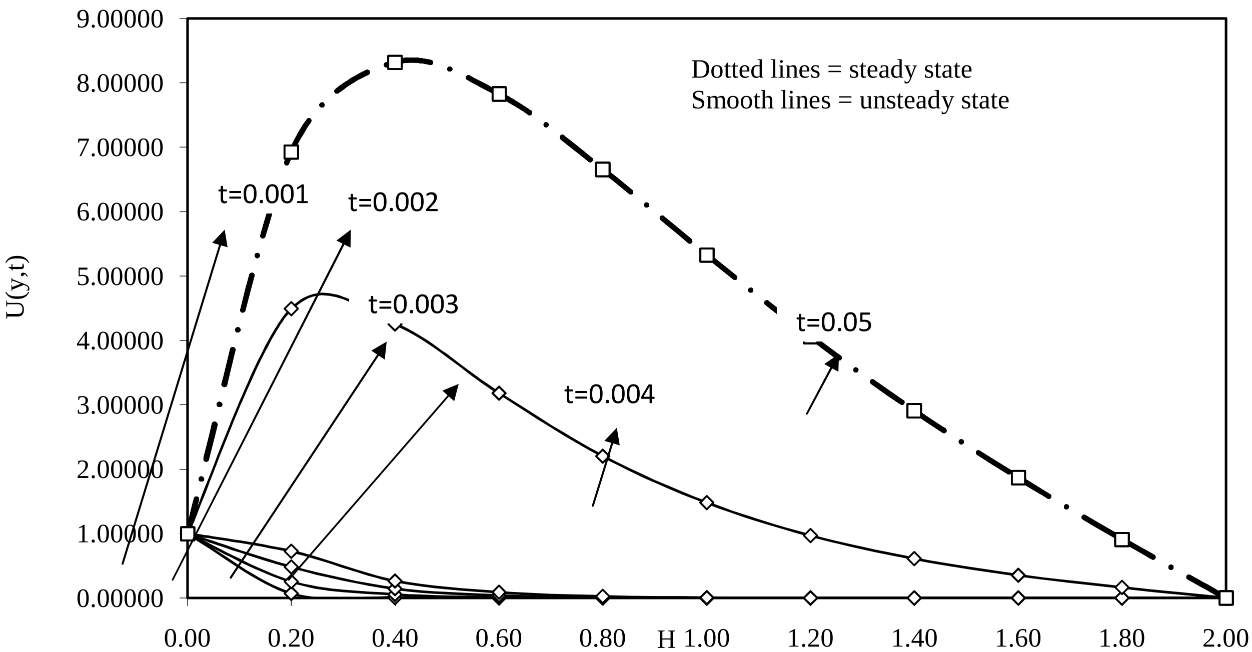

Fig. 10.0 Velocity profile with different t, Pr =0.044, S=1.5,γ=1.5,Da=0.05, Gr=385.0 and H = 2.0

