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Analytical Solution of Unsteady Gravity Flows of A Power-Law Fluid through A Porous Medium

***Peter B. A. and Ayeni R. O.**

Department of Mathematical Sciences,
Olabisi Onabanjo University, Ago – Iwoye.

Abstract

We present an analytical study of unsteady non-linear rheological effects of a power-law fluid under gravity. The fluid flows through a porous medium. The governing equations are derived and similarity solutions are determined. The results show the existence of traveling waves. It is assumed that the viscosity is temperature dependent. We investigate the effects of velocity on the temperature field. We investigate the power-law viscosity index, the Darcy parameter on the temperature profiles and the results were discussed.

Keywords: Unsteady gravity flows; Porous media; Non – Newtonian power- law fluid

1.0 Introduction

A particular difficulty of the flow in porous media is that it arises, often in subtly different forms, in several separate fields of natural science and in large number of branches of technology. The categories are far from clear-cut,but and enumeration must include fluid mechanics surface and colloid physical chemistry, physiology, rheology, soil physics, hydrology, soil mechanics, agricultural engineering, petroleum engine and chemical engine. Some scientists have studied gravity flows of a power-law fluid through a porous medium. These include, [1] investigated the effect of permeability on the temperature rise in a reacting porous medium. Cortell [5] presented a paper on unsteady gravity flows of a power-law fluid through a porous medium. Olajuwon and Ayeni [2] presented a note on the flow of a power-law fluid with memory past an infinite plate. Pascal and Pascal [3] also studied similarity solution to some gravity flows of non- Newtonian fluids through a porous media. Zueco [4] investigated the numerical solutions for unsteady rotating high-porosity medium.

2.0. Mathematical Formulation

The governing equations for the Mathematical formulation are continuity and momentum equation. Considering a two dimensional flow in the x – z plane where the free surface is a streamline at a point on the surface, we expressed the flow by a modified Darcy's law.

$$V=\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}}\frac{\partial h}{\partial s}\left|\frac{\partial h}{\partial s}\right|^{\frac{1-n}{n}} \tag{2.1}$$

Where S is measured along the streamline, since z = h on the free surface. The rheological parameter n is the power-law exponent which represents shear-thinning (i.e.n<1) and shear-thickening (i.e. n>1) fluids. k is the permeability, ρ is the density and μ_{ef} is the effective viscosity.

$$\frac{\partial h}{\partial s}\cong\frac{\partial h}{\partial x}$$

For small gradients which converts the problem into a one – dimensional problem. This approximation permits to assume a horizontal flow with $h=h(x,t)$ (t being the time) and equation (2.1) becomes

*Corresponding authors: B. A. Peter, E-mail Pbenjamin2008@yahoo.ca, Tel. +234 (0) 8056682388 / +234(0) 8060207359

$$v_x = -\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial x}\left|\frac{\partial h}{\partial x}\right|^{\frac{1-n}{n}} \qquad (2.2)$$

Whereas for radial axisymmetric flow

$$V_R = -\left(\frac{K\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial R}\left|\frac{\partial h}{\partial R}\right|^{\frac{1-n}{n}} \qquad (2.3)$$

Substituting equation (2.3) into the continuity equations we obtain

$$\frac{\partial(hv_R)}{\partial R} = -\Phi\frac{\partial h}{\partial t} \qquad (2.4)$$

Where Φ being the porosity

By cylindrical coordinate we obtain

$$\frac{1}{R}\frac{\partial}{\partial R}\frac{\partial h}{\partial R}\left(\left|\frac{\partial h}{\partial R}\right|^{\frac{1-n}{n}}\right) = \Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}\frac{\partial h}{\partial t} \qquad (2.5)$$

(Momentum)

Defining new variables

Let h (R, t) = $t^{\alpha} f(\eta); \eta = Rt^{\beta}$

Where η is a function of Rand t using similarity variable

We obtain

$$\frac{d}{d\eta}\left(\eta ff'|f'|^{\frac{1-n}{n}}\right) = a^2\eta\left(\alpha f - \frac{n+\alpha}{n+1}\eta\frac{df}{d\eta}\right) \qquad (2.6)$$

$$Where \quad a^2 = \Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}} \qquad (2.7)$$

