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Stability analysis of a predator prey system with disease in the prey

****Emmanuel O. Oghre and Dennis I. Egherha***

Department of Mathematics
University of Benin, Benin City. Nigeria

Abstract

It is more of biological significance to consider the effect of interacting species when we study the dynamical behaviour of epidemiological models. We have considered population in a predator prey system with disease in the prey that is divided into three classes: the Susceptible, the Infected but not infectious and the Infectious classes. Stability analysis shows that the disease free equilibrium is globally and asymptotically stable in the domain D if the reproduction number $R_0 < 1$. ***Its epidemiological implication is that the infected but not yet infectious and the infectious prey population vanishes so the disease in the prey die out.***

1.0 Introduction

The study of diseases in a prey-predator system has gained much interest in recent years [2, 3]. Species does not exist alone there is always interaction with other species. While species spreads the disease, it also competes with the other species for space or food, or is predated by other species. Therefore it is more of biological significance to consider the effect of interacting species when we study the dynamical behaviour of epidemiological models. Therefore an appropriate mathematical model is essential to study the effect of disease on interacting species [7]. Mathematical models have become important tools in analyzing the spread and control of infectious diseases and the consequent effect on population [3, 10, 11]. Mukherjee [6] analyzed a generalized prey-predator system with parasitic infection. Mukeheje [7] also investigated how the predation process influences the epidemic considering the case where the predator eats infected prey only. In epidemiology the population is divided into two classes Susceptible (S) and the Infected (I), however in practice this is not so as susceptible individual stays for some definite period after leaving the susceptible class and joining the infected class. This intermediate period may be termed as incubation period. The incubation period is defined as the time from exposure to onset of disease and when limited to infectious disease, corresponds to the time from infection with a microorganism to symptom development [1, 8]. This class is similar to the Exposed class in epidemiological models (see [5]). Therefore we consider a predator – prey population model in which a disease that can be transmitted by contact spreads among the prey. Unlike in the [7] model where the predator only preys on the infected, in our model we allow the predator to prey on susceptible, the infected not infectious and the infected class.

2.0 Model Formulation

We assume that the disease places new recruits from the susceptible class into infected but not yet infectious class (exposed) for a period of incubation. This set of new recruits we denote by φ(t). As usual $S(t)$, $I(t)$ and $P(t)$ represent respectively the susceptible prey, the infectious prey capable of transmitting the disease and the predators class. We assume the disease that can be transmitted among the prey by contact. The contact rate is taken in the form of bilinear mass action incidence law. The way an individual leaves the susceptible class is by becoming infected at the rate **a** or by being consumed by predators at the rate **c**. Each infected but not yet infectious φ(t) individual generates $I(t)$ new individual per unit time while $S(t)\ I(t)$ individuals generate the infected but not yet infectious class φ. The ways an individual in the infected but not yet infectious class φ can leave the class is by being infected at the rate β individuals per unit time or by being consumed at the rate **c**, which is the same as the rate the predators consume the susceptible. This is true because at this rate, the symptom has not developed, hence will not hinder its efficiency to avoid predators. Also we assume some of the individuals in the infectious class will die naturally, or die due to the infection at the rate μ **,** while others are being consumed

*Corresponding authors: Emmanuel O. Oghre, E-mail: eoghre@yahoo.com , Tel. +234 (0) 8033551266

by predator at the rate m. We also assume that the contact rate between the infected and not yet infectious class and the susceptible class cannot spread the disease. We define K as the carrying capacity of the prey and the predator's net gains for consuming the susceptible class, infected but not yet infectious class is θ_1, while θ_2 is the net gain for consuming the infectious class and that $\theta_1 > \theta_2$. Based on these assumptions, we have the following differential equations respectively for the susceptible class, infected but not yet infectious class, the infected class and the predator class

$$\left.\begin{aligned} \frac{dS}{dt} &= rS\left(1 - \frac{S}{K}\right) - cSP - a\,SI \\ \frac{d\phi}{dt} &= aSI - \beta\phi - c\phi\,P \\ \frac{dI}{dt} &= \beta\phi - mIP - \mu I \\ \frac{dP}{dt} &= \theta_1 SP + \theta_1 \phi P + \theta_2 IP - \delta P \end{aligned}\right\} \quad (1)$$