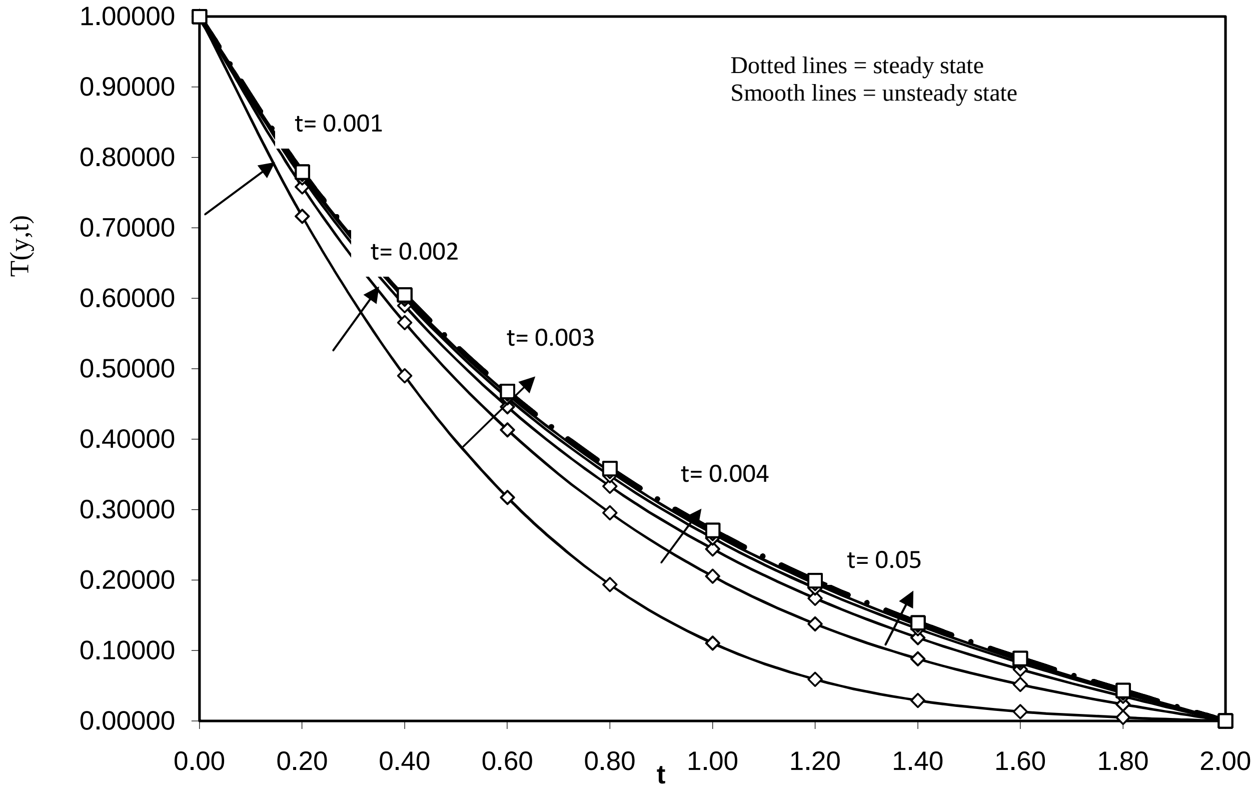

Fig. 11.0 Temperature profile with different t, Pr 0.044, S=1.5 and H = 2.0

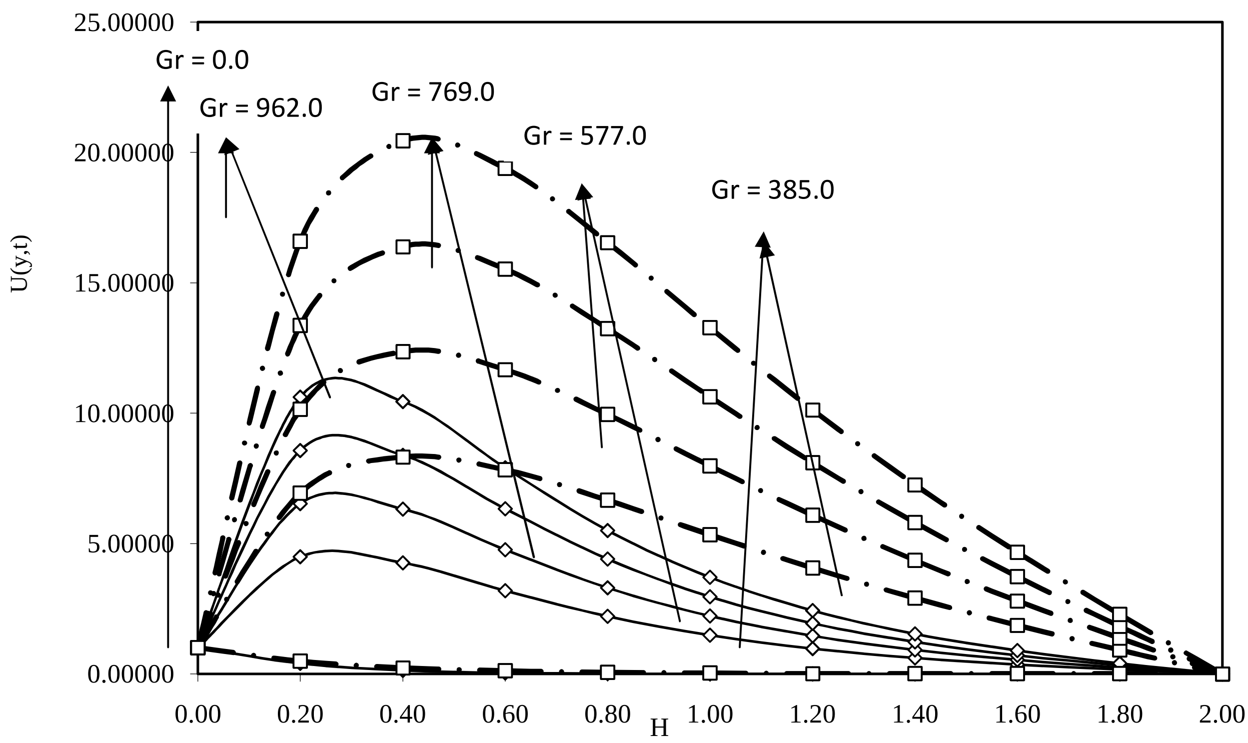

Fig.12.0 Velocity profile with different Gr, t = 0.003, Pr = 0.044, S=1.5,γ=1.5,Da=0.05, and H = 2.0

