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Energy Generation in the Human Body by the Human Cells

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Abstract

We adapted the thermodynamics equation for energy generation in a diesel engine in modeling energy generation in human body by the human cells by doing a thorough study on both systems and saw that the process of energy generation is the same in them. We equally saw that the stages involved in energy generation are similar. The adapted equations were properly modified to explain our current study. The resulting models were then solved both for the steady and non-steady cases.

Keywords: Thermodynamics, diesel engine, human cell, duhamel's principle.

1.0 Introduction

Food production by plants is by photosynthesis. This is the process whereby the green leaves of the plant in the presence of sun energy, CO_2 (carbondioxide) and water will produce the foods needed by both the plant itself and other animals. This food production takes place in the confined sites in the green leaves called the chloroplast. It is in the chloroplasts that we find the chlorophyll and the rest of the complex apparatus that carries out the light-dependent reactions of photosynthesis, those in which the light energy is absorbed and then stored in the form of NADPH (Nicotinamide Adenine Dinucleotide Phosphate) and ATP, AdenosineTriphosphete, [6].

However, one important fact to note is that in the course of food production by the plant, light energy is used in the process which is then held as a stored energy in the food only to be released when this food is oxidized. This stored energy is in the form of ATP and was used in the manufacture of storable carbohydrates which can be converted back to ATP when the need arises. Also, the chloroplast passes an electron transport system for producing ATP which is taken from water. During photosynthesis, CO_2 is reduced to carbohydrate by the energy obtained from the ATP [6, 12]. In general therefore, the reaction in photosynthesis that leads to carbohydrate production can be written in the form: $light\,energy + 6CO_2 + 12H_2O \rightarrow C_6H_{12}O_6 + 6H_2O + 6O_2$.

It has also to be noted that this reaction is possible with the reducing power of NADPH which are usually synthesized in the membranes of the photosynthesis apparatus through electron transport system which resembles those found in respiration [6, 12]. Therefore we can say that photosynthesis consists of a light-dependent process in which ATP and NADPH are generated and a light-independent process in which these are used to reduce CO_2 to produce carbohydrates.

In any living being, we find that it is made up of cells. It is these cells that utilize the available resources to generate the energy necessary for the maintenance of life of both the cells themselves and the organism in general. By energy here, we mean the necessary constituents of the body cells that enable the cells to continue living or being alive. In the absence of this energy, the cell will fail to function or die.

In a typical human cell, one of the major sources of energy is the sugar (glucose). This simple sugar (glucose) is the most important material catabolized by most cells [8]. This sugar is manufactured during photosynthesis by the plants and usually stored for usage during respiration. Many micro-organisms live on it and it is the main source of energy that animals carry in their blood to nourish their cells.

Once glucose gets into a cell, it will be oxidized primarily through the glycolytic or Embden-Mayerford pathway [13]. This glucose though full of energy, is not quite energetic enough to go through the initial reaction and so in the first few steps of the pathway, it has to be activated by the ATP. In general therefore, human cells generate energy through three steps: glycolysis, the Kreb's cycle and finally the electron transport chain.

Glycolysis must first take place to generate the initial ATP required for the other stages. In glycolysis, a total of four molecules of ATP are produced where two of them are used up by the process itself. The Kreb's cycle stage helps in producing ten molecules of NADH (Nicocinamide Adenine Dinucleotide Hydrolysed) and two molecules of $FADH_2$ (Flavin

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Adenine Dinucleotide). These two substances then drive the next stage which is the electron transport chain, the determinant of ATP production in the mitochondria, regarded as the power house of the human cell. It has to be stated clearly that ATP is the energy carrier and the amount of energy it carries is just the right amount for most biological reactions [8]. Because of the importance of ATP to life, it is regarded as the second most important macromolecule. ATP is the most widely distributed high energy compound within the human body [18]. Generally speaking, we can say that all fuel sources of nature, all foodstuffs of living things produce ATP which on its own powers virtually every activity of the cells and the entire organism.

