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A new Tripartite Randomized Response Technique

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Abstract

In any research involving sensitive questions, respondents tend to refuse to answer such questions or give evasive response. Many randomized response sampling techniques were developed but could only handle two answer options; 'Yes' and 'No', among which are [8], e.t.c. In this paper, a tripartite randomized response sampling technique was developed to handle the three answer options: 'Yes', 'No' and 'Undecided',based on the modification of [8] model.

Keywords: Stigmatized attributes, Tripartite Randomized Response Technique (TRRT), response and non response bias

1.0 Introduction

Randomized Response (RR) techniques were developed for the purpose of protecting surveyee's privacy and avoiding answer bias mainly. They were introduced by [9] as a technique to estimate the percentage of people in a population U that has a stigmatizing attribute A. In such cases respondents may decide not to reply at all or to incorrectly answer. The usual problem faced by researchers is to encourage participants to respond, and then to provide truthful response in surveys. The Randomized Response Technique is a survey method which appears to be particularly appropriate for the study of sensitive attitudes and behaviours. Probability theory is adopted to protect the privacy of an individual's response, and has been used successfully in the study of sensitive health behaviours such as rape, abortion and the use of contraceptives or condom e.t.c. With randomized response technique, a respondent is presented with a number of alternative questions or response options.

Warner [9] suggested an ingenious method to estimate the proportion of a sensitive characteristic like induced abortion, drug usage, tax evasion, shoplifting, cheating in exam, etc. To maintain the anonymity of the respondents [9] proposed to use a randomization device such as a deck of cards or a spinner. Greenberg et al. [5] borrowed the idea and extended it to the estimation of the mean of sensitive quantitative variables. Different modifications of [9] Randomized Response were further developed by authors including [1, 2, 3, 4, 6, 7, 8] among many others.

The primary focus of this paper is the modification of Hussain-Shabbir's dichotomous Randomized Response Technique (RRT) to extend beyond a dichotomous answer-options to a tripartite answer-options which may often enhance more honest answers to questions, reduces respondent misunderstanding, suspicion and confusion.

In section 2, we reviewed the Hussain and Shabbir's dichotomous Randomized Response Technique.

In section 3, the proposed Tripartite Randomized Response Technique was developed, and the variance was obtained.

In section 4, we conclude with some discussion about the Model developed.

2.0 Hussain-Shabbir's dichotomous Randomized Response Technique (RRT)

Hussain and Shabbir [8] proposed a Randomized Response Technique (RRT) based on the random use of one of the two randomization devices R_1 and R_2. In design, the two randomization devices R_1 and R_2 are the same as that of [9] device but with different probabilities of selecting the sensitive question. The idea behind this suggestion is to decrease the suspicion among the respondents by providing them choice to randomly choose the randomization device itself. As a result, respondents may divulge their true status. A simple random sample with replacement (SRSWR) sampling is assumed to select a sample of size n. Let $\propto$ and β be any two positive real numbers chosen such that $q = \frac{\alpha}{\propto+\beta}, (\alpha \neq \beta)$ is the probability of using R_1, where R_1 consists of the two statements of Warner's device but with preset probabilities P_1 and $1 - P_1$ and $1 - q = \frac{\beta}{\propto+\beta}$ is the probability of using R_2 ,where R_2 consists of the two statements of Warner's device also with preset probabilities P_2 and $1 - P_2$ respectively. For the i^{th} respondent, the probability of a "yes" response is given by

$$P(yes) = \emptyset = \frac{\alpha}{\alpha+\beta}[P_1\pi + (1 - P_1)(1 - \pi)] + \frac{\beta}{\alpha+\beta}[P_2\pi + (1 - P_2)(1 - \pi)] \qquad (2.1)$$

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$$\emptyset = \frac{\alpha[P_1\pi + (1 - P_1)(1-\pi)] + \beta[P_2\pi + (1 - P_2)(1-\pi)]}{\alpha + \beta} \qquad (2.2)$$

where π is the true population probability of yes response

By expanding and simplifying equation (2.2), we have

$$\emptyset = \frac{\pi[2\alpha P_1 - \alpha + 2\beta P_2 - \beta] + \alpha + \beta - P_1\alpha - P_2\beta}{\alpha + \beta} \qquad (2.3)$$

Substituting $P_1 = 1 - P_2$ into equation (2.3), we have

$$\emptyset = \frac{\pi[(\alpha - \beta)(1 - 2P_2)] + \beta + P_2\alpha - P_2\beta}{\alpha + \beta} \qquad (2.4)$$

$$\emptyset = \frac{\pi[(\alpha - \beta)(1 - 2P_2)] + P_2\alpha + P_1\beta}{\alpha + \beta} \qquad (2.5)$$

