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An Economic Production Quantity (EPQ) Model for Delayed Deteriorating Items with Stock-Dependent Demand Rate and Linear Time Dependent Holding Cost

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Abstract

In this paper, an economic production quantity (EPQ) model is presented for a single product with delayed deterioration in which the production rate is constant, demand rate is inventory level dependent in a linear functional form before and after production and the holding cost is a linear function of time. The main objective is to determine the optimal replenishment cycle time such that the total variable cost is minimised. Numerical examples are provided to illustrate the theoretical results.

Keywords: Inventory-production model, delayed deterioration, linear variable holding cost.

1.0 Introduction

An economic production quantity (EPQ) model is an inventory control model that determines the amount of product to be produced to minimize the total inventory cost so as to meet a deterministic demand.

An area of inventory management that has recently been receiving considerable attention involves situations in which the demand rate is dependent on the level of inventory, that is, the demand rate may go up or down with the on-hand stock level. As pointed out by [6] "at times, the presence of inventory has a motivational effect on the people around it. It is a common belief that large piles of goods displayed in a supermarket will lead the customers to buy more". Silver and Peterson [10] also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Due to these facts, a number of authors have developed the Economic Order Quantity (EOQ) models that focused on stock-dependent demand rate patterns. Gupta and Vrat [5] assumed that the demand rate was a function of initial stock level. The idea that that the demand rate would decline along with stock-level throughout the cycle was reflected first in the model developed by [1]. Misra [8] first studied the EPQ model for deteriorating items with varying and constant rate of deterioration. Datta and Pal [2] proposed deterministic Inventory Systems for Deteriorating Items with Inventory-level dependent Demand Rate and Shortages and [9] extended the model of [1] to the case of deteriorating items. Mandal and Phaujdar [7] then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand.

Gupta and Agarwal [4] studied EPQ model in which the production rate and demand rate during production remain constant and just as the production stops the demand rate becomes stock dependent. This continued up to a certain level of inventory and then the demand rate becomes constant for the rest of the cycle. Sugapriya and Jeyaraman [11] proposed an EPQ model for single product subject to non-instantaneous deterioration and in which the holding cost varies with time. Gary *et al* [3] obtained common production cycle time for an Economic Lot-Size Production (ELSP) with deteriorating items. Sugapriya and Jeyaraman [12] developed an ELSP model for non instantaneous deteriorating items using price discount and permissible delay in payments. Most recently, [14] proposed an optimal production – inventory model for deteriorating items with multiple-market demand. Tripathy *et al* [13] obtained an EPQ model for linear deteriorating item with variable holding cost.

In this paper an EPQ model is presented for a single item with delayed deterioration, i.e. non- instantaneous deterioration, in which the production rate is constant and demand rate is inventory level dependent in a linear functional form. The demand rate is linear during and after production run and the holding cost is also a linear function of time. The main objective is to determine optimal replenishment cycle time such that the total variable cost is minimised. Numerical examples are provided to illustrate the theoretical results.

The main difference between this paper and [11] is that [11] established an inventory model for non-instantaneous deteriorating item with constant demand rate for both in- production

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run and out-production run whereas, in this paper we have assumed the demand rate to be inventory level dependent, as a linear function of time for both in- production run and out-production run.

Notation And Modelling Assumptions

The inventory system is developed on the basis of the following model assumptions and notation:

Notation:

λ = production rate per unit time

C_0 = set up cost

θ = a constant deterioration rate (per unit time)

$C_h(t) = \delta_1 + \delta_2 t$, inventory carrying cost per unit per period (function of time), where δ_1 and δ_2 are positive constants

T = optimal production cycle time

u_1 = production run period

u_2 = time during which there is no production of the product and deterioration immediately sets in. Delay in deterioration is from time zero to u_1

$v_1(t)$ = inventory level for the product during the production period

$v_2(t)$ = inventory level for the product during the period when there is no production

V_m = maximum inventory level

K = unit production cost

TVC(T) = total inventory cost per unit time.

