

Effects of Diversification of Assets on Mean and Variance

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Abstract

Diversification is a means of minimizing risk and maximizing returns by investing in a variety of assets of the portfolio. This paper is written to determine the effects of diversification of three types of Assets; uncorrelated, perfectly correlated and perfectly negatively correlated assets on mean and variance. To go about this, useful data were collected for uncorrelated, perfectly correlated and perfectly negatively correlated assets. These data were computed for two assets, three assets and four assets model to know the effects of diversification of the three types of assets on mean and variance. It is observed that diversification of these three types of assets yield the same mean (returns). It is also noticed that Mean increases with the increasing number of diversification. And variance (risk) increases with mean for uncorrelated, perfectly correlated assets. But, for perfectly negatively correlated assets the variance decreases with its increasing mean. This implies that diversifying into perfectly negatively correlated assets (risky and riskless assets) together maximize mean (return) and minimize or cancel variance (risk).

1.0 Introduction

Markowitz [14] introduced modern portfolio theory. He formulated the portfolio problem as a choice of the mean and variance of a portfolio of assets. In [7] it was pointed out there that Markowitz established the fundamental theorem of mean variance of the portfolio theory, namely holding constant variance, maximize expected return, and holding constant expected return minimize variance. Tobin [9], Kraus and Litzenberger [11] and Lee [12] offered alternative portfolio theories that include more moments such as skewness or were accurate for more realistic descriptions of the return (see for example [6, 8]). Other useful models for measuring risks and returns have been discussed in [1, 10]. Interestingly, the work [13] gave an explicit characterization of risks aversion in terms of strength. Again, see [2, 3].

Diversification involves spreading investments around into many types of investments, including stocks mutual funds, bonds and cash. Money can also be diversified into different mutual fund investment strategies, including growths funds, balance funds, index funds, and sector-specific funds. Geographic diversification involves a mixture of domestic and international investment, also see [4, 5].

Diversification reduces the risk of a portfolio. It does not necessarily reduce the returns. This is why diversification is referred to as the only free lunch in finance.

Diversification can be quantified as the intra-portfolio correlation. This is a statistical measurement from negative one to one that measures the degree to which the various assets in a portfolio can be expected to perform in a similar fraction or not. Portfolio balance occurs as the sum of all intra-portfolio correlations approaches negative one. Diversification is thus defined as the intra-portfolio correlation or, more specifically, the weighted average intra-portfolio correlation. Maximum diversification occurs when the intra-portfolio correlation is minimized. Intra-portfolio correlation may be an effective risk management measurement. The computation may be expressed as:

$$Q = \frac{\sum_{i=1}^n \sum_{j=1}^n X_i X_j P_{ij}}{\sum_{j=1}^n \sum_{j=1}^n X_j X_j} \quad (1.1)$$

where, Q is the intra-portfolio correlation, X_i is the fraction invested in asset i, X_j is the fraction invested in asset j, P_{ij} is the correlation between assets i and j, and n is the number of different assets.

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Types of Diversification

- (i) Horizontal Diversification: is when you diversify between the same-type of investments. It can be a broad diversification (like investing in several companies) or more narrowed (investing in several stocks of the same branch or sector).
- (ii) Vertical Diversification: is investing between different types of investment. Again, can be a very broad diversification, like diversifying between bonds and stocks, or a more narrowed diversification, like diversifying between stocks of different branches.

While horizontal diversification lessens the risk of just investing all-in-one, a vertical diversification goes far beyond that and insures you against market and economical changes. Furthermore, the broader the diversification the lesser the risk.

2.0 Mathematical Models (c.f. Gillise (1999))

In this section, we wish to construct models to determine mean and variance of Assets. We shall construct two, three, and four assets models respectively that we shall compute with.

Two Assets Model

$$Rp = w_1R_1 + W_2R_2 \tag{2.1}$$

$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + 2w_1w_2\text{cov}_{1,2} \tag{2.2}$$

$$\text{cov}_{1,2} = \rho_{1,2}\delta_1\delta_2 \tag{2.3}$$

$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + 2w_1w_2r_{1,2}\delta_1\delta_2 \tag{2.4}$$

Three Assets Model

$$Rp = w_1R_1 + W_2R_2 + W_3R_3 \tag{2.5}$$

$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + W_3^2\delta_3^2 + 2w_1w_2r_{1,2}\delta_1\delta_2 + 2w_1w_3r_{1,3}\delta_1\delta_3 \tag{2.6}$$

$$+ 2w_2w_3r_{2,3}\delta_2\delta_3$$

Four Assets Model

$$Rp = w_1R_1 + W_2R_2 + W_3R_3 + W_4R_4 \tag{2.7}$$

$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + W_3^2\delta_3^2 + W_4^2\delta_4^2 + 2w_1w_2r_{1,2}\delta_1\delta_2 \tag{2.8}$$

$$+ 2w_1w_3r_{1,3}\delta_1\delta_3 + 2w_1w_4r_{1,4}\delta_1\delta_4 + 2w_2w_3r_{2,3}\delta_2\delta_3$$

$$+ 2w_2w_4r_{2,4}\delta_2\delta_4 + 2w_3w_4r_{3,4}\delta_3\delta_4.$$

where:

ρ = Correlation coefficient between assets

R_p = Returns of the portfolio

δ_p^2 = Variance of the portfolio

δ_1 = Standard deviation (risk) of equipment on lease to portfolio

δ_2 = Standard deviation (risk) of fixed assets to portfolio

W = Weighting or value of the assets.