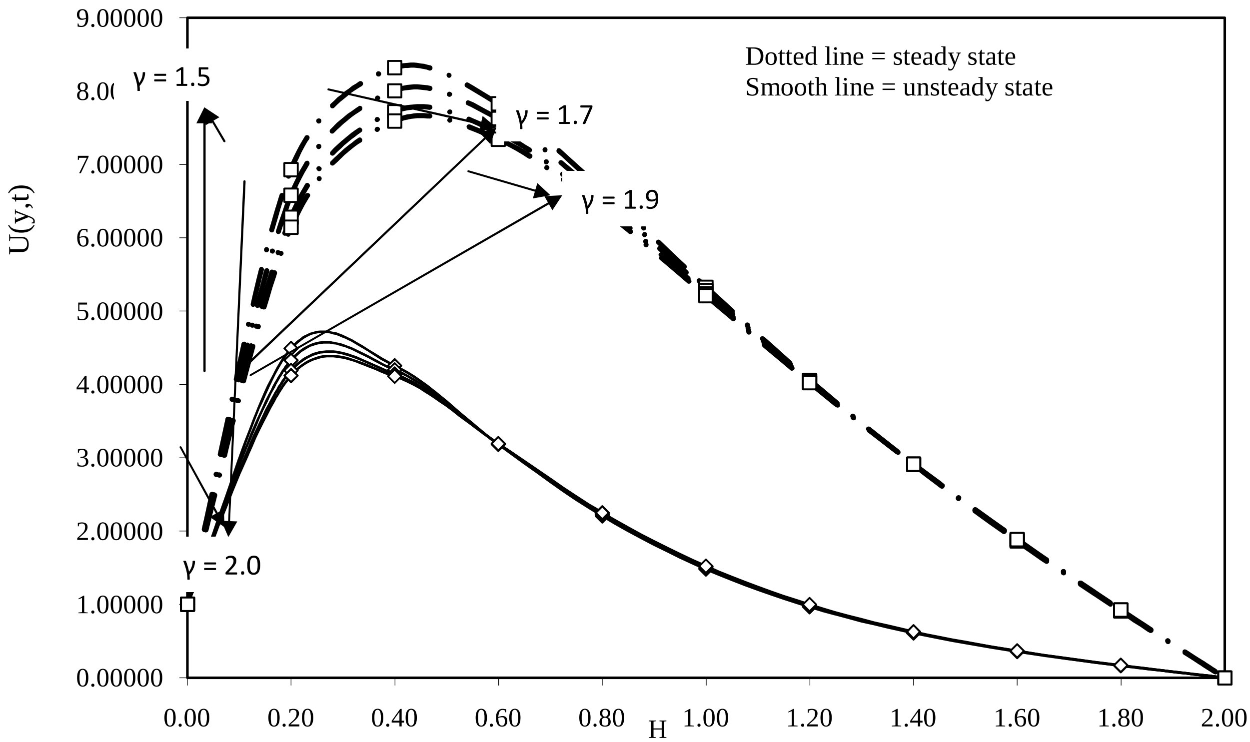

Fig. 13.0 Velocity profile with different γ, t = 0.003, Pr = 0.044, S=1.5, ,Da=0.05, Gr =385.0 and H = 2.0

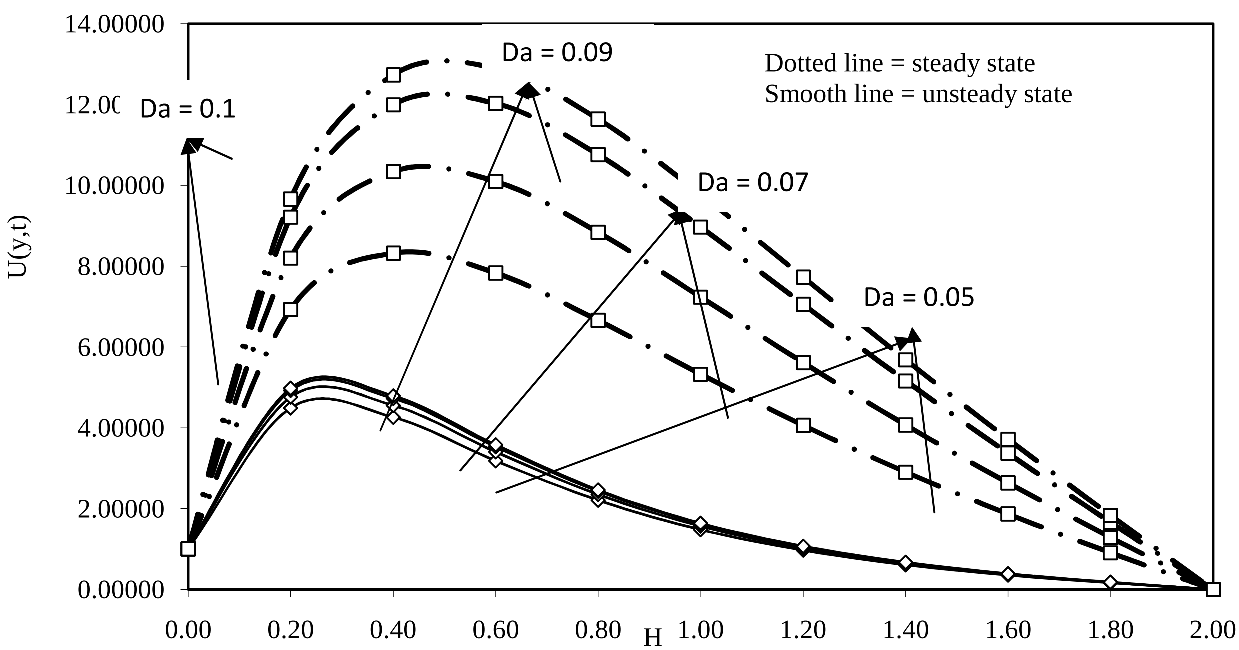

Fig. 14.0 Velocity profile with different Da and t=0.003,S=1.5,γ=1.5, Pr=0.044, Gr=385.0 and H=2.0