To provide the equal privacy protection in both the randomization devices R_1 and R_2, we put $P_2 = 1 - P_1$ into equation (2.5), to obtain:

$$\emptyset = \frac{\pi[(\alpha-\beta)(2P_1-1)]+P_1\beta+P_2\alpha}{\alpha+\beta} \qquad (2.6)$$

Hence,

$$\pi = \frac{\emptyset(\alpha + \beta) - P_1\beta - P_2\alpha}{(2P_1 - 1)(\alpha - \beta)}, P_1 \neq 1/2, \alpha \neq \beta \qquad (2.7)$$

The unbiased moment estimator of true probability of yes response (response rate) π was given by

$$\hat{\pi} = \frac{\hat{\emptyset}(\alpha + \beta) - P_1\beta - P_2\alpha}{(2P_1 - 1)(\alpha - \beta)} \qquad (2.8)$$

where $\hat{\pi}$ is the unbiased sample true probability of yes response of π

and $\hat{\emptyset} = \frac{y}{n}$; y is the number of respondents reporting a "yes" answer when $P_1 = 1 - P_2$. The variance of the estimator was given then by

$$V(\hat{\pi})_{conv} = \frac{\pi(1 - \pi)}{n} + \frac{(P_2\alpha + P_1\beta)(P_1\alpha + P_2\beta)}{n(2P_1 - 1)^2(\alpha - \beta)^2(\alpha + \beta)^2} \qquad (2.9)$$

3 *The proposed Randomized Response Technique (RRT)*

It has been discovered that despite the successful attempts by several authors in developing an efficient Randomized Response Techniques (RRTs), the developed techniques only considered a two-option of "yes" and "no" response. As a result of which we propose a new Randomized Response Technique (RRT) that will be based on the random use of one of the three randomization devices, R_1, R_2 and R_3. In design, the three randomization devices R_1, R_2 and R_3 are similar to that of Warner's device but with different probabilities of selection. In addition to α and β proposed earlier by Hussain and Shabbir, we introduce δ, a positive real number such that $q = \frac{\alpha}{\alpha+\beta+\delta}, \alpha \neq \beta \neq \delta$ is the probability of using R_1, where R_1 consists of the two statements of Warner's device and the new introduce device also with preset probabilities P_1, P_2 and P_3 respectively. By adopting Hussain and Shabbir's probability of a "yes" response for the i^{th} respondent, the probability of a "yes" response when the third option "undecided" is included is given by

$$Q(yes) = \varphi = \frac{\alpha}{\alpha+\beta+\delta}[P_1\pi + (1 - P_1)(1 - \pi)] + \frac{\beta}{\alpha+\beta+\delta}[P_2\pi + (1 - P_2)(1 - \pi)] + \frac{\delta}{\alpha+\beta+\delta}[P_3\pi + (1 - P_3)(1 - \pi)] \qquad (3.1)$$

$$= \frac{\alpha[P_1\pi + (1 - P_1)(1 - \pi)] + \beta[P_2\pi + (1 - P_2)(1 - \pi)] + \delta[P_3\pi + (1 - P_3)(1 - \pi)]}{\alpha + \beta + \delta} \qquad (3.2)$$

By expanding brackets and simplifying, equation (3.2) becomes

$$\varphi = \frac{[2\alpha P_1\pi - \alpha\pi + 2\beta P_2\pi - \beta\pi + 2\delta P_3\pi - \pi\delta] + [\alpha + \beta + \delta - P_1\alpha - P_2\beta - P_3\delta]}{\alpha+\beta+\delta} \qquad (3.3)$$

$$\pi = \frac{\varphi(\alpha + \beta + \delta) - [(\alpha + \beta + \delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \qquad (3.4)$$

Hence, the unbiased sample estimate of π is given as

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha + \beta + \delta) - [(\alpha + \beta + \delta) - P_1\alpha - P_2\beta - P_3\delta]}{2P_1\alpha + 2P_2\beta + 2P_3\delta - \alpha - \beta - \delta} \qquad (3.5)$$