$$f(\eta_1) = 0 \qquad (2.8)$$

$$\left(\frac{df}{d\eta}\right)\eta_1 = 0 \qquad (2.9)$$

3.0. Methods of Solution

From equation (2.7) when $n = \frac{1}{2}$

We obtain

$$\frac{d}{d\eta}\left(\eta ff'\left|\frac{df}{d\eta}\right|\right) = a^2\eta\left(\alpha f - \frac{2\alpha+1}{3}\eta f'\right) \qquad (3.1)$$

$$f(\eta) = 10 + \alpha\eta + \beta\eta^2 \qquad (3.2)$$

$$f(0) = 10 \qquad (3.3)$$

$$f'(0) = -0.3 \qquad (3.4)$$

$$f(1) = 2 \qquad (3.5)$$

We obtain

$$f(\eta) = 10 - 0.3\eta + 7.7\eta \qquad (3.6)$$

From equation (3.3) together with the boundary conditions

$$f(0)=10 \tag{3.7}$$

$$f(1)=2 \tag{3.8}$$

$$f'(0)=-0.3 \tag{3.9}$$

We obtain

$$f(\eta)=10-0.3\eta+8.7\eta^2 \tag{3.10}$$

From equation (2.7) let $a^2 \to \infty$

By using similarity variable we obtain

$$f(\eta)=2\eta^{\frac{\alpha(n+1)}{\alpha+n}} \tag{3.11}$$

From equation (2.7) assume $a^2 = 0$

By using similarity variable we obtain

$$f^{n+1}=-3.333\eta^{1-n}-2/3.333 \tag{3.12}$$

When $n=1/2$

$$f(\eta)=\left[-3.333\eta^{1/2}-0.6\right]^{2/3} \tag{3.13}$$

4.0 Results

The analytical solutions of equation (3.6), (3.10) and (3.11) were provided for various values of parameter α and n.

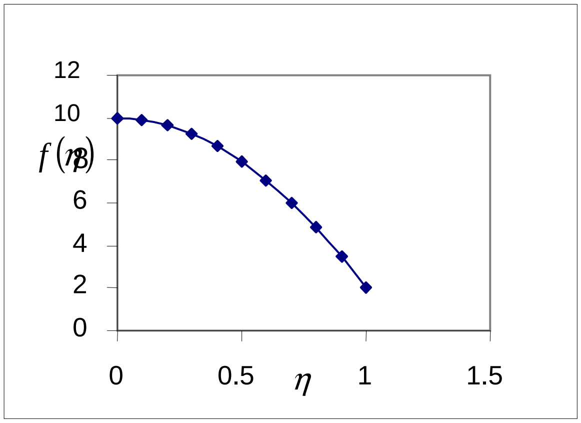

Fig4.1: Graph of the velocity function f against the similarity variable η when α = -0.3, β = -7.7 and n= ½.

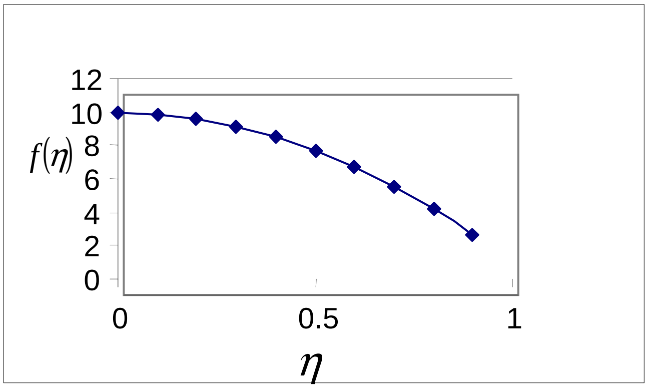

Fig4.2: Graph of the velocity function f against the similarity variable η when α = -0.3, β = -8.7 and n= ½.

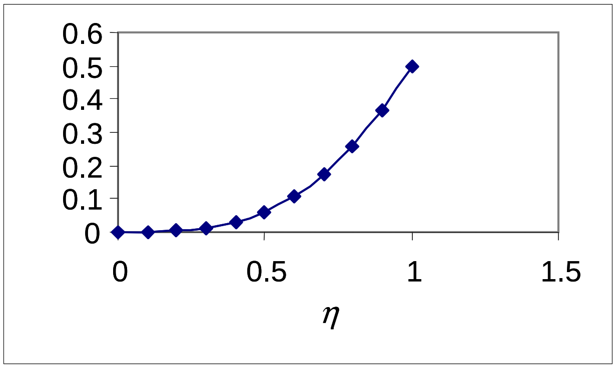

Fig 4.3: Graph of the velocity function f against the similarity variable η when α =½ and n= ½.

