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# The Unsteady Variable – Viscosity Free Convection Flow on a Porous Plate

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***Abstract***

***The unsteady variable-viscosity free convection flow of a viscous incompressible fluid near an infinite vertical plate (or wall) is investigated under an arbitrary time-dependent heating of the plates, and the governing equations of motion and energy transformed into ordinary differential equations. Employing asymptotic techniques, the resulting equations are solved numerically. It is shown that the velocity decreases as the ratio of the kinematic viscosity to the initial temperature increases.***

## 1.0 Introduction

The importance of studying natural convection in vertical channels arises from many engineering applications such as cooling of electric and electronic equipment, nuclear reactor, fuel elements, home ventilation and many others [1]. Some of the industrial applications in which natural convection in vertical channels formed by parallel plates receives significant attention include solar collections, fire research, aeronautics, chemical apparatus, building construction, etc.

Natural convection occurs in vertical parallel plate channels when at least one of the resulting buoyancy driven flow can be turbulent depending on the channel geometry, fluid properties and temperature difference between the plates and the ambient extensively presented the steady laminar free-convection flow of a viscous incompressible fluid between the vertical walls.

Our interest in this paper is to show how the fluid velocity varies with respect to the kinematic viscosity to the initial temperature, and our model is taken from the work of C.J. Toki et al [2] and A.C. Loyinmi [3].

## 2.0 Mathematical formulation

The free-convection flow is two-dimensional and it is considered with the coordinate origin at an arbitrary point on an infinite, porous limiting vertical plate or wall. The $x'$-axis is along the plate and in the upward direction and the $y'$-axis normal towards it. The fluid is viscous and incompressible. The flow is induced either by the motion of the plate or by heating it or both.

The plate is at rest with a constant temperature $T_\infty$, and it is suddenly moved with a constant velocity $u_\circ$. Its temperature is instantaneously increased (or decreased) by the quantity $\alpha\left(T'_w - T'_\infty\right)$ for $t \succ 0$, $T'_w$ $\left(\neq T_\infty\right)$ a constant temperature of the plate.

On the physical grounds of the present problem, all the quantities are assumed to be functions of the space coordinate $y'$ and $t'$, so that the vector of the velocity is given by ( $u', v', 0$ ).

Then the equation of continuity, on integration, gives $v' = cons\tan t = v'_\circ$ (say), where $v'_\circ$ is the normal velocity of suction or injection at the wall according as $v'_\circ \prec 0$ or $v'_\circ \succ 0$ respectively. $v'_\circ = 0$ represents the case of a non-permeable wall.

The corresponding equation of motion and energy for this case are respectively;

$$\rho\left[\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right] = \frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) + \frac{\rho g \beta'}{t}\left(T' - T'_\infty\right) \tag{1}$$

$$\frac{\partial T'}{\partial t'} + v'_\circ\frac{\partial T'}{\partial y'} = \frac{k}{\ell Cp}\cdot\frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho Cp}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

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where $\rho$ denotes the fluid density, $T'$ the temperature, g the acceleration due to gravity, $\beta'$ the coefficient of volume expansion, $k$ the thermal conductivity; and $Cp$ the specific heat at constant pressure.

We assume that the viscosity $\mu = \mu_\circ \left( \frac{T' - T'_\infty}{T'_w - T'_\infty} \right)^n$, where $n$ is a positive number.

We introduce the following non-dimensional variables and parameters;

$$t = \frac{t' u^2_\circ}{\upsilon}, \qquad y = \frac{y' u_\circ}{\upsilon}, \qquad v_\circ = \frac{v'_\circ}{u_\circ}, \tag{3}$$

Non-dimensional temperature, $\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$ (4)

Prandtl number, $P = \frac{\mu Cp}{k}$ (5)

Grashof number, $G = \frac{\upsilon g \beta' \left( T'_w - T''_\infty \right)}{u^3_\circ}$ (6)

The corresponding initial and boundary conditions of the system (1) and (2) after being non-dimensionalized are:

$$\theta(y,0) = 0, \ \theta(0,t) = \alpha, \qquad \theta(\infty,0) = 0 \tag{7}$$

$$u(y,0) = 0, \ u(0,t) = 0, \qquad u(\infty,0) = 0 \tag{8}$$

The boundary conditions (7) and (8) are solved asymptotically as

$$\theta = \theta_\circ + a\theta_1 + a^2\theta_2 + a^3\theta_3 + \ldots \quad \text{and} \quad u = u_\circ + au_1 + a^2u_2 + a^3u_3 + \ldots \tag{9}$$

Non – dimensionalization of (1) and (2) according to (3), (4), (5) and (6) gives;

$$\frac{\partial u}{\partial t} + v_\circ \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left( \theta^n \frac{\partial u}{\partial y} \right) + \frac{G}{t}\theta \tag{10}$$

$$\frac{\partial \theta}{\partial t} + v_\circ \frac{\partial \theta}{\partial y} = \frac{1}{P} \cdot \frac{\partial^2 \theta}{\partial y^2} + a\theta^n \left( \frac{\partial u}{\partial y} \right)^2 \tag{11}$$

where $a = \frac{u^2}{Cp\left( T'_w - T'_\infty \right)}$

Solving (10) and (11) using the similarity variable $\eta = \frac{y}{2\sqrt{vt}}$ where $u_\circ = u(\eta)$ and $\theta_\circ = \theta(\eta)$