Substituting $P_3 = 1 - P_1 - P_2$ into equation (3.5), we have

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha + \beta + \delta) - [(\alpha + \beta + \delta) - P_1\alpha - P_2\beta - \delta(1 - P_1 - P_2)]}{2P_1\alpha + 2P_2\beta + 2\delta(1 - P_1 - P_2) - \alpha - \beta - \delta} \qquad (3.6)$$

$$= \frac{\hat{\varphi}(\alpha + \beta + \delta) - [(\alpha + \beta) - P_1\alpha - P_2\beta + P_1\delta + P_2\delta)]}{2P_1\alpha + 2P_2\beta - 2P_1\delta - 2P_2\delta - \alpha - \beta + \delta} \qquad (3.7)$$

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha + \beta + \delta) - [(\alpha + \beta) + P_1(\delta - \alpha) + P_2(\delta - \beta)]}{2P_1(\alpha - \delta) + 2P_2(\beta - \delta) - (\alpha + \beta - \delta)} \qquad (3.8)$$

Remark: If we set δ to zero in equation (3.8), we recover the unbiased estimate of true probability of yes response given by Hussain and Shabbir in equation (2.8). Thus, equation (3.8) is the proposed response sampling technique which could be called a"Tripartite Randomized Response Technique (TRRT)". In deriving the variance of this response sampling, we obtain the following result:

Lemma: When $P_1 = 1 - P_2 - P_3$, the variance of the tripartite RRT is given by

$$V(\hat{\pi}) = \frac{(\alpha+\beta+\delta)^2\varphi(1-\varphi)}{n[2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)]^2} \qquad (3.9)$$

$$= \frac{\pi(1-\pi)}{n} + \frac{(P_1\alpha+P_2\beta+P_3\delta)(P_3\alpha+P_2\beta+P_1\delta)}{n[2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \qquad (3.10)$$

Proof:

By definition

$$\hat{\pi} = \frac{\hat{\varphi}(\alpha+\beta+\delta)-[(\alpha+\beta+\delta)-P_1\alpha-P_2\beta-P_3\delta]}{2P_1\alpha+2P_2\beta+2P_3\delta-\alpha-\beta-\delta} \qquad (3.11)$$

$$\hat{\pi}_{prop} = \frac{\hat{\varphi}(\alpha+\beta+\delta)-[(\alpha+\beta)+P_1(\delta-\alpha)+P_2(\delta-\beta)]}{2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)} \qquad (3.12)$$

$$V(\hat{\pi}) \equiv V\left[\frac{\hat{\varphi}(\alpha+\beta+\delta)-[(\alpha+\beta)+P_1(\delta-\alpha)+P_2(\delta-\beta)]}{2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)}\right] \qquad (3.13)$$

$$V(\hat{\pi}) = \frac{(\alpha+\beta+\delta)^2\varphi(1-\varphi)}{n[2P_1(\alpha-\delta)+2P_2(\beta-\delta)-(\alpha+\beta-\delta)]^2} \qquad (3.14)$$

$$\varphi^2 = \left[\frac{\begin{matrix}2\alpha P_1\pi-\alpha\pi+\alpha-\alpha P_1+2\beta P_2\pi\\ -\beta\pi+\beta-P_2\beta+2\delta P_3\pi-\delta\pi+\delta\\ -P_3\delta\end{matrix}}{\alpha+\beta+\delta}\right]\left[\frac{\begin{matrix}2\alpha P_1\pi-\alpha\pi+\alpha-\alpha P_1+2\beta P_2\pi\\ -\beta\pi+\beta-P_2\beta+2\delta P_3\pi-\delta\pi+\delta\\ -P_3\delta\end{matrix}}{\alpha+\beta+\delta}\right] \qquad (3.15)$$