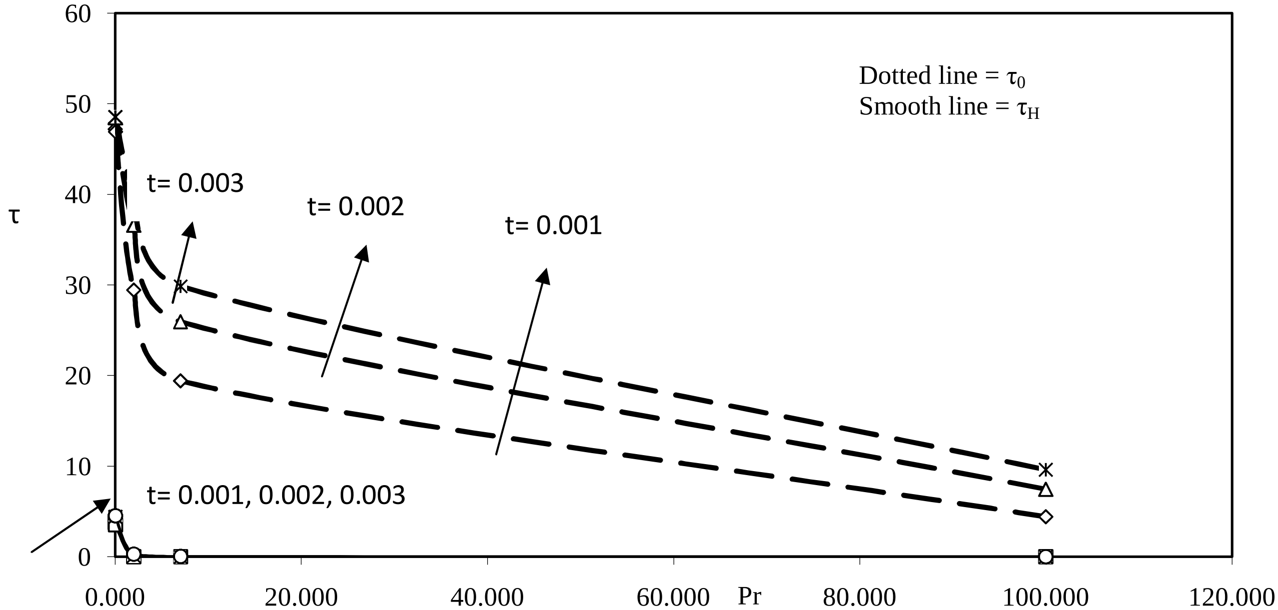

Fig. 15.0 Skin friction with different Pr, t and Gr =385.0,S=1.5,Da=0.05,γ=1.5 and H=2.0