The initial conditions are given as

$$S(0) \geq 0\,,\ \phi(0) \geq 0\,,\ I(0) \geq 0\,,\ P(0) \geq 0 \quad (2a)$$

$$r > 0,\ a > 0,\ \mu > 0,\ \beta > 0,\ \theta_1 > 0,\ \theta_2 > 0,\ c > 0,\ \delta > 0,\ m > 0\,. \quad (2b)$$

[9] had considered the stability of system of equation (1) and determined the condition for epidemic outbreak. Haque et al [4] had pointed out also that there are three biological relevant equilibria E_0, E_1, E_2 for a system such as in equation (1).

3.1 The Steady States

The steady state occurs when

$$\frac{\partial S}{\partial t} = \frac{\partial \phi}{\partial t} = \frac{\partial I}{\partial t} = \frac{\partial P}{\partial t} = 0 \quad (3)$$

This reduces our system (1) (when the variables are distinguished with asterisk) to

$$rS^*\left(1 - \frac{S^*}{k}\right) - cS^*P^* - aS^*I^* = 0 \quad (4)$$

$$aS^*I^* - \beta\phi^* - c\phi^*P^* = 0 \quad (5)$$

$$\beta\phi^* - mI^*P^* - \mu I^* = 0 \quad (6)$$

$$\theta_1 S^*P^* + \theta_1\phi^*P^* + \theta_2 I^*P^* - \delta P^* = 0 \quad (7)$$

From equation (7),

$$P^*\left(\theta_1 S^* + \theta_1\phi^* + \theta_2 I^* - \delta\right) = 0 \quad (8)$$

Equation (8) implies that

$$P^* = 0 \text{ or } \left(\theta_1 S^* + \theta_1\phi^* + \theta_2 I^* - \delta\right) = 0$$

When $P^* = 0$,

This reduces equations (4) – (7) (when the variables are distinguished with subscript 0)to