A look at how energy is transferred from the ATP to the cells will justify the need for proper understanding of how this takes place, the danger of interrupting any of the steps or even attempting to alter any of the chemicals needed in the process of ATP production and energy release. ATP reacts with substances in the cell and this removes one of the phosphate-oxygen groups and therefore reduces to ADP (Adenosine Diphosphate). In the process, energy is usually liberated. This ADP is immediately recycled in the mitochondria where it is recharged and it comes out of it again as ATP [20]. In a given human being, we have about one hundred trillion cells of which each cell contains about one billion ATP. The amount of this ATP produced by all these cells is just sufficient for the cell's needs for only a few minutes and therefore must be recharged for further production. Therefore, we discover that for each ATP, the terminal phosphate is added and removed thrice in a minute [11]. In general, the total body content of ATP is only about 50 grams and this must be recycled constantly every day. Therefore, for one to continue the supply of energy, food must be eaten. ATP is simply the carrier and regulation-storage unit of energy.

ATP PRODUCTION IN THE CELL.

As said, we have three steps of energy (ATP) generation in the human cells. We shall now consider how and the quantity of ATP produced in each of these steps.

a) GLYCOLYSIS:

This is a series of reactions which converts glucose to two identical C_3 units (pyruvic acid or pyruvate) of lower free energy in a process that harnesses the released free energy to synthesize ATP from ADP and phosphates. In the first case, glucose has to enter the cell before any reaction will take place [6, 8, and 13]. The glycolytic pathway requires a pathway of chemically coupled phosphoryl-transfer reactions of which the chemical strategy is to:

1) add phosphoryl groups to glucose;
2) chemically convert phosphorylated intermediates into compounds with high phosphate group-transfer potentials;
3). chemically couple the subsequent hydrolysis of reaction substances to ATP synthesis.

In glycolysis, each molecule of glucose is oxidized to yield two molecules of pyruvic acid along side two ATPs and two NADH. In the absence of oxygen, anaerobic condition, homolactic fermentation of pyruvate occurs in the muscles while under aerobic condition (presence of oxygen), pyruvate is oxidized to acetyl coenzymes A and this reacts with oxoloacetate to begin the next stage of ATP production called the Kreb's cycle. Further details of glycolysis can be found in any good and standard biology text. In chemical terms, the equation of reaction is:

$$Glu\cos e + 2NAD^{+} + 2ADP + 2P_i \rightarrow 2NADH + 2Pyruvate + 2ATP + 2H_2O + 4H^{+}$$

a) The Kreb's Cycle:

This was discovered by a German British Biochemist Hans Kreb. He discovered that the pyruvic acid molecules are converted to carbondioxide (CO_2) while two more ATP molecules are produced for each mole of glucose. Detailed reactions and treatments that led to this result can be found in [2, 4, 8, 15, and 21]. The Kreb's cycle reactions take place at the mitochondrial matrix. In general, one glucose molecule causes two turns of Kreb's cycle which results in the production of six NADH, two $FADH_2$, two ATP and four CO_2 molecules. It is these NADH and $FADH_2$ produced in this Kreb's cycle that drives the next stage of aerobic respiration which is the Electron Transport Chain.

b) Electron Transport Chain:

This is where most of the energy transfer from glucose to ATP actually occurs. In this stage, many of the compounds that make up the electron transport chain belong to a special group of chemicals called the Cytochromes. Thus, in electron transport chain, we find a system of electron carriers embedded in the inner membrane of mitochondria in eukaryotes. It is made of several kinds of carriers that can be reduced and oxidized reversibly. Without O_2, H^+ will be idle and cannot react with the cytochromes to form water. In the absence of oxygen, NADH cannot be oxidized to NAD^+ and thus the Kreb's cycle will stop, resulting to eventual non-production of ATP. Note that in this stage where bulk of the ATP is produced, there is a type of transport just like the transportation of fuel or diesel from the tank to the carburetor as found in the car using engine. In the electron transport chain stage which also takes place in the mitochondria, energy is harvested and stored as ATP as electron are passed from one molecule to the next in the chain. It was discovered that for each molecule of

NADH that puts its two electrons into the reaction, approximately three molecules of ATP are produced while for each molecule of $FADH_2$, about two molecules of ATP are also produced. Detailed study of the ATP production in the mitochondria can be found in [4, 9, 10, 14, and 16].

In general therefore, most eukaryotic cells produce about 38 ATP molecules during aerobic respiration. Having seen this, let us now try to model the equations describing the change in the volume of ATP, the concentration of the glucose/sugar and the eventual energy release in the cell.

