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Chaos Control and Synchronization in Generalized Lorenz Systems With fully Uncertain Parameters

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Abstract

In this paper, recursive and adaptive backstepping techniques are respectively employed to design control functions for chaos control and synchronization of the generalized Lorenz systems (GLS) of Oldroyd-B fluids with fully unknown parameters. The designed recursive backstepping nonlinear controllers are capable of stabilizing the chaotic GLS at any position. The designed adaptive backtstepping controllers are effective in globally synchronizing two identical GLS evolving from different initial conditions. The feasibility of the designed controllers is illustrated with numerical simulations.

Keywords: Chaos Control, Synchronization, Generalized Lorenz system and backstepping techniques.

1.0 Introduction

Nature is intrinsically nonlinear. So it is not surprising that most of the systems encountered in the real world are nonlinear. And what is interesting is that nonlinear systems can exhibit a variety of dynamical behaviours including chaos. In both scientific and technological realms, chaos may be a desirable trait in some systems and can be applied in other areas of research. So, intensive research activities have been devoted to this area of study; and various dynamical phenomena including, bifurcation analysis, chaos control, chaos synchronization, time series analysis, basin bifurcation leading to multistability have been studied and reported extensively in the literatures. For a more detailed and comprehensive description of some of these dynamical phenomena as applied to physical, biological, and chemical systems, the reader can refer to the book by Strogatz [1]

The year 1990 witnessed the publication of two seminar papers [2, 3]. Promising wide applications outside the traditional scope of chaos and nonlinear dynamics research, these two papers immediately received a great deal of attention and have led to the establishment of two active areas of research. On one hand, chaos control refers to manipulating the dynamical behaviours of a chaotic system in which the goal is to suppress chaos when it is harmful or to enhance or create chaos when it is beneficial. Chaos synchronization on the other hand, deals with the task of enabling the dynamical synchrony of several connected chaotic systems by means of control techniques or through specifically designed coupling configurations. The idea of chaos control have been shown to have its origin in control theory, synchronization of chaos has evolved somewhat in its own right. Research efforts [4 - 6] showed that the problem of chaos synchronization could be associated with nonlinear control theory. This unifies the study of chaos control and chaos synchronization under the same rubric.

In view of its potential applications, especially in chemical reactions, power converters, biological systems, information processing, secure communication and surveillance; a variety of techniques have been proposed for the synchronization and control of chaos in both low-dimensional and high-dimensional systems. Some of this control approaches include backstepping control [7 – 11], linear state control [12 – 14], variable substitution control [15 – 17], active control [18], sinusoidal error feedback control [19] etc. Outstanding among these techniques is the backstepping control approach which has been shown to possess many advantages over other control methods.

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The backstepping nonlinear control technique is a systematic design approach which consists in a recursive procedure that skillfully interlaces the choice of Lyapunov function with the control. It has the ability to achieve global stability, tracking, and transient performance for a broad class of strict feedback nonlinear systems [20, 21]. Furthermore, it has the advantages of applicability to a variety of chaotic systems whether they contain external excitation or not; needs only one controller to achieve synchronization between two chaotic systems, thereby reducing controller complexity; there are no derivatives in the controller [20]; the controller is singularity free from the nonlinear term of quadratic type , gives flexibility to construct a control law which can be extended to higher dimensional chaotic systems, and the closed-loop system is globally stable [21].

By utilizing the classical Lorenz system [22] which was originally derived from the study of Rayleigh-Bennard convection, Khayat [23, 24] reported Rayleigh-Bennard thermal convection of viscoelastic fluids using Oldroyd-B fluids to obtain a four-dimensional nonlinear system which describes some mechanical properties of some polymeric fluids. However, Khayat's system [23,24] was derived by severe truncation of conservation equations. In order to account for the physical meanings of the variables of the system, Yang and Zhou [25] derived a new four-dimensional system - the Generalized Lorenz System (GLS) of the Oldroyd-B fluids - in a clearer and more concise way; and the system was shown to exhibit rich dynamical behaviours including chaos, limit cycle and Hopf bifurcation. It is well known that understanding the synchronization behaviour of coupled or driven oscillators is very relevant for various scientific and engineering applications, and in particular, its diverse applications ranging from secure communications to the monitoring of dynamical systems and control [26,27] cannot be overemphasized. To the best of our knowledge, chaos control and synchronization dynamics of GLS with fully unknown parameters have not been reported in the literature. The aims of the present paper are therefore; to design via recursive backstepping technique, nonlinear control functions that can stabilize the chaotic GLS at any position; and to design via adaptive backstepping approach nonlinear controllers that can synchronize the GLS with fully unknown parameters.