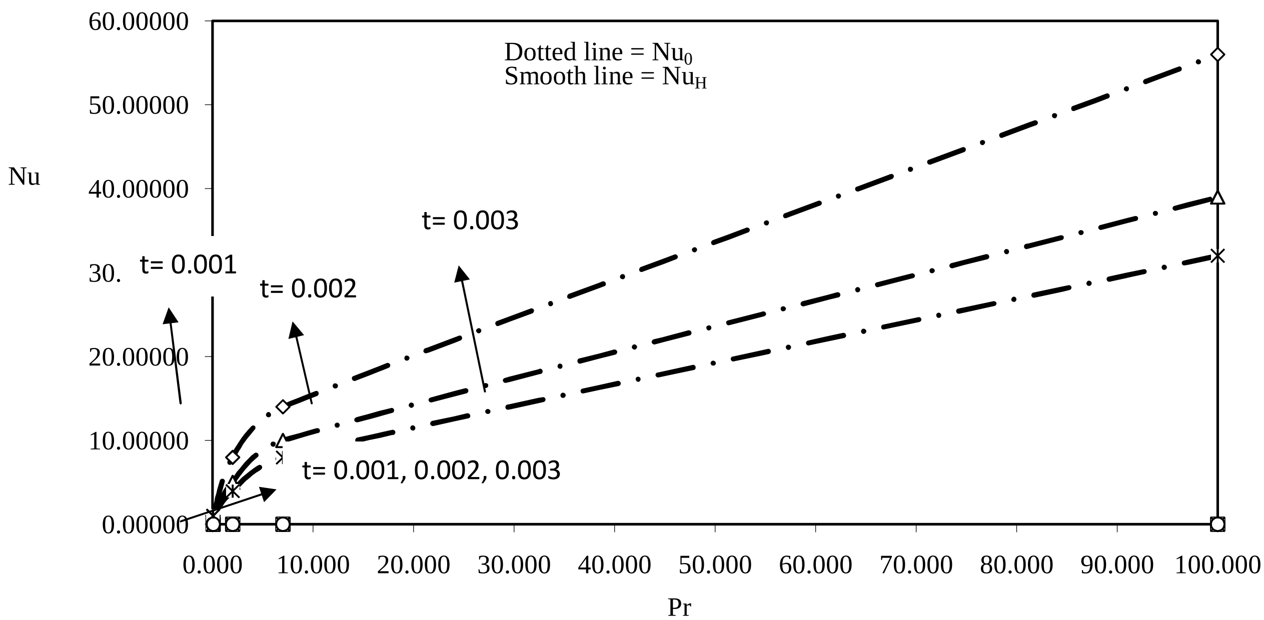

Fig. 16.0 Rate of heat transfer with different Pr, t, S=1.5 and H=2.0

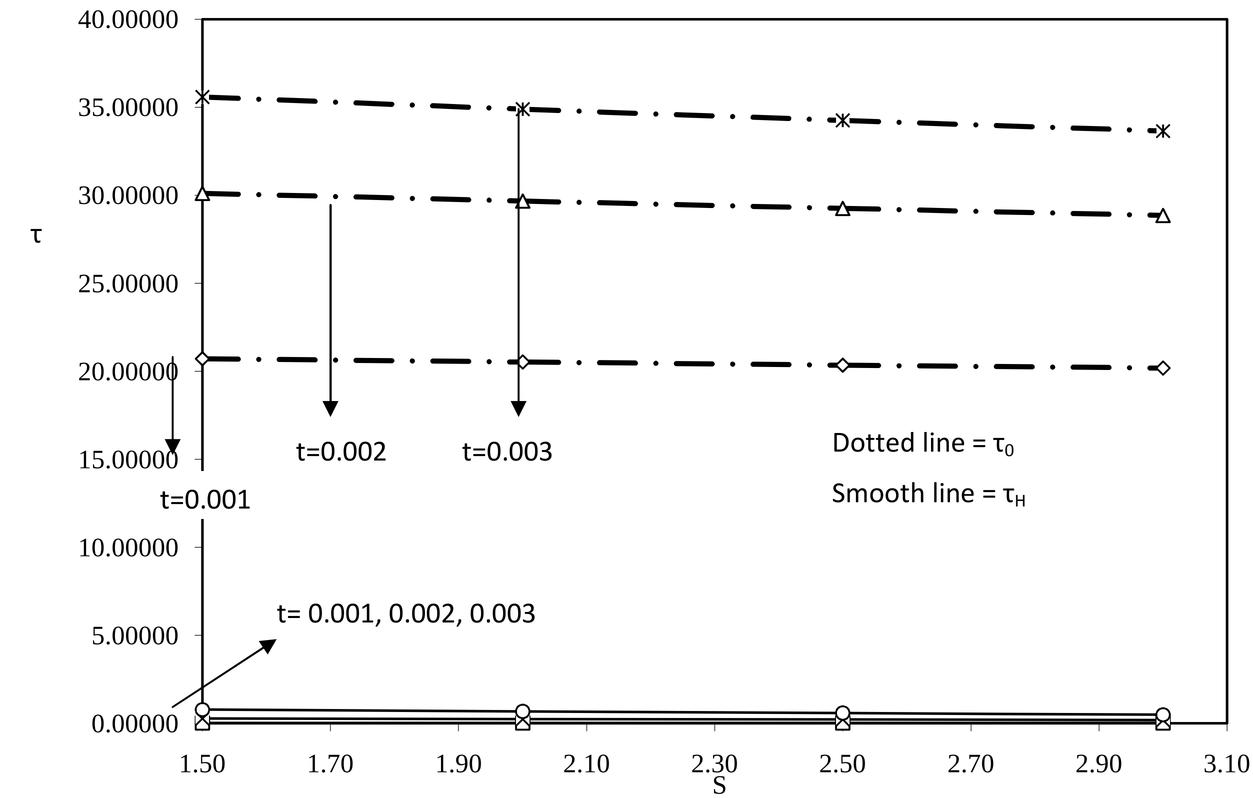

Fig. 17.0 Skin friction with different Pr, t and Gr =385.0,S=1.5,Da=0.05,γ=1.5 and H=2.0