For the analysis of this paper, we collected useful data from Investment Banking and Trust Company PLC (IBTC). The assets are equipment on lease and fixed assets from 2001 to 2002. Hence, we shall use the data in Table 1 to see the effect of these assets on percentage returns and risk to the portfolio within this period.

Table 1: Data for Equipment on Lease and Fixed Asset Assets

Year	R ₁ on equipment on lease (%)	δ_1 (%)	R ₂ on fixed Asset (%)	δ_2 (%)	W (%)
2001	0.2	9.9	5.0	0.8	50.0
2002	0.8	5.5	4.3	7.7	50.0

Using the equations (2.1) – (2.8) we obtained the results in Table 2 below.

Table 2: Results of the effects of Diversification on Mean and Variance

Diversification	Mean or (Returns)	Variance (Risk)		
		Uncorrelated Asset	Perfectly Correlated Asset	Perfectly negatively Correlated Asset
2	2.6	24.7	28.6	20.7
3	9.6	124.7	235.6	13.7
4	15.6	227.7	650.2	-190.9

The result in figure 1 is obtained by plotting the data in column 1 (Diversification) against column 2 (Mean or Returns) in Table 2. Similarly, the result in figure 1 is obtained by plotting the data in column 1 (Diversification) against column 2 (Mean or Returns) in Table 2.

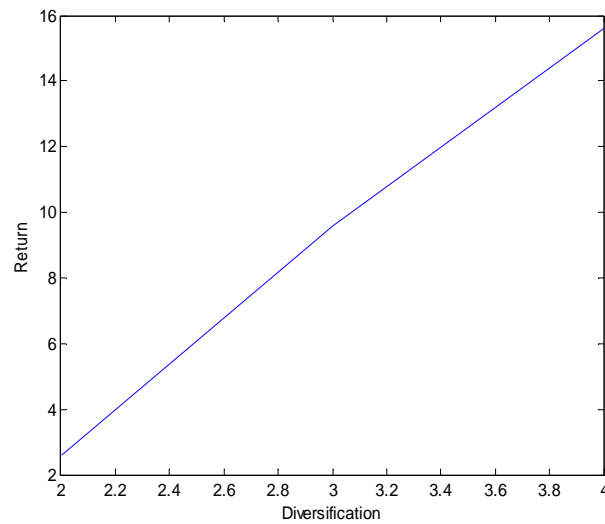


Fig. 1: The effect of diversification on returns.

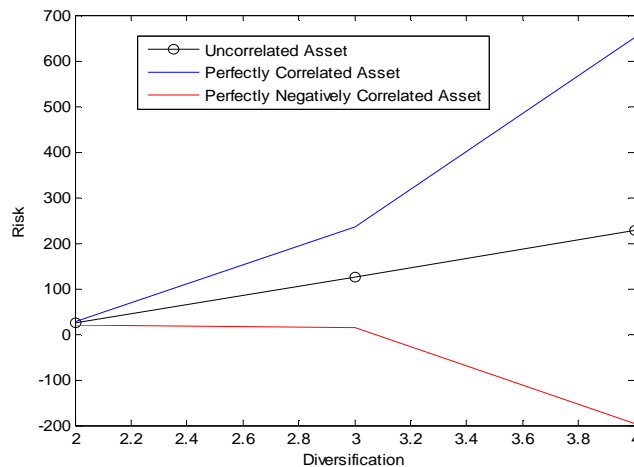


Fig. 2: The effect of diversification on risk of uncorrelated, perfectly correlated and perfectly negatively correlated assets.

4.0 Conclusion and Recommendation

This study was carried out to determine the effect of diversification of three types of assets: uncorrelated, perfectly correlated and perfectly negatively correlated assets. Uncorrelated asset means diversifying into different assets that are not similar in characteristics; perfectly correlated assets implies diversifying into different assets with similar characteristics while perfectly negatively correlated assets is diversifying into different assets that are risky and riskless in nature.

From the Table 2, it was discovered that the mean (returns) of the three types of assets are increasing with the increasing number of diversification and they are the same. Also, the variances (risks) of uncorrelated and perfectly correlated assets are increasing with increasing number of diversification. But, the variance of perfectly negatively correlated asset is decreasing with the increasing number of diversification. This shows that mean (return) increases as one increases diversification for any type of asset. Also, variance for Uncorrelated and perfectly correlated increases as diversification increases. But variance for perfectly negatively correlated asset decreases rapidly as diversification increases.

From this study we discovered that the risk of perfectly negatively correlated asset decreases/cancels as one is increasing diversification of asset. So in view of the result of the study, I wish to recommend that organization should diversify more in perfectly negatively correlated assets that is, risky and riskless assets. The securities of this asset are high in the portfolio.

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