Assumptions:

The following are the assumptions applied in the development of the model:

1 The demand rate for the product is linear with time.
2 Shortage is not allowed and replenishment is finite
3 Rate of inflation is constant.
4 The time horizon of the inventory system is infinite. Only a typical planning schedule of length is considered, all remaining cycles are identical.
5 Once a unit of the product is produced, it is available to meet the demand.
6 Once the production is terminated the product starts deterioration.
7 There is no replacement or repair for a deteriorated item.

Model Development

The objective of the inventory problem here is to determine the optimal order quantity so as to keep the total relevant cost as low as possible.

At start t = 0, the inventory level is zero. The production and supply start simultaneously and the production stops at $t = u_1$ during which the maximum inventory V_m is reached. The demand rate during production is assumed to be $\alpha + \beta v_1(t)$, where, $0 < \beta < 1$, and so the inventory built up is at the rate $\left(\lambda - \left(\alpha + \beta v_1(t)\right)\right) > 0$ in the interval $[0, u_1]$, and in that interval there is no deterioration. After the time u_1, the produced units start deteriorating. There is no fall in demand, when the inventory reduces to zero level and production run begins. The inventory level of the product at time t over period $[0, T]$ can be represented by the following differential equations:

$$\frac{dv_1(t)}{dt} = \lambda - \left(\alpha + \beta v_1(t)\right), \; for \; 0 \le t \le u_1 \tag{1}$$

$$\frac{dv_2(t)}{dt} + \theta v_2(t) = -\left(\alpha + \beta v_2(t)\right), \; for \; u_1 \le t \le T \Leftrightarrow 0 \le t \le u_2 \tag{2}$$

That is the demand rate after production is $\alpha + \beta v_2(t)$ and the product has then started deteriorating at the rate of θ.

The pictorial representation of the model is given in Figure 1.

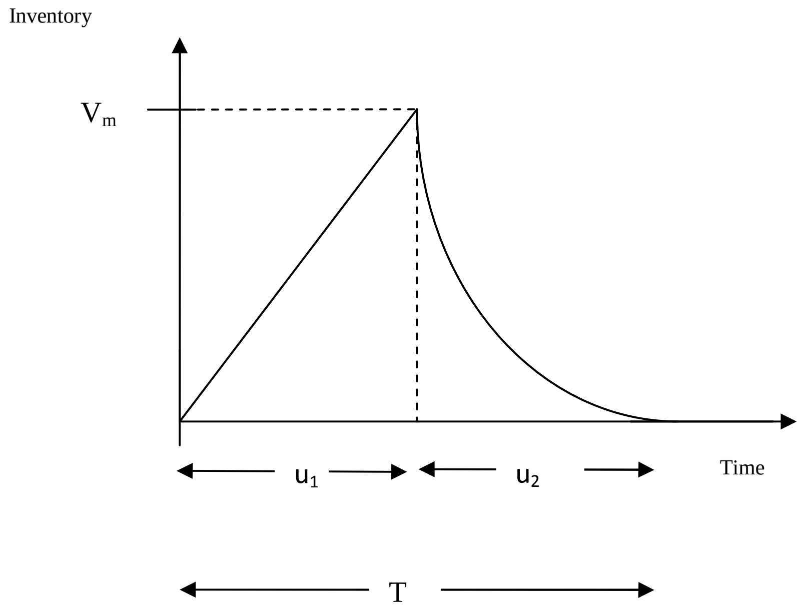

Fig.1.The inventory system of the situation

Analysis

On integrating (1) with respect to t, we have

$$v_1 e^{\beta t} = \frac{(\lambda-\alpha)e^{\beta t}}{\beta} + k_1 \quad (3)$$

where k_1 is a constant of integration.