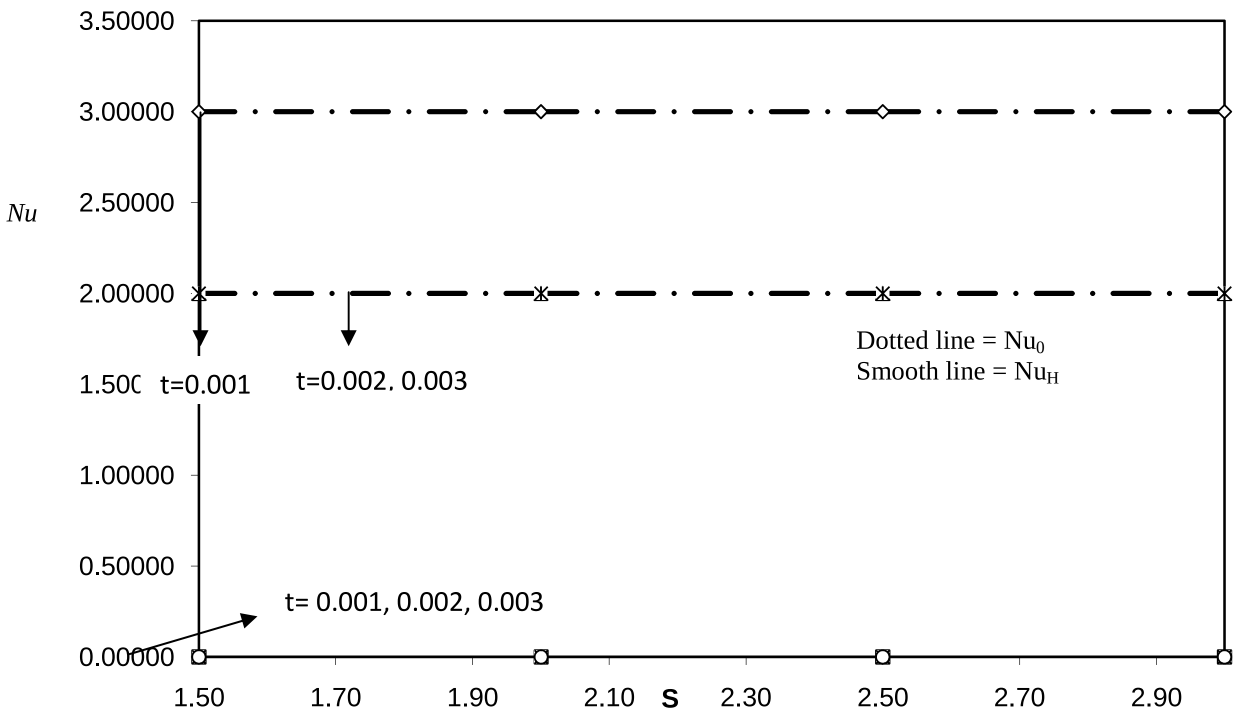

Fig. 18.0 Rate of heat transfer with different S, t and Pr=0.044, H=2.0

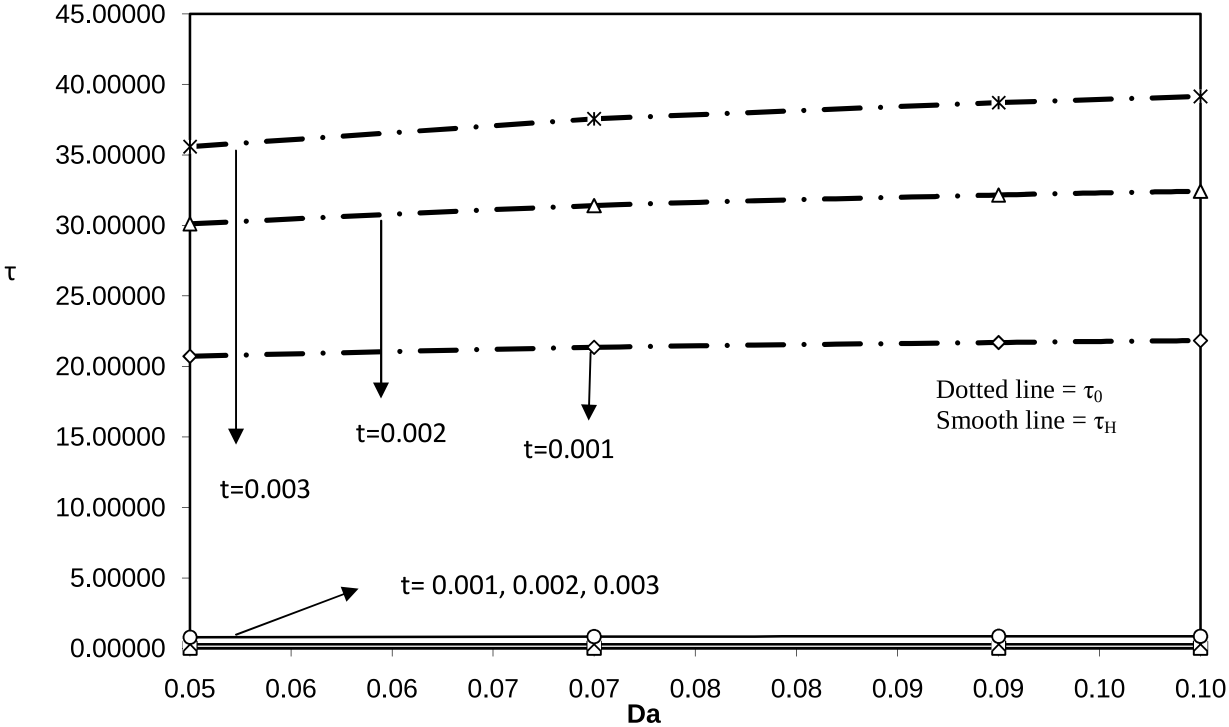

Fig. 19.0 Skin friction with different Da, t and Gr =385.0,S=1.5,Pr=0.044,γ=1.5 and H=2.0