The rest of the paper is structured as follows. In the following section, we give a brief description of the model that we studied in this paper. Section 3 presents a novel recursive nonlinear controller with numerical simulation and section 4 presents adaptive synchronization controller with corresponding parameter update laws to synchronize two GLS evolving from different initial conditions as well as numerical simulations results that demonstrate the effectiveness of the designed controllers and the paper is concluded in section 5.

2.0 Model Description

The GLS was derived from the classical Lorenz system of Rayleigh-Bennard thermal convection of Oldroyd-B fluids in a closed loop. The GLS can be described by the following set of autonomous differential equations [25]:

$$\begin{aligned} \dot{x} &= p(y-w) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -z + xy \\ \dot{w} &= m[(x-w) + pn(y-w)] \end{aligned} \tag{1}$$

where p is the counterpart of the Prandtl number for a fully treated convection problem, r is the counterpart for Rayleigh number, Deborah number, m, is a measure of relaxation time of the fluid relative to the characteristic time of this problem and Deborah number, n, is a measure of retardation time of the fluid relative to the characteristics time. The model is considered as a new chaotic system since it has an attractor that is completely different from the popular Lorenz [22], Chen [28], Lu [29], Rossler [30] attractors; and reflects nonlinear characteristics of thermal convection of Oldroyd-B fluids in a closed loop.

In Ref. [25], Yang and Zhou showed that the values of m and n greatly influence the dynamics of the GLS. For example, m was shown to precipitate the onset of periodic behaviour while n impedes the onset of chaotic dynamics in the system. The GLS governed by Eq. 1 has been shown to exhibit rich varieties of dynamical behaviour including chaotic motion - depicted in Fig. 1 - with the following parameter settings p=10, r =20 and m = n =0.1

3.0 Chaos Control in GLS

3.1 *Design of the recursive backstepping controllers*

Let us express system (1) in the following form:

$$\begin{aligned}\dot{x} &= p(y-w)+u_1(t)\\ \dot{y} &= rx-y-xz+u_2(t)\\ \dot{z} &= -z+xy+u_3(t)\\ \dot{w} &= m[(x-w)+pn(y-w)]+u_4(t)\end{aligned} \qquad (2)$$

where $u_i(t)$, i=1,2,3,4 are control inputs to be determined such that the state variables x,y,z and w of system (2) can take desired values x_d, y_d, z_d and w_d respectively. We define the error states between the state variables and desired values as:

$$\begin{aligned}e_x &= x-x_d\\ e_y &= y-y_d\\ e_z &= z-z_d\\ e_w &= w-w_d\end{aligned} \qquad (3)$$

In order to design control functions u_i, i=1,2,3,4, that can control system (2), let

$$\begin{aligned}x_d &= f(t)\\ y_d &= c_1e_x\\ z_d &= c_2e_x+c_3e_y\\ w_d &= c_4e_x+c_5e_y+c_6e_z\end{aligned} \qquad (4)$$

where c_i, i=1,2,…6 are arbitrary control parameters to be chosen appropriately. By substituting (4) into (3), one readily obtains the following error dynamics:

$$\begin{aligned}\dot{e}_x &= p(e_y+c_1e_x-e_w-c_4e_x-c_5e_y-c_6e_z)-\dot{f}(t)+u_1(t)\\ \dot{e}_y &= r(e_x+f(t))-(e_y+c_1e_x)-(e_x+f(t))(e_z+c_2e_x+c_3e_y)-c_1\dot{e}_x+u_2(t)\\ \dot{e}_z &= -e_z-c_2e_x-c_3e_y+(e_x+f(t))(e_y+c_1e_x)-c_2\dot{e}_x-c_3\dot{e}_y+u_3(t)\\ \dot{e}_w &= m\{(e_x+f(t)-e_w-c_4e_x-c_5e_y-c_6e_z)+pn(e_y+c_1e_x-e_w-c_4e_x-c_5e_y-c_6e_z)\}-\\ &\quad c_4\dot{e}_x-c_5\dot{e}_y-c_6\dot{e}_z+u_4(t)\end{aligned} \qquad (5)$$