The Model

A comparatively good description of what happens in the human system is the process of energy generation in a diesel engine which generates the energy that sustains the running of the engine. The diesel engine has a carburetor (equivalent to the mitochondria) where the diesel is ignited and burnt to release heat energy. We see here that just like the reaction in the Kreb's cycle and mitochondria, oxygen is very important since we know that there would be no combustion in the absence of oxygen. The detailed functioning of a diesel engine have been discussed by several authors [4, 7, 19]. Comparing these two processes of energy generation in two different systems, we find that they are alike in many ways. The energy sources are similar- hydrocarbon. The initial source of energy to enable subsequent energy generation that sustains the systems was provided by the glycolytic pathway and the battery respectively. The burning of the sources of energy (glucose and fuel or diesel) is provided with chambers (mitochondria and carburetor) that are highly regulated. This is seen by comparing the inflow of glucose (in the form of acetyl CoA) into the mitochondria which is regulated by the electron transport chain and the fuel droplets into the carburetor which on its own is regulated by the metering processes. Based on these similarities in the two systems on their energy generation processes and requirements, we can liken the combustion in the carburetor to the ATP production in the mitochondria. We shall then adopt the models describing the non-homogeneous combustion reaction in the carburetor for our energy generation in the mitochondria with proper adjustments to take care of the nature of the systems involved in the reactions.

Thus, the combustion equations according to [1] are:

$$P_g C_{pg} \alpha_g \frac{\partial T_g}{\partial t} = \lambda_g \frac{\partial^2 T_g}{\partial x^2} + C_f Q_f \alpha_g \mu_f A \exp[-\frac{E}{R} T_g] - 4\pi R_d n_d (T_g - T_0) - 4\pi R_d^2 \sigma_1 n_d (T_g^4 - T_0^4) \quad (1)$$

$$\frac{d(R_d^2)}{dt} = -\frac{2\lambda_g}{\rho L}(T_g - T_0) - \frac{4\pi R_d \sigma_1}{L\rho}(T_g^4 - T_0^4) \quad (2)$$

$$\alpha_g \frac{\partial C_f}{\partial t} = D_f \frac{\partial^2 C_f}{\partial x^2} - C_f \alpha_g \mu_f A \exp[-\frac{E}{R} T_g] + \frac{4\pi R_d \lambda_g n_d (T_g - T_0)}{L\mu_g \alpha_g} + \frac{4\pi R_d^2 \sigma_1 n_d (T_g^4 - T_0^4)}{L\mu_g \alpha_g} \quad (3)$$

together with the initial and boundary conditions

$$T_g = T, C_f = C_{f_0}, R_d = R_{d_0}$$

$$T_g = T_0, C_f = C_{f_i}, at\ x = \pm 1, i = 1 \quad \text{where:}$$

T = temperature
E = activation energy
L = liquid evaporation energy
C =reactant
R_d = radius of drops
Q = heat released per unit mass

$$\sigma_1 = \frac{2\sigma\varepsilon_d}{2 - \varepsilon_d}$$

σ = Stefan – Boltzmann constant
ε_d = emissivity of the droplets surface
μ = molar mass
ρ = density
α = volumetric phase constant
n = number of drops per unit volume
λ = thermal conductivity
A =Pre – exponential factor

R = universal gas constant
Subscripts: g = gas mixture
l = liquid
f = combustible gas component of the mixture
d = liquid drops.

Since not much heat is released ordinarily in the human system, we shall be interested particularly about equations (2) and (3). However, these equations have to be modified to truly reflect and or represent the system being studied. From biological studies, glycolysis occurs at physiologically constant temperature which is 37^0C. Thus, our term T_g- T_0 can be simply represented by T. Also the activation energy can be taken as a constant since glycolysis is a spontaneous process and does not require any extra energy order than that present at the initial time. The process generates subsequent energy it needs to continue the process. Thus, the activation energy in this work will then be taken as the energy supplied by the glycolytic pathway which is the 2ATP and as such, E is constant per mole of glucose. Since the process occurs in the cell and considering the nature of the cell, evaporation does not occur so that the liquid evaporation energy is constant if at all it is required. The reactants in our study here are the sugar (glucose) and oxygen. The radius of drop can be similarly taken here to mean the volume of the glucose that enters into reaction which can be measured in mole.