$$rS_0\left(1 - \frac{S_0}{K}\right) - aS_0 I_0 = 0 \quad (9)$$

$$aS_0 I_0 - \beta\phi_0 = 0 \quad (10)$$

$$\beta\phi_0 - \mu I_0 = 0 \quad (11)$$

From equation (11),

$$\phi_0 = \frac{\mu I_0}{\beta} \quad (12)$$

Using equation (12), equation (10) becomes

$$I_0\left(aS_0 - \mu\right) = 0 \quad (13)$$

This implies that

$$I_0 = 0 \quad (14)$$

or

$$S_0 = \frac{\mu}{a} = \frac{1}{R_0} \tag{15}$$

From equation (14), (12) becomes

$$\phi_0 = 0 \tag{16}$$

Using equation (14), equation (9) becomes

$$rS_0\left(1 - \frac{S_0}{K}\right) = 0$$

$$\Rightarrow S_0 = 0 \ \ or \ S_0 = K$$

$$for \ S_0 = 0,$$

$$\mathrm{E}_0 \equiv (0,0,0,0) \text{ at the origin} \tag{17}$$

When $S_0 = K$, $P_0 = 0,$

Equation (9), (10), and (11) becomes

$$-aKI_1 = 0 \tag{18}$$

$$aKI_1 - \beta\phi_1 = 0 \tag{19}$$

$$\beta\phi_1 - \mu I_1 = 0 \tag{20}$$

From equations (18) (19) and (20), we have that

$$I_1 = 0$$

$$\phi_1 = 0$$

We then have equilibrium

$$E_1 \equiv (K, 0, 0, 0) \tag{21}$$

When $P = 0$, $S_0 = \frac{\mu}{a}$,

Equations (4) – (7) reduce to

$$r\frac{\mu}{a}\left(1 - \frac{\mu}{aK}\right) - \mu I_2 = 0 \tag{22}$$

$$\mu I_2 - \beta\phi_2 = 0 \tag{23}$$

$$\beta\phi_2 - \mu I_2 = 0 \tag{24}$$

From equation (22), we have,

$$I_2 = \frac{r}{a}\left(1 - \frac{\mu}{aK}\right) \tag{25}$$

Using equation (25), equation (23) becomes

$$\phi_2 = \frac{\mu}{\beta}\left[\frac{r}{a}\left(1 - \frac{\mu}{aK}\right)\right] \tag{26}$$

So that

$$E_2 \equiv \left(\frac{\mu}{a}, \ \frac{\mu}{\beta}\left(\frac{r}{a} - \frac{r\mu}{a^2K}\right), \ \frac{r}{a}\left(1 - \frac{\mu}{aK}\right), \ 0\right) \tag{27}$$

We now consider the case when $P \neq 0$ *i.e*

When $(\theta_1 S^* + \theta_1\phi^* + \theta_2 I^* - \delta) = 0$

So that our equations (4-7) become

$$rS_3\left(1 - \frac{S_3}{K}\right) - cSP_3 - aS_3I_3 = 0 \tag{28}$$

$$aS_3I_3 - \beta\phi_3 - c\phi_3P_3 = 0 \tag{29}$$

$$\beta\phi_3 - mI_3P_3 - \mu I_3 = 0 \tag{30}$$

$$\theta_1 S_3 P_3 + \theta_1 \phi_3 P_3 + \theta_2 I_3 P_3 - \delta P_3 = 0 \tag{31}$$

From equation (28), if we set $S_3 = 0$, equation (28), (29), (30) and (31) becomes

$$S_3 = 0 \tag{32}$$

$$-\beta\phi_3 - c\phi_3 P_3 = 0 \tag{33}$$

$$\beta\phi_3 - mI_3 P_3 - \mu I_3 = 0 \tag{34}$$

$$\theta_1 \phi_3 + \theta_2 I_3 - \delta = 0 \tag{35}$$

From equation (33), we have

$$P_3 = -\frac{\beta}{c} \tag{36}$$

Using equation (36), (34) becomes

$$\beta\phi_3 + \frac{\beta m}{c} I_3 - \mu I_3 = 0$$

This reduces to

$$I_3 = -\frac{c\beta\phi_3}{\beta m - c\mu} \tag{37}$$

Using equation (37), (35) becomes

$$\theta_1 \phi_3 - \frac{c\theta_2 \beta\phi_3}{\beta m - c\mu} - \delta = 0$$

When simplified gives

$$\phi_3 = \frac{\delta(\beta m - c\mu)}{[(\beta m - c\mu)\theta_1 - c\beta\theta_2]} \tag{38}$$

Substituting equation (38) in (37) gives

$$I_3 = -\frac{c\delta\beta}{(\beta m - c\mu)\theta_1 - c\beta\theta_2} \tag{39}$$

So that

$$(S_3, \phi_3, I_3, P_3) = \left[0, \frac{\delta(\beta m - c\mu)}{[(\beta m - c\mu)o_1 - c\beta o_2]}, -\frac{c\delta\beta}{[(\beta m - c\mu)\theta_1 - c\beta\theta_2]}, -\frac{\beta}{c}\right] \tag{40}$$

3.2 Stability of The Steady States

At steady state, our system (1) reduces to

$$\left.\begin{aligned} rS^*\left(1 - \frac{S^*}{k}\right) - cS^*P^* - aS^*I^* &= 0 \\ aS^*I^* - \beta\phi^* - c\phi^*P^* &= 0 \\ \beta\phi^* - mI^*P^* - \mu I^* &= 0 \\ \theta_1 S^*P^* + \theta_1\phi^*P^* + \theta_2 I^*P^* - \delta P^* &= 0 \end{aligned}\right\} \tag{41}$$