$$= \frac{\left[\begin{matrix}
4\alpha^2P_1^2\pi^2-2\alpha^2P_1\pi^2+2\alpha^2P_1\pi-2\alpha^2P_1^2\pi+4\alpha\beta P_1P_2\pi^2-2\alpha\beta P_1\pi^2+2\alpha\beta P_1\pi\\
-2\alpha\beta P_1P_2\pi+4\alpha\delta P_1P_3\pi^2-2\alpha\delta P_1\pi^2+2\alpha\delta P_1\pi-2\alpha\delta P_1P_3\pi-2\alpha^2P_1\pi^2+\alpha^2\pi^2\\
-\alpha^2\pi+\alpha^2P_1\pi-2\alpha\beta P_2\pi^2+\alpha\beta\pi^2-\alpha\beta\pi+\alpha\beta P_2\pi-2\alpha\delta P_3\pi^2+\alpha\delta\pi^2-\alpha\delta\pi\\
+\alpha\delta P_3\pi+2\alpha^2P_1\pi-\alpha^2\pi+\alpha^2-\alpha^2P_1+2\alpha\beta P_2\pi-\alpha\beta\pi+\alpha\beta-\alpha\beta P_2\\
+2\alpha\delta P_3\pi-\alpha\delta\pi+\alpha\delta-\alpha\delta P_3-2\alpha^2P_1^2\pi+\alpha^2P_1\pi-\alpha^2P_1+\alpha^2P_1^2\\
-2\alpha\beta P_1P_2\pi+\alpha\beta P_1\pi-\alpha\beta P_1+\alpha\beta P_1P_2-2\alpha\delta P_1P_3\pi+\alpha\delta P_1\pi-\alpha\delta P_1+\alpha\delta P_1P_3\\
+4\alpha\beta P_1P_2\pi^2-2\alpha\beta P_2\pi^2+2\alpha\beta P_2\pi-2\alpha\beta P_1P_2\pi+4\beta^2P_2^2\pi^2-2\beta^2P_2\pi^2\\
+2\beta^2P_2\pi-2\beta^2P_2^2\pi+4\beta\delta P_2P_3\pi^2-2\beta\delta P_2\pi^2+2\beta\delta P_2\pi-2\beta\delta P_2P_3\pi-2\alpha\beta P_1\pi^2\\
+\alpha\beta\pi^2-\alpha\beta\pi+\alpha\beta P_1\pi-2\beta^2P_2\pi^2+\beta^2\pi^2-\beta^2\pi+\beta^2P_2\pi-2\beta\delta P_3\pi^2+\beta\delta\pi^2\\
-\beta\delta\pi+\beta\delta P_3\pi+2\alpha\beta P_1\pi-\alpha\beta\pi+\alpha\beta-\alpha\beta P_1+2\beta^2P_2\pi-\beta^2\pi+\beta^2\\
-\beta^2P_2+2\beta\delta P_3\pi-\beta\delta\pi+\beta\delta-\beta\delta P_3-2\alpha\beta P_1P_2\pi+\alpha\beta P_2\pi-\alpha\beta P_2\\
+\alpha\beta P_1P_2-2\beta^2P_2^2\pi+\beta^2P_2\pi-\beta^2P_2+\beta^2P_2^2-2\beta\delta P_2P_3\pi+\beta\delta P_2\pi-\beta\delta P_2\\
+\beta\delta P_2P_3+4\alpha\delta P_1P_3\pi^2-2\alpha\delta P_3\pi^2+2\alpha\delta P_3\pi-2\alpha\delta P_1P_3\pi+4\beta\delta P_2P_3\pi^2-2\beta\delta P_3\pi^2\\
+2\beta\delta P_3\pi-2\beta\delta P_2P_3\pi+4\delta^2P_3^2\pi^2-2\delta^2P_3\pi^2+2\delta^2P_3\pi-2\delta^2P_3^2\pi\\
-2\alpha\delta P_1\pi^2+\alpha\delta\pi^2-\alpha\delta\pi+\alpha\delta P_1\pi-2\beta\delta P_2\pi^2+\beta\delta\pi^2-\beta\delta\pi\\
+\beta\delta P_2\pi-2\delta^2P_3\pi^2+\delta^2\pi^2-\delta^2\pi+\delta^2P_3\pi+2\alpha\delta P_1\pi-\alpha\delta\pi+\alpha\delta\\
-\alpha\delta P_1+2\beta\delta P_2\pi-\beta\delta\pi+\beta\delta-\beta\delta P_2+2\delta^2P_3\pi-\delta^2\pi+\delta^2\\
-\delta^2P_3-2\alpha\delta P_1P_3\pi+\alpha\delta P_3\pi-\alpha\delta P_3+\alpha\delta P_1P_3-2\beta\delta P_2P_3\pi+\beta\delta P_3\pi\\
-\beta\delta P_3+\beta\delta P_2P_3-2\delta^2P_3^2\pi+\delta^2P_3\pi-\delta^2P_3+\delta^2P_3^2
\end{matrix}\right]}{(\alpha+\beta+\delta)^2} \qquad (3.16)$$