In the human cell, we consider the thermal conductivity such that its value is zero, although we know that excess heat is removed from the body by perspiration or sweating. We shall equally drop the subscripts in the terms since sugar (glucose) does not appear in gaseous forms. Thus, the modified equations will be:

$$\frac{d(V^2)}{dt} = \frac{-4\pi V \sigma_1 T^4}{G\rho} \tag{4}$$

$$\alpha \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - C\alpha\mu A \exp - [\frac{E}{M}T] + \frac{4\pi V^2 \sigma_1 m T^4}{G\mu\alpha} \tag{5}$$

together with the initial and boundary conditions

$V = V_o$

$C = C_i$ at $x = \pm i$, $i = 1$

where V is the volume of glucose that enters into the mitochondria in the oxidized form.

T = temperature (constant)

$$\sigma_1 = \frac{2\sigma\varepsilon}{2-\varepsilon}$$

σ = Stefan – Boltzmann constant
ε = dissociative constant
G = dissociation energy of the glucose molecule
m = number of glucose molecule per unit volume
α = volumetric phase constant
E = activation energy in the form ATP (constant)
M = molar mass of the hydrocarbon
A = pre – exponential factor
C = concentration of the reactant
D = diffusion coefficient
ρ = density

It is these two equations that describe the change in the volume of the reactants that enters the mitochondria in the oxidized forms over a period of time as well as the change in the concentration over a period of time.

Solutions:

We shall solve the equation (4) first. Thus we have:

$$\frac{d(V^2)}{dt} = \frac{-4\pi V \sigma_1 T^4}{G\rho}$$

but $$\frac{d(V^2)}{dt} = 2V\frac{dV}{dt}$$

$$\Rightarrow 2V\frac{dV}{dt} = \frac{-4\pi V\sigma_1 T^4}{G\rho}$$

$$\frac{dV}{dt} = \frac{-2\pi\sigma_1 T^4}{G\rho}$$

Integrating gives

$$V = \frac{-2\pi\sigma_1 T^4}{G\rho}t + K$$

Applying the initial condition:

At t = 0, $V = V_o$

$$\Rightarrow \mathrm{K} = \mathrm{V_o}$$

$$\Rightarrow \quad V = \frac{-2\pi\sigma_1 T^4}{G\rho}t + V_0$$

$$\therefore\ V_0 - V = \frac{2\pi\sigma_1 T^4}{G\rho}t \qquad (S_1)$$

This implies that $V_0 > V$ and a valid result at $t > 0$.

For equation (5), we shall solve it for two different states to check the validity of each state. The states are:

1.) The steady state.
2.) The non-steady state.

CASE 1: The steady state:

Here, we try to see if the concentration remains the same at all times but probably varies depending on the location of interest at the mitochondria. Thus, for a steady state, $\frac{\partial C}{\partial t} = 0$ so that our equation (5) becomes:

$$\frac{\partial^2 C}{\partial x^2} = \frac{\alpha}{D}\frac{\partial C}{\partial t} + \frac{C\alpha\mu A \exp -[\frac{E}{M}T]}{D} - \frac{4\pi V^2(t)\sigma_1 m T^4}{G D \mu\alpha} \qquad (8)$$

Putting $\frac{\alpha}{D} = K_1$, we have

$$\frac{\alpha\mu}{D} A \exp -[\frac{E}{M}T] = K_2 \text{ and}$$

$$\frac{4\pi\sigma_1 m T^4}{G D \mu\alpha} = K_3$$

Then our equation (8) becomes:

$$\Rightarrow \frac{d^2C}{dx^2} - K_2 C = -K_3 V^2$$

Solving this, we obtain:

$$\Rightarrow C(x) = Ae^{\sqrt{K_2}x} + Be^{-\sqrt{K_2}x} + \frac{K_3 V^2}{K_2} \qquad (S_2)$$

where *A* and *B* are constants of integration.

CASE II: The non-steady state:

In this, we shall solve equation (5) the way it is .Hence, using the above substitutions, equation (5) will become:

$$\Rightarrow \quad K_1\frac{\partial C}{\partial t} - \frac{\partial^2 C}{\partial x^2} = +K_3 V^2(t) - K_2 C$$

Dividing through by K_1 gives

$$\frac{\partial C}{\partial t} - \frac{1}{K_1}\frac{\partial^2 C}{\partial x^2} = \frac{1}{K_1}\{K_3 V^2(t) - K_2 C\} \qquad (9)$$

Theorem: Duhamel's principle [17]