The Jacobian matrix J_0 is given as

$$J_0 = \begin{pmatrix} r - \frac{2rS_0}{k} - cP_0 - aI_0 & 0 & -aS_0 & 0 \\ aI_0 & -\beta & -aS_0 & -c\phi_0 \\ 0 & \beta & -mP_0 - \mu & -mI_0 \\ \theta_1 P_0 & \theta_1 P_0 & \theta_2 P_0 & -\delta \end{pmatrix} \tag{42}$$

Evaluated at $(S_0, \phi_0, I_0, P_0) = (0,0,0,0)$, J_0 reduces to

$$J_0 = \begin{pmatrix} r & 0 & 0 & 0 \\ 0 & -\beta & 0 & 0 \\ 0 & \beta & -\mu & 0 \\ 0 & 0 & 0 & -\delta \end{pmatrix}$$

The characteristic equation for J_0 is given by

$$\begin{vmatrix} r-\lambda & 0 & 0 & 0 \\ 0 & -\beta-\lambda & 0 & 0 \\ 0 & \beta & -\mu-\lambda & -\delta-\lambda \end{vmatrix} = 0$$

$$= (r-\lambda)(-\beta-\lambda)(-\mu-\lambda)(-\delta-\lambda) = 0 \quad (43)$$

Our eigenvalues can be obtained from the expression

$$\left.\begin{aligned} \lambda_{01} &= r \\ \lambda_{02} &= -\beta < 0 \\ \lambda_{03} &= -\mu < 0 \\ \lambda_{04} &= -\delta < 0 \end{aligned}\right\} \quad (44)$$

Since all the eigenvalues are not negative, it implies that the disease free equilibrium will be locally asymptotically stable if $r < 0$. But $r > 0$, hence the system is unstable at this point.

The Jacobian matrix J_1 at evaluated at $(S_1, \phi_1, I_1, P_1) = (k, 0, 0, 0)$, J_1 reduces to

$$J_1 = \begin{pmatrix} -r & 0 & -ak & 0 \\ 0 & -\beta & -ak & 0 \\ 0 & \beta & -\mu & 0 \\ 0 & 0 & 0 & -\delta \end{pmatrix}$$

The characteristic equation for J_1 is given by

$$\begin{vmatrix} -r-\lambda & 0 & -\lambda k & 0 \\ 0 & -\beta-\lambda & -\lambda k & 0 \\ 0 & \beta & -\mu-\lambda & 0 \\ 0 & 0 & 0 & -\delta-\lambda \end{vmatrix} = 0$$

$$= (-r-\lambda)(-\beta-\lambda)(-\mu-\lambda)(-\delta-\lambda) = 0 \quad (45)$$

This implies that

$$\left.\begin{aligned} \lambda_{11} &= -r < 0 \\ \lambda_{12} &= -\beta < 0 \\ \lambda_{13} &= -\mu < 0 \\ \lambda_{14} &= -\delta < 0 \end{aligned}\right\} \quad (46)$$

Since all the Eigen values are negative, it therefore implies that the disease free equilibrium is locally asymptotically stable.

In similar manner, for Jacobian matrix J_2,

$$J_2 = \begin{pmatrix} r - \frac{2rS_2}{k} - cP_2 - aI_2 & 0 & -aS_2 & 0 \\ aI_2 & -\beta & -aS_2 & 0 \\ 0 & \beta & -mP_2 - \mu & -mI_2 \\ \theta_1 P_2 & \theta_1 P_2 & \theta_2 P_2 & -\delta \end{pmatrix} \quad (47)$$