Using the boundary condition $v_1(0)=0$ in equation (3), we have

$$v_1 e^{\beta t} = \frac{(\lambda-\alpha)e^{\beta t}}{\beta} - \frac{(\lambda-\alpha)}{\beta} \quad , \ 0 \le t \le u_1$$

and from which

$$v_1(t) = \frac{(\lambda-\alpha)}{\beta}\left(1 - e^{-\beta t}\right)$$

Using the Taylor series expansion for exponential function, we have

$$v_1(t) = \left(\frac{\lambda-\alpha}{\beta}\right)\left\{1-\left(1-\beta t+\frac{(\beta t)^2}{2}.....\right)\right\}$$

Approximating the value by neglecting those terms in βt (since $0<\beta<1$) of degree greater than 2, we have

$$v_1(t) = (\lambda-\alpha)\left(t-\frac{\beta t^2}{2}\right), \qquad 0 \le t \le u_1 \qquad (4)$$

From (2) we have

$$\frac{dv_2(t)}{dt} + (\theta+\beta)v_2(t) = -\alpha$$

which on integration with respect to t gives

$$v_2 e^{(\theta+\beta)t} = \frac{-\alpha e^{(\theta+\beta)t}}{\theta+\beta} + k_2 \tag{5}$$

where k_2 is a constant of integration.

And using the condition $v_2(u_2) = 0$ in (5) gives

$$v_2 e^{(\theta+\beta)t} = \frac{-\alpha e^{(\theta+\beta)t}}{\theta+\beta} + \frac{\alpha e^{(\theta+\beta)u_2}}{\theta+\beta}$$

Thus

$$v_2(t) = \frac{\alpha}{\theta+\beta}\left[e^{(\theta+\beta)(u_2-t)} - 1\right], \quad 0 \le t \le u_2 \quad ,$$

Using the Taylor series expansion for exponential function, we have

$$v_2(t) = \frac{\alpha}{\theta+\beta}\left\{ \left(1 + (\theta+\beta)(u_2-t) + \frac{((\theta+\beta)(u_2-t))^2}{2} \ .... \right) - 1 \right\} ,$$

Approximating the value by neglecting those terms in βt (since $0 < \beta < 1$) of degree greater than 2, we have

$$v_2(t) = \alpha\left\{ (u_2 - t) + \frac{(\theta+\beta)(u_2-t)^2}{2} \right\}, 0 \le t \le u_2 \tag{6}$$

The holding cost per period C_p is given by

$$C_p = \int_0^{u_1} C_h(t) v_1(t) dt + \int_0^{u_2} C_h(t) v_2(t) dt$$

$$= \int_0^{u_1} (\delta_1 + \delta_2 t)(\lambda - \alpha)\left(t - \frac{\beta t^2}{2} \right) dt + \int_0^{u_2} (\delta_1 + \delta_2 t)\alpha \left\{ (u_2 - t) + \frac{(\theta+\beta)(u_2-t)^2}{2} \right\} dt$$

$$= (\lambda - \alpha)\int_0^{u_1} (\delta_1 + \delta_2 t)\left(t - \frac{\beta t^2}{2} \right) dt + \alpha \int_0^{u_2} (\delta_1 + \delta_2 t)(u_2 - t) dt + \frac{\alpha(\theta+\beta)}{2}\int_0^{u_2} (\delta_1 + \delta_2 t)(u_2 - t)^2 \ dt$$

$$= (\lambda - \alpha)\int_0^{u_1} \left(\delta_1 t - \frac{\delta_1 \beta t^2}{2} + \delta_2 t^2 - \frac{\delta_2 \beta t^3}{2} \right) dt + \alpha \int_0^{u_2} \left(\delta_1 u_2 - \ \delta_1 t + \delta_2 u_2 t - \delta_2 t^2 \right) dt$$

$$+ \frac{\alpha(\theta+\beta)}{2}\int_0^{u_2} \left(\delta_1 u_2^2 - \ 2\delta_1 u_2 t + \delta_1 t^2 + \delta_2 u_2^2 t - 2\delta_2 u_2 t^2 + \delta_2 t^3 \right) dt$$

$$= (\lambda - \alpha)\left(\frac{\delta_1 u_1^2}{2} - \frac{\delta_1 \beta u_1^3}{6} + \frac{\delta_2 u_1^3}{3} - \frac{\delta_2 \beta u_1^4}{8} \right) + \alpha\left(\frac{\delta_1 u_2^2}{2} + \frac{\delta_2 u_2^3}{6} \right)$$

$$+ \frac{\alpha(\theta+\beta)}{2}\left(\frac{\delta_1 u_2^3}{3} + \frac{\delta_2 u_2^4}{12} \right)$$