$$= \frac{\left[\begin{matrix}
4\alpha^2P_1^2\pi^2+4\delta^2P_3^2\pi^2+4\beta^2P_2^2\pi^2-4\alpha^2P_1\pi^2+6\alpha^2P_1\pi-4\alpha^2P_1^2\pi\\
+8\alpha\beta P_1P_2\pi^2-4\alpha\beta P_1\pi^2+6\alpha\beta P_1\pi-4\alpha\beta P_1P_2\pi+8\alpha\delta P_1P_3\pi^2\\
-4\alpha\delta P_1\pi^2+6\alpha\delta P_1\pi-4\alpha\delta P_1P_3\pi+\alpha^2\pi^2+\beta^2\pi^2+\delta^2\pi^2\\
-2\alpha^2\pi-4\alpha\beta P_2\pi^2+2\alpha\beta\pi^2-4\alpha\beta\pi+6\alpha\beta P_2\pi-4\alpha\delta P_3\pi^2+2\alpha\delta\pi^2\\
-4\alpha\delta\pi+6\alpha\delta P_3\pi+\alpha^2+\beta^2+\delta^2-2\alpha^2P_1+2\alpha\beta-2\alpha\beta P_2\\
+2\alpha\delta-2\alpha\delta P_3+\alpha^2P_1^2-2\alpha\beta P_1+2\alpha\beta P_1P_2-2\alpha\delta P_1+2\alpha\delta P_1P_3\\
-4\alpha\beta P_1P_2\pi-4\beta^2P_2\pi^2+6\beta^2P_2\pi-4\beta^2P_2^2\pi+8\beta\delta P_2P_3\pi^2-4\beta\delta P_2\pi^2\\
+6\beta\delta P_2\pi-8\beta\delta P_2P_3\pi-2\beta^2\pi-4\beta\delta P_3\pi^2+2\beta\delta\pi^2-4\beta\delta\pi+6\beta\delta P_3\pi\\
-2\beta^2P_2+2\beta\delta-2\beta\delta P_3+\beta^2P_2^2-2\beta\delta P_2+2\beta\delta P_2P_3\\
-4\alpha\delta P_1P_3\pi-4\delta^2P_3\pi^2+6\delta^2P_3\pi-4\delta^2P_3^2\pi-2\delta^2\pi-2\delta^2P_3+\delta^2P_3^2
\end{matrix}\right]}{(\alpha+\beta+\delta)^2} \qquad (3.17)$$

$$\varphi-\varphi^2 = \frac{\left[\begin{matrix}
-4\alpha^2P_1^2\pi^2-4\delta^2P_3^2\pi^2-4\beta^2P_2^2\pi^2+4\alpha^2P_1\pi^2-4\alpha^2P_1\pi+4\alpha^2P_1^2\pi\\
-8\alpha\beta P_1P_2\pi^2+4\alpha\beta P_1\pi^2-4\alpha\beta P_1\pi+8\alpha\beta P_1P_2\pi-8\alpha\delta P_1P_3\pi^2\\
+4\alpha\delta P_1\pi^2-4\alpha\delta P_1\pi+4\alpha\delta P_1P_3\pi-\alpha^2\pi^2-\beta^2\pi^2-\delta^2\pi^2\\
+\alpha^2\pi+4\alpha\beta P_2\pi^2-2\alpha\beta\pi^2+2\alpha\beta\pi-4\alpha\beta P_2\pi+4\alpha\delta P_3\pi^2-2\alpha\delta\pi^2\\
+2\alpha\delta\pi-4\alpha\delta P_3\pi+\alpha^2P_1+\alpha\beta P_2+\alpha\delta P_3-\alpha^2P_1^2+\alpha\beta P_1\\
-2\alpha\beta P_1P_2+\alpha\delta P_1-2\alpha\delta P_1P_3+4\beta^2P_2\pi^2-4\beta^2P_2\pi+4\beta^2P_2^2\pi\\
-8\beta\delta P_2P_3\pi^2+4\beta\delta P_2\pi^2-4\beta\delta P_2\pi+8\beta\delta P_2P_3\pi+\beta^2\pi+4\beta\delta P_3\pi^2\\
-2\beta\delta\pi^2+2\beta\delta\pi-4\beta\delta P_3\pi+\beta^2P_2+\beta\delta P_3-\beta^2P_2^2+\beta\delta P_2\\
-2\beta\delta P_2P_3+4\alpha\delta P_1P_3\pi+4\delta^2P_3\pi^2-4\delta^2P_3\pi+4\delta^2P_3^2\pi\\
+\delta^2\pi+\delta^2P_3-\delta^2P_3^2
\end{matrix}\right]}{(\alpha+\beta+\delta)^2} \qquad (3.18)$$