Since $(S_2, \varphi_2, I_2, P_2) = \left[\frac{1}{R_0}, \left(\frac{r}{\beta} - \frac{r}{R_0 k}\right)\frac{1}{R_0}, \frac{r}{a}\left(1 - \frac{1}{R_0 k}\right), 0\right]$

where $R_0 = \frac{a}{\mu}$,

Equation (47) becomes

$$J_2 = \begin{pmatrix} r - \frac{2r}{k}\bullet\frac{1}{R_0} - r\left(1-\frac{1}{R_0 k}\right) & 0 & -\mu & 0 \\ r\left(1-\frac{1}{R_0 k}\right) & -\beta & -\mu & 0 \\ 0 & \beta & -\mu & -\frac{mr}{a}\left(1-\frac{1}{R_0 k}\right) \\ 0 & 0 & 0 & -\delta \end{pmatrix} \tag{48}$$

The characteristics equation for J_2 is given by

$$(-\delta-\lambda)\left(\left[-\lambda^3-\left(\frac{r}{R_0 k}+\beta+\mu\right)\lambda^2-\left(\frac{r\beta}{R_0 k}+\frac{r\mu}{R_0 k}+\mu\beta\right)\lambda-r\mu\beta\left(1-\frac{1}{R_0 k}\right)\right]\right)=0 \tag{49}$$

To determine the stability, we apply Routh-Hurwitz criteria for cubic polynomial which states that if we have the characteristic equation of a Jacobian Matrix in the form

$$(-\lambda-a)\left(\lambda^3+A\lambda^2+B\lambda+C\right)=0 \tag{50}$$

But $\lambda=-a$ is negative, then the stability is satisfied if and only if $A>0, B>0, AB>C.$ Comparing equation (49) and (50), we see that

$$A=\left(\frac{r}{R_0 k}+\beta+\mu\right)>0$$

$$B=\frac{r}{R_0 k}(\beta+\mu)+\mu\beta>0$$

So that

$$AB-C=\left(\frac{r}{R_0 k}+\beta+\mu\right)\left(\frac{r}{R_0 k}(\beta+c)+\mu\beta\right)-\left(1-\frac{1}{R_0 k}\right)r\mu\beta$$

$$=\left(\frac{r}{R_0 k}+\beta+\mu\right)\left(\frac{r\beta}{R_0 k}+\frac{rc}{R_0 k}+\mu\beta\right)-r\mu\beta\left(1-\frac{1}{R_0 k}\right)>0$$

Since

$$r\mu\beta>\frac{r\mu\beta}{R_0 k} \tag{51}$$

Clearly, we say that all the eigen-values of the Jacobian matrix have negative real part provided $\lambda=-\delta, r>0$, hence the disease free equilibrium corresponding to J_2 is locally asymptotically stable.

The Jacobian matrix J_3 is given as

$$J_3 = \begin{pmatrix} r-\frac{2rS_3}{K}-cP_3-aI_3 & 0 & -aS_3 & -cS_3 \\ aI_3 & -\beta & -aS_3 & -c\phi_3 \\ 0 & \beta & -mP-\mu & -mI \\ \theta_1 P_3 & \theta_1 P_3 & \theta_2 P_3 & -\delta \end{pmatrix} \tag{52}$$

Evaluated at

$$(S_3,\phi_3,I_3,P_3)=\left(0,\frac{\delta(\beta m-c\mu)}{(\beta m-c\mu)\theta_1-c\beta\theta_2},-\frac{c\delta\beta}{(\beta m-c\mu)\theta_1-c\beta\theta_2},-\frac{\beta}{c}\right) \tag{53}$$

gives

$$J_3 = \begin{pmatrix} r+\beta+\frac{ac\delta\beta}{(\beta m-c\mu)\theta_1-c\beta\theta_2} & 0 & 0 & 0 \\ -\frac{ac\delta\beta}{(\beta m-c\mu)\theta_1-c\beta\theta_2} & -\beta & 0 & -\frac{c\delta(\beta m-c\mu)}{(\beta m-c\mu)\theta_1-c\beta\theta_2} \\ 0 & \beta & m\frac{\beta}{c}-\mu & \frac{mc\delta\beta}{(\beta m-c\mu)\theta_1-\beta\theta_2} \\ -\theta_1\frac{\beta}{c} & -\theta_1\frac{\beta}{c} & -\theta_2\frac{\beta}{c} & -\delta \end{pmatrix}$$