$$= (\lambda-\alpha)\left(\frac{\delta_1 u_1^{2}}{2}-\frac{\delta_1\beta u_1^{3}}{6}+\frac{\delta_2 u_1^{3}}{3}-\frac{\delta_2\beta u_1^{4}}{8}\right)+\alpha\left(\frac{\delta_1(T-u_1)^2}{2}+\frac{\delta_2(T-u_1)^3}{6}\right)$$

$$+\frac{\alpha(\theta+\beta)}{2}\left(\frac{\delta_1(T-u_1)^3}{3}+\frac{\delta_2(T-u_1)^4}{12}\right) \quad (7)$$

The demand M_2 in the time period u_2 is given by

$$M_2 = \int_0^{u_2}(\alpha+\beta t)\,dt = \alpha u_2+\frac{\beta u_2^{2}}{2} \quad (8)$$

The number of Deteriorated items per cycle is given by

$$I_d = v_2(0)-M_2 = \alpha\left\{u_2+\frac{(\theta+\beta)u_2^{2}}{2}\right\}-\left(\alpha u_2+\frac{\beta u_2^{2}}{2}\right) = \frac{1}{2}(\alpha\theta+\alpha\beta-\beta)\,u_2^{2} \quad (9)$$

The total variable inventory cost in a period TRC is given by
TRC = set up cost (C_o) + cost of deteriorated items (kI_d) +holding cost(C_p).
The average total variable inventory cost per unit time, TRC(T), is given by

$$TRC(T) = \frac{1}{T}\left(C_0+k\,I_d+C_p\right)$$

$$= \frac{C_0}{T}+\frac{k}{T}(\alpha\theta+\alpha\beta-\beta)\frac{(T-u_1)^2}{2}+\frac{1}{T}(\lambda-\alpha)\left(\frac{\delta_1 u_1^{2}}{2}-\frac{\delta_1\beta u_1^{3}}{6}+\frac{\delta_2 u_1^{3}}{3}-\frac{\delta_2\beta u_1^{4}}{8}\right)$$

$$+\frac{\alpha}{T}\left(\frac{\delta_1(T-u_1)^2}{2}+\frac{\delta_2(T-u_1)^3}{6}\right)+\frac{\alpha(\theta+\beta)}{2T}\left(\frac{\delta_1(T-u_1)^3}{3}+\frac{\delta_2(T-u_1)^4}{12}\right) \quad (10)$$

To minimize the total variable cost per unit time TRC (T), we differentiate TRC (T) with respect to T and set the result to zero. A positive value of T for which sufficient condition $\frac{d^{2}(TRC(T))}{dT^{2}}>0$ gives a minimum for the total average cost function TRC (T).Using this value of T, the optimal values of TRC (T) and V_m can, in theory, be calculated. However, because of the complexity of the model, it is checked that the optimal numerical solution of T satisfies the sufficient condition.

$$\frac{d}{dT}(TRC(T)) = C_0\frac{d}{dT}\left(\frac{1}{T}\right)+k\,(\alpha(\theta+\beta)-\beta)\frac{d}{dT}\left(\frac{(T-u_1)^2}{2T}\right)+(\lambda-\alpha)\left(\frac{\delta_1 u_1^{2}}{2}-\frac{\delta_1\beta u_1^{3}}{6}+\frac{\delta_2 u_1^{3}}{3}-\frac{\delta_2\beta u_1^{4}}{8}\right)\frac{d}{dT}\left(\frac{1}{T}\right)$$

$$+\frac{d}{dT}\left(\frac{\alpha}{T}\left(\frac{\delta_1(T-u_1)^2}{2}+\frac{\delta_2(T-u_1)^3}{6}\right)\right)+\frac{d}{dT}\left(\frac{\alpha(\theta+\beta)}{2T}\left(\frac{\delta_1(T-u_1)^3}{3}+\frac{\delta_2(T-u_1)^4}{12}\right)\right)$$