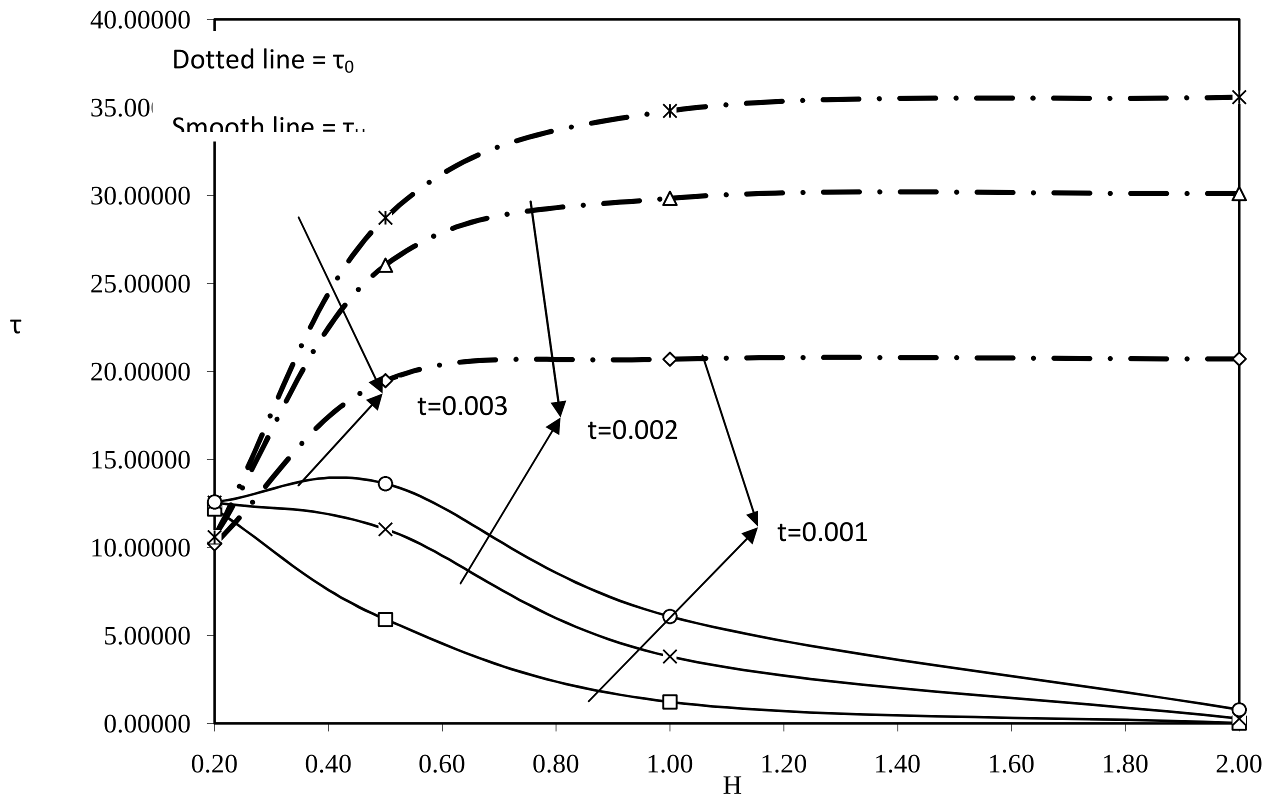

Fig. 20.0 Skin friction with different H, t and Gr =385.0,S=1.5,Pr=0.044,γ=1.5 and Da=0.05

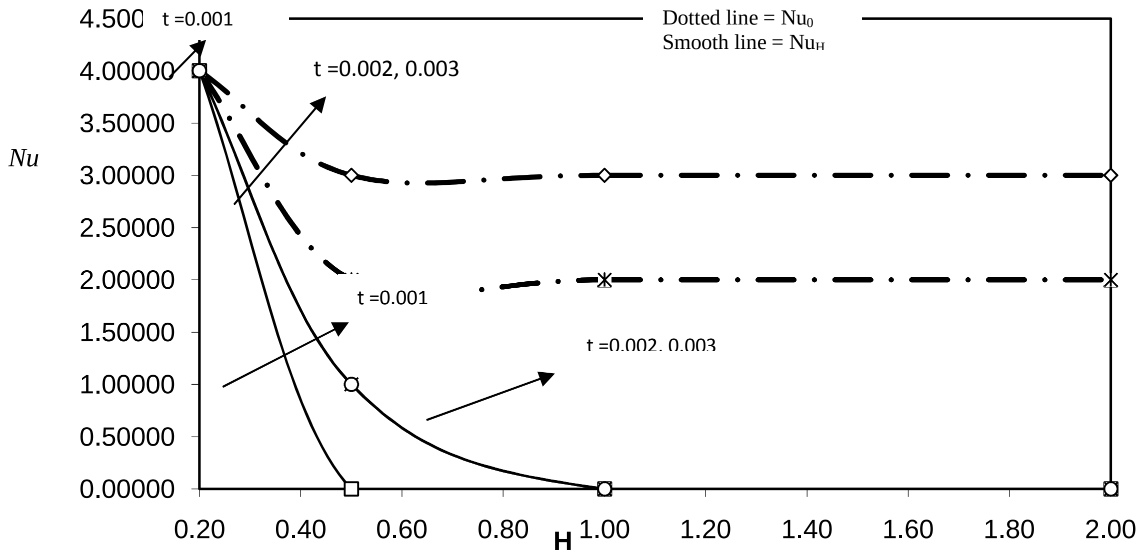

Fig. 21.0 Rate of heat transfer with different H, t and Pr=0.044, S = 1.5

5.0 CONCLUSION

From the solutions of the mathematical model capturing the problem, the emerging parameters are Da, Pr, S, γ, H and Gr. Analysis of the effect of the variation of these parameters as presented graphically leads to the following conclusions:

(1)The fluid velocity is increased by increasing Da and Gr but decreases with $\gamma, S\ and\ Pr$.

(2) Also the fluid motion as well as the temperature of the fluid can be suppressed or improved by decreasing or increasing the distance H between the plates respectively.

(3) The rate of heat transfer decreases with $H\ and\ Pr$ but increase with S on the heated plate.

(4) The absence of the convection current ($Gr = 0$) does not result to flow stagnation. The equation representing such flow has been obtained.

Appendix I

NOMENCLATURE

C_p	specific heat at constant pressure
Da	Darcy number
Gr	Grashof number
g	acceleration due to gravity
H	distance between the plates
K	Permeability of the porous media
k	thermal conductivity
Nu	Nusselt number
Pr	Prandtl number
S	dimensionless heat generating/absorbing parameter
T	dimensionless temperature of the fluid
t	dimensionless time
T'	temperature of the fluid
T_w	temperature of the plate at $y' = 0$
T_h	temperature of the plate at $y' = h$
t'	dimensional time
U'	dimensional velocity of the fluid
U	dimensionless velocity of the fluid
U_0	a constant with the dimension of velocity
Q_o	dimensional heat generating/absorbing parameter

Greek symbols

β	coefficient of thermal expansion

τ skin friction
υ_{eff} effective viscosity of the saturated porous media
υ kinematic viscosity of the fluid
γ ratio of viscosities

Appendix II

(a) Table 1 showing the values of Grashof number (Gr)

$T_w - T_h$	$Gr = \frac{g\beta(T_w - T_h)\upsilon}{U_0^3}$
0°C	0.0
100°C	385.0
150°C	577.0
200°C	769.0
250oC	962.0