$$\therefore \quad \left(r + \beta + \frac{ac\,\delta\beta}{(\beta m - c\mu)\theta_1 - c\beta\theta_2} - \lambda \right)\left(\lambda^3 + A\lambda^2 + B\lambda + C\right) = 0 \tag{54}$$

where,

$$A = \delta + \beta + \mu - \frac{m\beta}{c} \tag{55}$$

$$B = \frac{\beta}{c}\frac{c\delta}{(\beta m - c\mu)\theta_1 - c\beta\theta_2}(\theta_2 m\beta - \theta_1(\beta m - c\mu)) - \mu\beta - \delta\mu - \delta\beta \tag{56}$$

$$C = \delta\beta\left(\mu - \frac{\beta m}{c}\right) + \theta_2\frac{\beta^2}{c}\left(\frac{mc\,\delta\beta}{(\beta m - c\mu) - c\beta\theta_2}\right) - \left[\theta_2\frac{\beta^2}{c} - \left(\theta_1\frac{\beta}{c}\right)\left(\frac{m\beta}{c} - \mu\right)\right]\frac{c\delta(\beta m - c\mu)}{(\beta m - c\mu)\theta_1 - c\beta\theta_2} \tag{57}$$

Since λ is of degree three, we apply the Routh–Hurwitz criteria to determine the stability. If λ is negative, then the stability is satisfied if

$$A > 0, B > 0, AB > C. \tag{58}$$

Equation (52) will be stable if

$$r + \beta + \frac{ac\,\delta\beta}{(\beta m - c\mu)\theta_1} < 0 \tag{59}$$

and inequality (58) is satisfied.

4. Global Stability

The global stability of the disease.-free equilibrium can be analysed. But we need to choose a suitable domain that is positively invariant for our analysis. We follow the method in [7];

We define a domain

$$D = \{(S, \phi, I) : S \geq 0, \phi \geq 0, I \geq 0\} \tag{60}$$

The domain is positive because no solution paths leave through any boundary. Solution exists for all positive time $t > 0.$ We need to show the disease free equilibrium is globally asymptotically stable in D if the upper limit on the number of infected prey $R_o < 1.$

By Lyapunov functional,

$$V = \frac{1}{a}S(t) + \left(\frac{1}{\mu}\right)[\phi(t) + I(t)] \tag{61}$$

Where $V > 0$ since, $S > 0, \phi \geq 0, I \geq 0, \mu > 0, a > 0,$.

Next we obtain the Lyapunov derivative of V i.e differentiating the Lyapunov functional V with respect to time t. i.e,

$$\dot{V} = \frac{1}{a}\dot{S} + \frac{1}{\mu}\dot{\phi} + \frac{1}{\mu}\dot{I}$$

Where

$$\dot{V} = \frac{dV}{dt}, \dot{S} = \frac{dS}{dt}, \dot{\phi} = \frac{d\phi}{dt}, \dot{I} = \frac{dI}{dt}$$

Then,

$$\frac{dV}{dt} = \frac{1}{a}\frac{dS}{dt} + \frac{1}{\mu}\frac{d\phi}{dt} + \frac{1}{\mu}\frac{dI}{dt} \tag{62}$$