Hence, we have

$$V(\hat{\pi}) = \frac{(\alpha+\beta+\delta)^2\begin{bmatrix}\pi(4\alpha^2P_1^2 - 4\alpha^2P_1 - 4\alpha\beta P_1 + 8\alpha\beta P_1P_2 - 4\alpha\delta P_1 \\ +8\alpha\delta P_1P_3 + \alpha^2 + 2\alpha\beta - 4\alpha\beta P_2 + 2\alpha\delta \\ -4\alpha\delta P_3 - 4\beta^2P_2 + 4\beta^2P_2^2 - 4\beta\delta P_2 + 8\beta\delta P_2P_3 \\ +\beta^2 + 2\beta\delta - 4\beta\delta P_3 - 4\delta^2P_3 + 4\delta^2P_3^2 + \delta^2) \\ -\pi^2(4\alpha^2P_1^2 - 4\alpha^2P_1 - 4\alpha\beta P_1 + 8\alpha\beta P_1P_2 \\ -4\alpha\delta P_1 + 8\alpha\delta P_1P_3 + \alpha^2 + 2\alpha\beta - 4\alpha\beta P_2 \\ +2\alpha\delta - 4\alpha\delta P_3 - 4\beta^2P_2 + 4\beta^2P_2^2 - 4\beta\delta P_2 \\ +8\beta\delta P_2P_3 + \beta^2 + 2\beta\delta - 4\beta\delta P_3 - 4\delta^2P_3 \\ +4\delta^2P_3^2 + \delta^2)\end{bmatrix} \begin{array}{c} +[\alpha^2P_1 + \alpha\beta P_2 + \alpha\delta P_3 - \alpha^2P_1^2 + \alpha\beta P_1 - 2\alpha\beta P_1P_2 \\ +\alpha\delta P_1 - 2\alpha\delta P_1P_3 + \beta^2P_2 + \beta\delta P_3 - \beta^2P_2^2 + \\ \beta\delta P_2 - 2\beta\delta P_2P_3 + \delta^2P_3 - \delta^2P_3^2]\end{array}}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \quad (3.19)$$

Substituting $P_3 = 1 - P_1 - P_2$ into equation (3.19) and in line with Hussain and Shabbir (2007), it thus follows that

$$[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2 = 4\alpha^2P_1^2 - 4\alpha^2P_1 - 4\alpha\beta P_1 + 8\alpha\beta P_1P_2 - 4\alpha\delta P_1 + 8\alpha\delta P_1P_3 + \alpha^2 + 2\alpha\beta - 4\alpha\beta P_2 + 2\alpha\delta - 4\alpha\delta P_3 - 4\beta^2P_2 + 4\beta^2P_2^2 - 4\beta\delta P_2 + 8\beta\delta P_2P_3 + \beta^2 + 2\beta\delta - 4\beta\delta P_3 - 4\delta^2P_3 + 4\delta^2P_3^2$$

Hence,

$$V(\hat{\pi}) = \frac{\pi(1-\pi)}{n} + \frac{\begin{array}{c}\alpha^2P_1 + \alpha\beta P_2 + \alpha\delta P_3 - \alpha^2P_1^2 + \alpha\beta P_1 - 2\alpha\beta P_1P_2 \\ +\alpha\delta P_1 - 2\alpha\delta P_1P_3 + \beta^2P_2 + \beta\delta P_3 - \beta^2P_2^2 + \beta\delta P_2 \\ -2\beta\delta P_2P_3 + \delta^2P_3 - \delta^2P_3^2\end{array}}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \quad (3.20)$$

Factoring the numerator and setting $P_3 = 1 - P_1 - P_2$ in equation (3.20) gives

$$V(\hat{\pi}) = \frac{\pi(1-\pi)}{n} + \frac{(P_1\alpha + P_2\beta + P_3\delta)(P_3\alpha + P_2\beta + P_1\delta)}{n[2P_1(\alpha-\delta) + 2P_2(\beta-\delta) - (\alpha+\beta-\delta)]^2(\alpha+\beta+\delta)^2} \quad (3.21)$$

4. Conclusion

In this paper, the work of Hussain and Shabbir (2007) was reviewed. However, a major lapse in Hussain-Shabbir's dichotomous Randomized Response Technique (RRT) is that it did not consider all the three response options which are not trivial in real life situation. In consideration of this lapse, we proposed a new Randomized Response Technique (RRT) called "Tripartite Randomized Response Technique " which considers all the three response options by modifying Hussain-Shabbir's dichotomous Randomized Response Technique (RRT) . Therefore, Numerical Comparison of the proposed Randomized Response Technique and some existing Techniques can be a promising future study.

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