$$= -C_0\left(\frac{1}{T^{2}}\right)+k\,(\alpha\theta+\alpha\beta-\beta)\left(\frac{T^{2}-u_1^{2}}{2T^{2}}\right)-(\lambda-\alpha)\left(\frac{\delta_1 u_1^{2}}{2}-\frac{\delta_1\beta u_1^{3}}{6}+\frac{\delta_2 u_1^{3}}{3}-\frac{\delta_2\beta u_1^{4}}{8}\right)\left(\frac{1}{T^{2}}\right)$$

$$+\frac{\alpha\delta_1}{2}\left(1-\frac{u_1^{2}}{T^{2}}\right)+\frac{\alpha\delta_2}{6}\left(2T-3u_1+\frac{u_1^{3}}{T^{2}}\right)+\left(\frac{\alpha\delta_1(\theta+\beta)}{6}\left(2T-3u_1+\frac{u_1^{3}}{T^{2}}\right)\right)$$

$$+\left(\frac{\alpha(\theta+\beta)\delta_2}{24}\left(3T^2-8Tu_1+6u_1^2-\frac{u_1^4}{T^2}\right)\right)$$

$$=-C_0\left(\frac{1}{T^2}\right)+k\left(\alpha\theta+\alpha\beta-\beta\right)\left(\frac{T^2-u_1^2}{2T^2}\right)-\left(\lambda-\alpha\right)\left(\frac{\delta_1u_1^2}{2}-\frac{\delta_1\beta u_1^3}{6}+\frac{\delta_2u_1^3}{3}-\frac{\delta_2\beta u_1^4}{8}\right)\left(\frac{1}{T^2}\right)$$
$$+\frac{\alpha\delta_1}{2T^2}\left(T^2-u_1^2\right)+\frac{\alpha\delta_2}{6T^2}\left(2T^3-3T^2u_1+u_1^3\right)+\left(\frac{\alpha\delta_1(\theta+\beta)}{6T^2}\left(2T^3-3T^2u_1+u_1^3\right)\right)$$
$$+\left(\frac{\alpha(\theta+\beta)\delta_2}{24T^2}\left(3T^4-8T^3u_1+6T^2u_1^2-u_1^4\right)\right)$$

and setting it to zero, we have

$$-C_0\left(\frac{1}{T^2}\right)+k\left(\alpha\theta+\alpha\beta-\beta\right)\left(\frac{T^2-u_1^2}{2T^2}\right)-\left(\lambda-\alpha\right)\left(\frac{\delta_1u_1^2}{2}-\frac{\delta_1\beta u_1^3}{6}+\frac{\delta_2u_1^3}{3}-\frac{\delta_2\beta u_1^4}{8}\right)\left(\frac{1}{T^2}\right)$$
$$+\frac{\alpha\delta_1}{2T^2}\left(T^2-u_1^2\right)+\frac{\alpha\delta_2}{6T^2}\left(2T^3-3T^2u_1+u_1^3\right)+\left(\frac{\alpha\delta_1(\theta+\beta)}{6T^2}\left(2T^3-3T^2u_1+u_1^3\right)\right)$$
$$+\left(\frac{\alpha(\theta+\beta)\delta_2}{24T^2}\left(3T^4-8T^3u_1+6T^2u_1^2-u_1^4\right)\right)=0$$

or

$$-24C_0+12k\left(\alpha\theta+\alpha\beta-\beta\right)\left(T^2-u_1^2\right)-\left(\lambda-\alpha\right)\left(12\delta_1u_1^2-4\delta_1\beta u_1^3+8\delta_2u_1^3-3\delta_2\beta u_1^4\right)$$
$$+12\alpha\delta_1\left(T^2-u_1^2\right)+4\alpha\delta_2\left(2T^3-3T^2u_1+u_1^3\right)+\left(4\alpha\delta_1(\theta+\beta)\left(2T^3-3T^2u_1+u_1^3\right)\right)$$
$$+\alpha(\theta+\beta)\delta_2\left(3T^4-8T^3u_1+6T^2u_1^2-u_1^4\right)=0$$