Where $\frac{\partial}{\partial t} = \frac{\partial \eta}{\partial t} . \frac{d}{d\eta} = \frac{\eta}{2t} . \frac{d}{d\eta}$ $\quad \frac{\partial}{\partial y} = \frac{\partial \eta}{\partial y} . \frac{d}{d\eta} = \frac{1}{2\sqrt{vt}} . \frac{d}{d\eta}$ $\quad \frac{\partial^2}{\partial y^2} = \frac{1}{4vt} . \frac{d^2}{d\eta^2}$,

we have

$$-\frac{\eta}{2} . \frac{du_o}{d\eta} = \frac{1}{4v} . \frac{d}{d\eta}\left( \theta_o^n \frac{du_o}{d\eta} \right) + G\theta_o \tag{12}$$

$$\frac{d^2\theta_o}{d\eta^2} + 2vp\eta \frac{d\theta_o}{d\eta} = 0 \tag{13}$$

On integrating (13) twice, we have $\theta(\eta) = A\int \ell^{-vp\eta^2} d\eta + k_2$ (14)

And on applying (8) to (14) we have $\theta(\eta) = aerf_c(\eta)$ (15)

Putting (15) into (12) simply gives;

$$\frac{d^2u_o}{d\eta^2} + \frac{2\eta v}{\alpha^n}\left( erf_c(\eta) \right)^{-1} \frac{du_o}{d\eta} = \frac{2n}{\sqrt{\pi}}\left( erf_c(\eta) \right)^{-1} \ell^{-n^2} - 4v\alpha^{1-n}\left( erf_c(\eta) \right)^{1-n} \tag{16)}$$

which is of the form

$$\frac{d^2u_o}{d\eta^2} + P(\eta)\frac{du_o}{d\eta} + Q(\eta) = f(\eta), \;\; u(0) = 1, \;\; u(1) = 0 \tag{17}$$

Equation (14) is an error function whose points are plotted as shown in Figure1.
Equation (17) is solved numerically using the forward difference method with
$u_o 1 = \alpha = 1$, and the computation domain is divided into meshes of dimension $\Delta\eta = 0.1$ and the result shown in Figure2.

## 3. Conclusion

The velocity of the fluid decreases as $\beta$, the ratio of the Kinematic viscosity to energy of the fluid decreases, and the energy decreases as $\eta = (y, \upsilon, t)$ increases.