It states that if $f : \overline{Q^+} \to \Re$ is continuous and $u : \overline{Q^+} \to \Re$ is a solution of the following non – homogeneous initial/boundary value problem:

$$u_t(x,t) - u_{xx}(x,t) = f(x,t) \qquad (x,t) \in Q^+$$
$$u(x,0) = 0 \qquad x \in [0,1]$$
$$u(0,t) = u(1,t) = 0 \qquad t \in [0,\infty)$$

where

$$\overline{Q^+} = \{(x,t) \in \Re^2 : 0 < x < 1, 0 < t < \infty\}$$

and $\overline{Q^+}$ = closure of Q^+

then for each $P \in [0, \infty)$, let H (x, t, P) be the solution of the following pulse problem:

$$H_t - H_{xx} = 0, \quad (x,t) \in [0,1] \times [p, \infty)$$
$$H(x,t,p) = f(x,p) \qquad x \in [0,1]$$
$$H(0,t,p) = H(1,t,p) = 0 \qquad t \in [p, \infty)$$

then u and H satisfies

$$u(x,t) = \int_0^t H(x,t,p)\,dp$$

Applying this theorem or principle in our problem here, we set:

$$\frac{1}{K_1}\{K_3 V^2(t) - K_2 C\} = f(x,t),$$

$$u(x,t) = C(x,t) \qquad \text{and} \quad C(x,t) = \int_0^t H(x,t,p)\,dp \ .$$

Then, equation (9) becomes:

$$\Rightarrow \quad \frac{\partial C}{\partial t} - \frac{1}{K_1}\frac{\partial^2 C}{\partial x^2} = f(x,t)$$

This implies that

$$H_t - \frac{1}{k_1} H_{xx} = 0 \qquad (x,t) \in (0,1) \times [p, \infty)$$

Now, let

$$H(x,t,p) = X(x)T(t)P(p)$$

This implies that

$$H_t = X(x)T'(t)P(p)$$
$$H_{xx} = X''(x)T(t)P(p)$$

So that

$$XT'P - \frac{1}{K_1} X''TP = 0$$

$$\Rightarrow \quad XT' - \frac{1}{K_1} X''T = 0$$

$$\Rightarrow \quad \frac{T'}{T} = \frac{1}{K_1}\frac{X''}{X} = \lambda$$

$$\Rightarrow \quad \frac{T'}{T} = \lambda \qquad \text{(a)}$$

and

$$\Rightarrow \quad \frac{X''}{X} = K_1 \lambda \qquad \text{(b)}$$

Solving (a) and (b), we have;

$$\Rightarrow \quad T = Re^{\lambda t}, \; R = e^{\zeta_1} \quad \text{where } \zeta_1 \text{ is a constant.}$$

and $$\therefore \quad X(x) = \zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}$$

Thus we have;

$$\therefore \; H(x,t,p) = X(x)T(t)P(p) = (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) Re^{\lambda t} P(p).$$

Since $P \in [0,\infty)$ *and* $U(x,0) = 0$ *for* $x \in [0,1]$, then we can regard $U_p = 0$ to imply that U(*x, t, p*) = constant and as such, we can say that U varies only with respect to *x* and *t*.Thus, we have;

$$\Rightarrow \quad H(x,t,p) = (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) Re^{\lambda t} \beta$$

$$\therefore \; C(x,t) = \int_0^t H(x,t,p)\, dp$$

$$= \int_0^t (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) Re^{\lambda t} \beta \, dp$$

$$= (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) Re^{\lambda t} \beta \int_0^t dp$$

$$= Re^{\lambda t} \beta (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) \, p \, |_0^t$$

this implies that

$$\Rightarrow \quad C(x,t) = Re^{\lambda t} \beta t (\zeta_2 e^{\sqrt{k_1 \lambda}\, x} + \zeta_3 e^{-\sqrt{k_1 \lambda}\, x}) \qquad (S_3)$$

Using the boundary conditions in the theorem, we can see that they can only be satisfied fully if λ is a negative number. This further requires that the diffusion coefficient be negative. This again is valid since the H^+ are transported against the normal diffusion principle, that is, from area of lower concentration to that of higher concentration

Conclusion

We have successively demonstrated by modeling that the generation of energy in the human system can be correctly likened to the energy generation in a car although with appropriate moderation to suit human system. From the equations, we can easily verify the effect of concentration of the food taken to the level of energy generation in the system. Similarly, the effect of the quantity of food taken in terms of the volume can also be verified. We successfully obtained solutions to these equations that demonstrated the variations in the volumes and concentrations of the food taken. In our further work, we shall graphically show the various inputs of these volumes and concentrations on the level of energy generated.

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