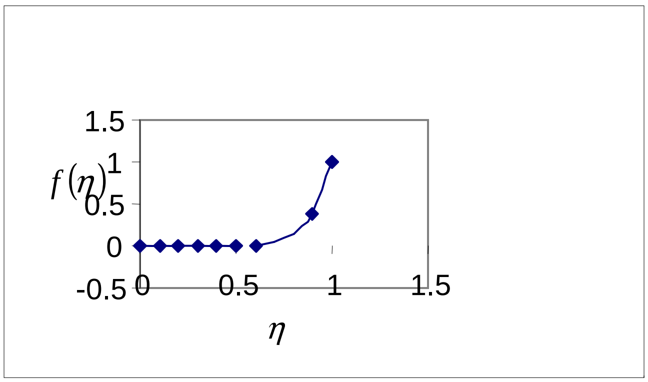

Fig 4.4: Graph of the velocity function f against the similarity variable η when α =1½ and n= ½.

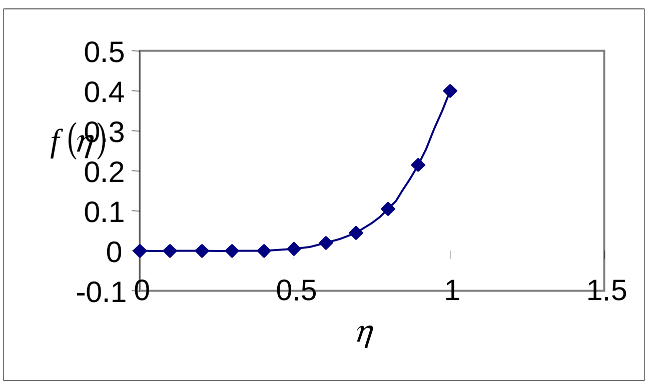

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Fig 4.5: Graph of the velocity function f against the similarity variable η when α =2 and n= ½.

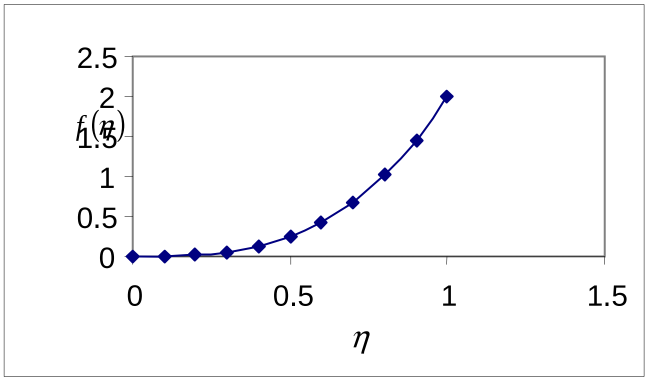

Fig 4.6: Graph of the velocity function f against the similarity variable η when α =-1 and n= ½.

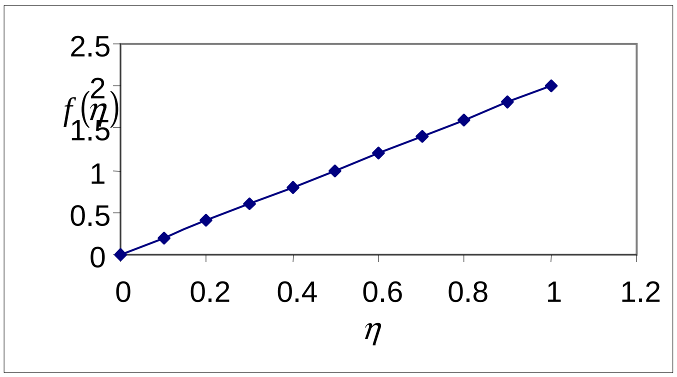

Fig 4.7: Graph of the velocity function f against the similarity variable η when α =1 and n= ½.