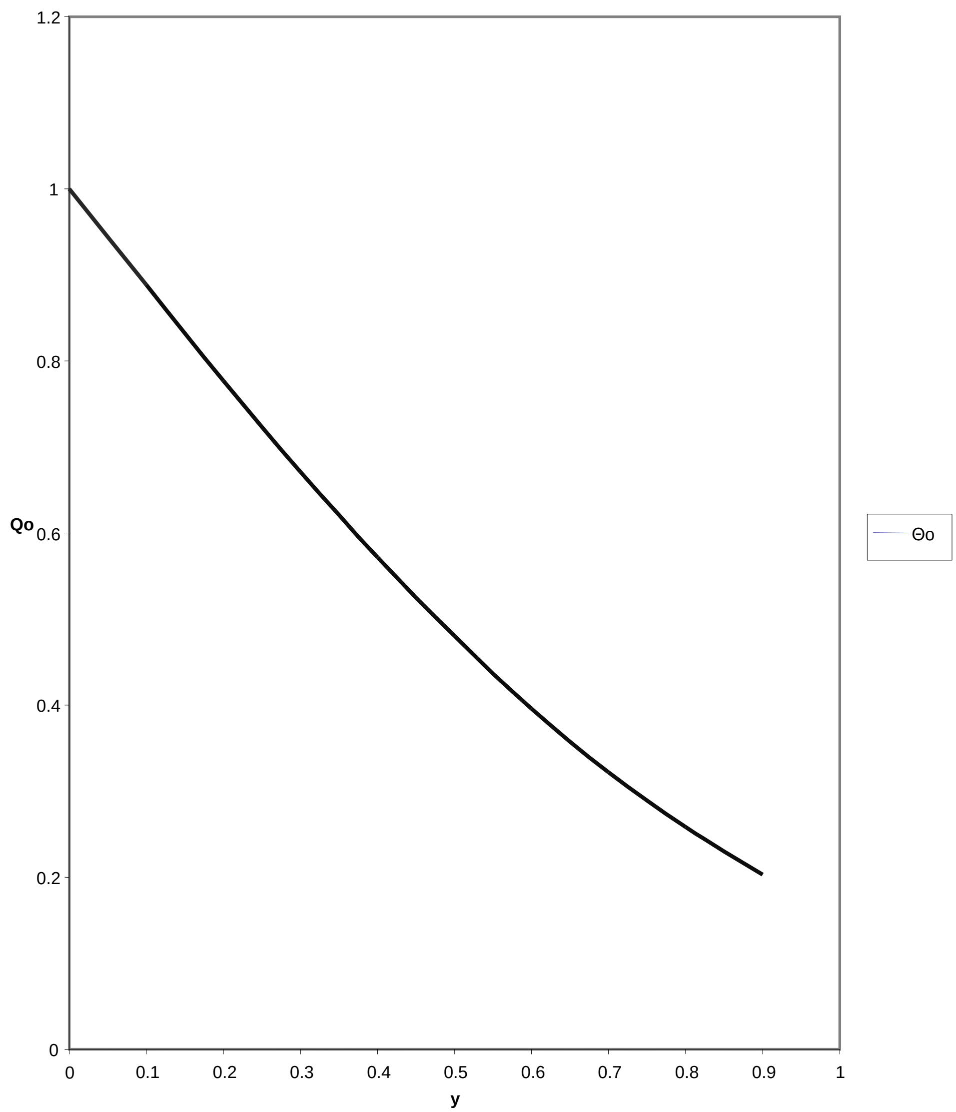


**Fig.1:** The graph of $\theta_0$ against y for the unsteady case

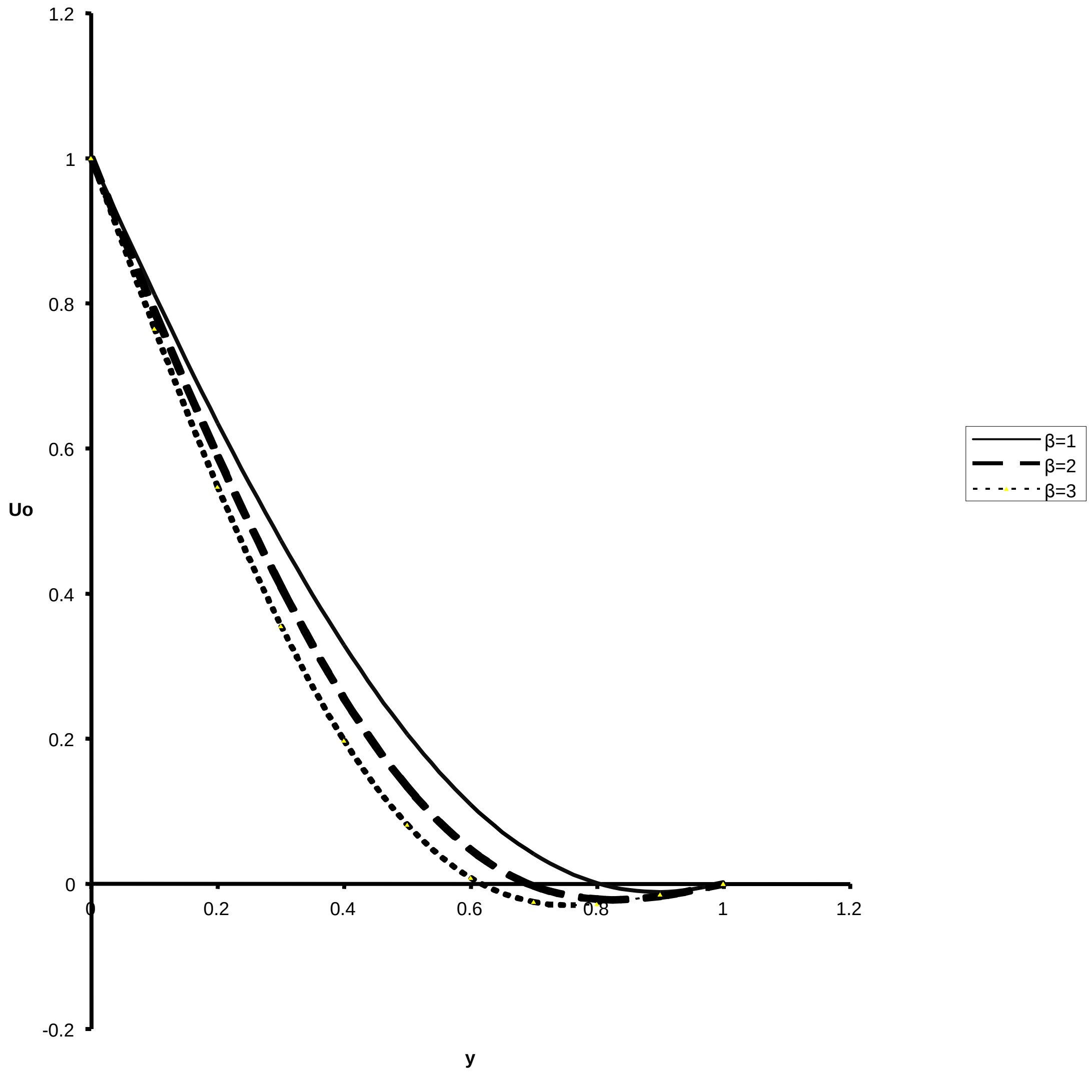


**Figure2:** The graph of $U_o$ against y for the unsteady case

## References.


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