(b) List of constants used in the work

$a = 2mH + y, b = 2mH + 2H - y,$ $c = \frac{s}{Pr}, c_1 = \gamma Pr - 1, c_2 = \frac{1 - srDa}{C_1 Da}$

$c_3 = 2mH, c_4 = 2mH + 2H, c_5 = 2mH + H, m_1 = t + \frac{a\sqrt{Da}}{\sqrt{t}}$

$m_2 = t - \frac{a\sqrt{Da}}{\sqrt{t}}$, $m_3 = \frac{1}{\sqrt{\gamma\pi t}}, m_4 = \frac{\sqrt{Pr}}{\sqrt{t\pi}}, m_5 = \frac{1}{\sqrt{\pi Da}}, m_6 = \frac{\sqrt{Da}}{2\sqrt{t}}, m_7 = \sqrt{s}$

$m_8 = \exp(c_2 t), m_9 = \sqrt{\frac{1}{\gamma}(c_2 + \frac{1}{Da})}, m_{10} = \sqrt{\Pr(c_2 + c)}, \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-z^2}dz$

(c) Functionals used to define velocity, temperature, skin friction and rate of heat transfer

$f_1(d_1, d_2, d_3, d_4, d_5) = \exp(d_1\sqrt{d_5})\, erfc(\frac{d_1}{2}\sqrt{\frac{d_3}{d_2}} + \sqrt{d_2 d_4}) +$

$\exp(-d_1\sqrt{d_5})\, erfc(\frac{d_1}{2}\sqrt{\frac{d_3}{d_2}} - \sqrt{d_2 d_4})$

$f_2(d_1, d_2, d_3, d_4, d_5) = \exp(d_2 d_3)\, [\exp\left(d_1\sqrt{\frac{1}{d_4}(d_3 + \frac{1}{d_5}}\right) erfc(\frac{d1}{2\sqrt{d_4 d_2}} + \sqrt{d_2(d_3 + \frac{1}{d_5}}) +$

$\exp\left(-d_1\sqrt{\frac{1}{d_4}(d_3 + \frac{1}{d_5}}\right) erfc(\frac{d1}{2\sqrt{d_4 d_2}} - \sqrt{d_2(d_3 + \frac{1}{d_5}})$

$f_3(d_1, d_2, d_3, d_4, d_5) = \exp(d_2 d_3)\, [\exp\left(d_1\sqrt{d_5(d_3 + d_4}\right) erfc(\frac{d1}{2}\sqrt{\frac{d_5}{d_2}} + \sqrt{d_2 d_5(d_3 + d_4}) +$

$\exp\left(-d_1\sqrt{d_5(d_3 + d_4}\right) erfc(\frac{d1}{2}\sqrt{\frac{d_5}{d_2}} - \sqrt{d_2 d_5(d_3 + d_4})$

$f_4(d_1, d_2, d_3, d_4) = \exp\left(\frac{d_1}{\sqrt{d_3 d_4}}\right) erfc(\frac{d_1}{2\sqrt{d_2 d_3}} + \sqrt{\frac{d_2}{d_4}}) + \exp\left(-\frac{d_1}{\sqrt{d_3 d_4}}\right) erfc(\frac{d_1}{2\sqrt{d_2 d_3}} - \sqrt{\frac{d_2}{d_4}})$

$f_5(d_1, d_2, d_3, d_4, d_5) = \exp\,(\frac{d_1}{\sqrt{d_3(d_4 + d_5)}} - \left(\frac{d_1}{2\sqrt{d_2 d_3}} + \sqrt{(d_5 + \frac{1}{d_4})d_2)^2}\right) +$

$\exp\,(-\frac{d_1}{\sqrt{d_3(d_4 + d_5)}} - \left(\frac{d_1}{2\sqrt{d_2 d_3}} - \sqrt{(d_5 + \frac{1}{d_4})d_2)^2}\right)$

$f_6(d_1, d_2, d_3, d_4, d_5) = \exp\,(d_1\sqrt{d_3(d_4 + d_5}) - \left(\frac{d_1\sqrt{d_3}}{2\sqrt{d_2}} + \sqrt{(d_5 + d_4})d_2)^2\right) +$

$\exp\,(-d_1\sqrt{d_3(d_4 + d_5}) - \left(\frac{d_1\sqrt{d_3}}{2\sqrt{d_2}} - \sqrt{(d_5 + d_4})d_2)^2\right)$

$f_7(d_1, d_2, d_3, d_4) = \exp\,(\frac{d_1}{\sqrt{d_3 d_4}} - \left(\frac{d_1}{2\sqrt{d_2 d_3}} + \sqrt{\frac{d_2}{d_4}})\right)^2\right) + \exp\,(-\frac{d_1}{\sqrt{d_3 d_4}} - \left(\frac{d_1}{2\sqrt{d_2 d_3}} - \sqrt{\frac{d_2}{d_4}})\right)^2\right)$

$f_8(d_1, d_2, d_3, d_4) = \exp\left(\frac{d_1}{\sqrt{d_3 d_4}}\right) erfc\left(\frac{d_1}{2\sqrt{d_2 d_3}} + \sqrt{\frac{d_2}{d_4}}\right) - \exp\left(-\frac{d_1}{\sqrt{d_3 d_4}}\right) erfc\left(\frac{d_1}{2\sqrt{d_2 d_3}} - \sqrt{\frac{d_2}{d_4}}\right)$

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