which simplifies to

$$3\alpha\delta_2(\theta+\beta)T^4+\left\{8\alpha\delta_2+8\alpha\delta_1(\theta+\beta)-8\alpha\delta_2u_1(\theta+\beta)\right\}T^3$$
$$+\left\{12k(\alpha\theta+\alpha\beta-\beta)+12\alpha\delta_1-12\alpha\delta_2u_1-12\alpha\delta_1u_1(\theta+\beta)+6\alpha u_1^2\delta_2(\theta+\beta)\right\}T^2$$
$$+\left\{-24C_0-12u_1^2k(\alpha\theta+\alpha\beta-\beta)-\left(\lambda-\alpha\right)\left(12\delta_1u_1^2-4\delta_1\beta u_1^3+8\delta_2u_1^3-3\delta_2\beta u_1^4\right)\right.$$
$$\left.-12\alpha\delta_1u_1^2+4\alpha\delta_2u_1^3+4\alpha\delta_1(\theta+\beta)u_1^3-\alpha\delta_2(\theta+\beta)u_1^4\right\}=0$$
$$\psi_1T^4+\psi_2T^3+\psi_3T^2+\psi_4=0 \quad (11)$$

where

$$\psi_1=3\alpha\delta_2(\theta+\beta)$$
$$\psi_2=8\alpha\delta_2+8\alpha\delta_1(\theta+\beta)-8\alpha\delta_2u_1(\theta+\beta)$$
$$\psi_3=12k(\alpha\theta+\alpha\beta-\beta)+12\alpha\delta_1-12\alpha\delta_2u_1-12\alpha\delta_1u_1(\theta+\beta)+6\alpha u_1^2\delta_2(\theta+\beta)$$
$$\psi_4=\left\{-24C_0-12u_1^2k(\alpha\theta+\alpha\beta-\beta)-\left(\lambda-\alpha\right)\left(12\delta_1u_1^2-4\delta_1\beta u_1^3+8\delta_2u_1^3-3\delta_2\beta u_1^4\right)\right.$$
$$\left.-12\alpha\delta_1u_1^2+4\alpha\delta_2u_1^3+4\alpha\delta_1(\theta+\beta)u_1^3-\alpha\delta_2(\theta+\beta)u_1^4\right\}$$

Equation (11) is solved numerically by using any suitable numerical method such as Newton – Raphson method.

NUMERICAL EXAMPLES

Table 1: Some numerical examples to illustrate the theoretical results

S/No	α	β	θ	δ_1	δ_2	C_0	u_1	λ	k	T	TVC	EPQ(V_m)
1	75	0.70	0.1	30	40	1200	29.20 days	200	15	81.03 days	5649.81	2.38
2	50	0.60	0.2	40	50	1100	32.85 days	180	20	75.56 days	4659.84	13.65
3	40	0.50	0.05	30	40	1000	36.50 days	160	25	80.67 days	4652.28	11.70
4	90	0.04	0.08	60	75	1400	54.75 days	250	10	75.92 days	7357.61	20.47
5	80	0.10	0.02	75	95	1300	73.00 days	220	5	100.01 days	5690.19	27.72

The results as tabulated in Table 1 reveals that for various values of inventory level dependent demand rate parameter $(\beta = 0.70, 0.60, 0.50, 0.04, 0.10)$ and deterioration rates $(\theta = 0.10, 0.20, 0.05, 0.08, 0.02)$, the optimum values of production quantity and average total cost function per unit time have been determined using the expressions for (4) and (10) respectively. An increase in θ, β or both leads to an increase in the total system cost under optimal production quantity. Based on the model developed, both the optimal production quantity and the production cycle time T increase when the holding cost increase. Also, It can be readily seen in the table that as the inventory level dependent demand rate increases the optical production quantity deceases, however, holding cost increases with increasing cycle time.

Conclusions

In this article, an economic production quantity (EPQ) model is presented for single product with delayed deterioration in which the production rate is constant, demand rate is inventory level dependent in a linear functional form before and after production and the holding cost is a linear function of time. An analytic formulation of the problem on the framework described above has been provided and also an optimal solution procedure with the help of Newton-Raphson method was used to find the optimal replenishment policy. Some numerical examples are also provided to illustrate the theoretical results.

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