So that from system (1),

$$\dot{S} = \frac{dS}{dt} = r\left(S - \frac{S}{K}\right) - cSP - aSI$$

$$\dot{\phi} = \frac{d\phi}{dt} = aSI - \beta\phi - c\phi P$$

$$\dot{I} = \frac{dI}{dt} = \beta\phi - mIP - \mu I$$

This gives,

$$\dot{V} = \frac{1}{a}\left(rS - \frac{r}{K}S^2 - cSP - aSI\right) + \frac{1}{\mu}(aSI - \beta\phi - c\phi P) + \frac{1}{\mu}(\beta\phi - mIP - \mu I)$$

$$= \frac{r}{a}S - \frac{r}{ak}S^2 - \frac{c}{a}SP - SI + \frac{a}{\mu}SI - \frac{\beta}{\mu}\phi - \frac{c}{\mu}\phi P + \frac{\beta}{\mu}\phi - \frac{m}{\mu}IP - I$$

$$= -\left(1-\frac{a}{\mu}\right)SI - \left(\frac{c}{a}S + \frac{m}{\mu}I\right)P - \frac{r}{a}\left(\frac{S}{K}-1\right)S - I - \frac{c}{\mu}\phi P \quad (63)$$

Since $\frac{a}{\mu} = R_0$, and in the disease free state, $S = K$,

$$\dot{V} = -(1-R_0)KI - \left(\frac{c}{a}K + \frac{m}{\mu}I\right)P - I - \frac{c}{\mu}\phi P \quad (64)$$

We have shown that the Lyapunov derivative is less than zero. i.e

$$\dot{V} < 0 \quad \text{Provided} \quad R_0 < 1.$$

The condition for $\dot{V} < 0$ is that $a > 0, \mu > 0, c > 0, m > 0$

We see that the value of $\dot{V}$ strictly lies on the value of R_0

Thus if

$$R_0 > 1, \text{ then } \dot{V} > 0$$

$$R_0 < 1, \text{ then } \dot{V} < 0$$

Clearly, the set $V = 0$ gives the face of D with $\phi = 0$ but $\frac{d\phi}{dt} = aSI$ on this face so that ϕ moves to the face unless $I = 0$. When $\phi = I = 0$,

$$\frac{dS}{dt} = rS\left(1 - \frac{S}{K}\right) - cSP \quad (65)$$

5. Conclusion

The implication of equation (65) is that the disease free equilibrium is globally asymptotically stable in the domain D if $R_0 < 1$. Its epidemiological implication is that the infected but not yet infectious and the infectious prey population vanishes so the disease in the prey die out.

References

[1] Dhar J., Sharma A. K. (2009). "The Role of the Incubation Period in A Disease model". *Applied Mathematics E-Note.*, 2009; *9: 146-153*

[2] Freedman H. I. "A model of Predator-Prey dynamics as modified by the action of a parasite" *Math. Biosci* , 1990; *99:143-155.*

[3] Hadeler K. P., Freedman H. I. "Predator-Prey Populations with Parasitic Infection". *Mathematical Biology,* 1989; ***27:*** *609- 631.*

[4] Haque, M., Zhen J., Venturino E. "An Ecoepidemiological Predator- Prey Model with Standard Disease incidence". *Mathematical Methods in the Applied Sciences.* 2008; *1071*

[5] Hethcote, H. W. "The mathematics of Infectious Diseases". *SIAM Review 2000; 42: 599-653*

[6] Mukherjee D. "Uniform persistence in a generalized prey-predator system with parasite infection". *Biosystems 1998; 47: 149-155*

[7] Mukherjee D. "Stability Analysis of a stochastic model for Predator- Prey System with Disease in the Prey". *Nonlinear Analysis: Modeling and control* 2003; ***8:*** *83-92.*

[8] Nishiura, H. "Early efforts in modeling the incubation period of infectious diseases with an acute course of illness". *Emerging Themes in Epidemiology*. 2007; 4: 2

[9] Oghre, E. O., Egberha, D. I. "Disease dynamics in predator-prey population with disease in the prey". (Submitted for publication in this edition in 2011)

[10] Xiao Y., Chen, L., "Modeling and analysis of a predator-prey model with disease in the prey". *Math. Biosci*. 2001a; 171: 59 - 82

[11] Xiao Y., Chen, L., "Analysis of a three species Eco-Epidemiological model" . *J Math. Anal. Appl*. 2001b; 258: 733 - 754