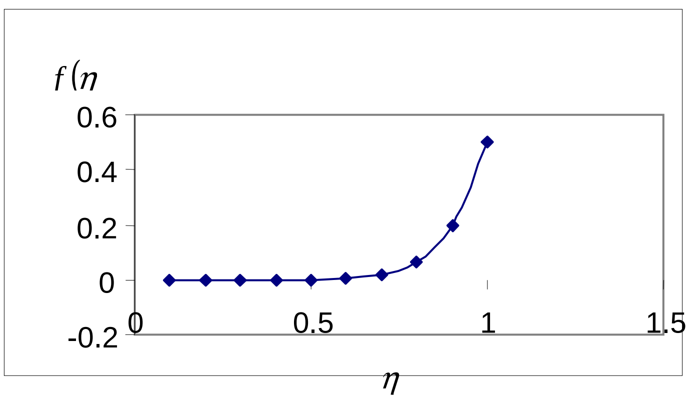

Fig 4.8: Graph of the velocity function f against the similarity variable η when α =-1½ and n= ½.

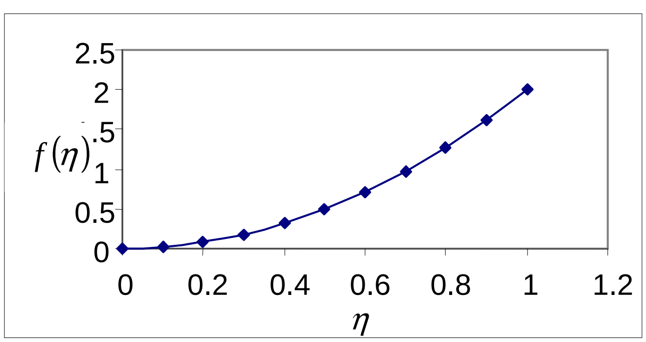

Fig 4.9: Graph of the velocity function f against the similarity variable η when α =-2 and n= ½.

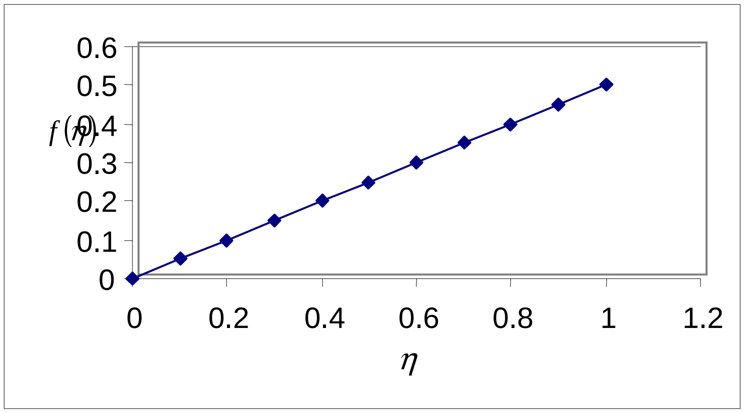

Fig 4.10 : Graph of the velocity function f against the similarity variable η when $\alpha = {}^1/_{10}$ and n= ½.

5.0 Discussion/Conclusion

We proposed suitable expression for unsteady gravityflows of a power-law fluid through a porous medium. The unsteady profile and the velocity profile were studied for various values of α and power-law viscosity index n are emphasized on the flow behaviour. The fluid is Pseudo-plastic for n<1.We plot the graph of velocity function f against the similarity variable η. It is seen from figs 4.1-4.2 that the effect of the power-law index n is to decrease the flow characteristics. It is seen from figs 4.3-4.5that the effect of the power-law index n is to increase the flow characteristics. On the other hand, from analytical calculations we also see that the parameter n affects both the flow characteristics and the accuracy of the approximate solutions significantly.

Reference

[1] A.J.Omowaye and R.O.Ayeni(2005):The effect of permeability on the on the temperature rise in a reacting porous medium. Appreciating the future in Maths pp85.

[2] B.I.Olajuwon and R.O.Ayeni(2004):Flow of a power-law fluid with memory past an infinite plate. Journal of the Nigerian Association of Mathematical Physics (NAMPS), Vol.8 pp 161.

[3] J.P. Pascal and H. Pascal (1993): Similarity solutions to some gravity flows of non-Newtonian fluids through a porous medium, Int. J. Non-Linear Mech. 38 pp 157 – 167.

[4] J.Zueco (2009): Numerical solutions for unsteady rotating high porosity medium. Journal of Physical Sciences Vol.80, Number 3.

[5] R. Cortell Bataller (2008): Unsteady gravity flows of a power-law fluid through a porous medium. Applied Mathematics and Computation 196 pp356-362.