In order to stabilize the error system (5) at the equilibrium position, let us consider a Lyapunov function of the form

$$V=\frac{1}{2}\left(k_xe_x^2+k_ye_y^2+k_ze_z^2+k_we_w^2\right) \qquad (6)$$

where k_i , i=x,y,z,w , are positive constants. The differential of the Lyapunov function along the trajectory of the error system (5) is

$$\dot{V}=k_xe_x\dot{e}_x+k_ye_y\dot{e}_y+k_ze_z\dot{e}_z+k_we_w\dot{e}_w \qquad (7)$$

To satisfy the condition for asymptotic stability of the error dynamics (5) necessary for controlling chaos , we substitute (5) into (7) and choose u_i, i=1,2,3,4 such that the derivative of the Lyapunov function is negative definite as follows:

$$\begin{aligned}u_1(t) &= \dot{f}(t)-p(e_y+c_1e_x-e_w-c_4e_x-c_5e_y-c_6e_z)-e_x\\ u_2(t) &= c_1\dot{e}_x+(e_x+f(t))(e_z+c_2e_x+c_3e_y)+e_y+c_1e_x-r(e_x+f(t))-e_y\\ u_3(t) &= c_2\dot{e}_x+c_3\dot{e}_y+(e_z+c_2e_x+c_3e_y)-(e_x+f(t))(e_y+c_1e_x)-e_z\\ u_4(t) &= c_4\dot{e}_x+c_5\dot{e}_y+c_6\dot{e}_z-m[(e_x+f(t)-e_w-c_4e_x-c_5e_y-c_6e_z)+\\ &\quad pn(e_y+c_1e_x-e_w-c_4e_x-c_5e_y-c_6e_z)]-e_w\end{aligned} \qquad (8)$$

We have observed from the results of computations carried out, that system (2) is effectively controlled with only $c_1 = c_3 = 1$ and so we set $c_2 = c_4 = c_5 = c_6 = 0$ which simplify the controllers in (8) to

$$
\begin{aligned}
u_1(t) &= \dot{f}(t) - p(e_x + e_y - e_w) - e_x \\
u_2(t) &= \dot{e}_x + e_x + e_y + (e_x + f(t))(e_y + e_z) - r(e_x + f(t)) - e_y \\
u_3(t) &= \dot{e}_y + e_y + e_z - (e_x + f(t))(e_x + e_y) - e_z \\
u_4(t) &= -m[(f(t) + e_x - e_w) + pn(e_x + e_y - e_w] - e_w
\end{aligned}
\tag{9}
$$

3.2 *Numerical results*

In this sub-section, we will show numerical simulation results to demonstrate the feasibility of the designed controllers. Using the Fourth order Runge-Kutta routine with time step of 0.001 and initial conditions (x, y, z, w) = (0.8, 0.4, -0.8, -0.2), and fixing the parameter values of p, r, m, and n as in fig. 1 to ensure chaotic dynamics of the state variables, we solved system (2) with the controllers u_i, i = 1,2,3,4 as defined in (9). The state trajectory of the system depicts irregular pattern when the controllers are deactivated and when the controllers are switched on at t=50, the state variables are controlled to the equilibrium position. We note that the controller in (9) is capable of controlling the dynamics of the chaotic GLS (i.e. Eq. 2) to stabilize it at any position ε (case $f(t) = \varepsilon$) and when $\varepsilon = 0$, the system becomes stabilized at the origin.

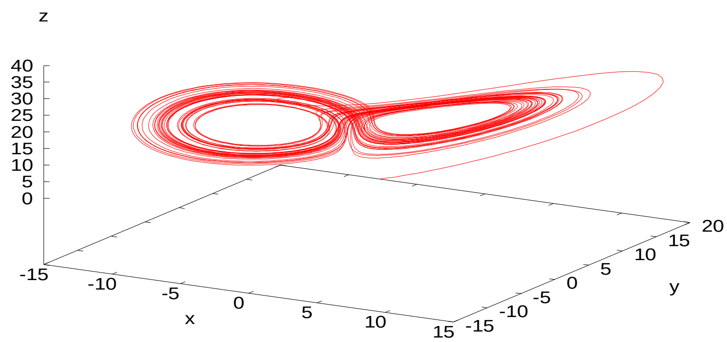

Fig. 1: Three dimensional phase portrait of the generalized Lorenz system with the following parameter settings: p=10, r=20, m= 0.1 and n = 0.1.

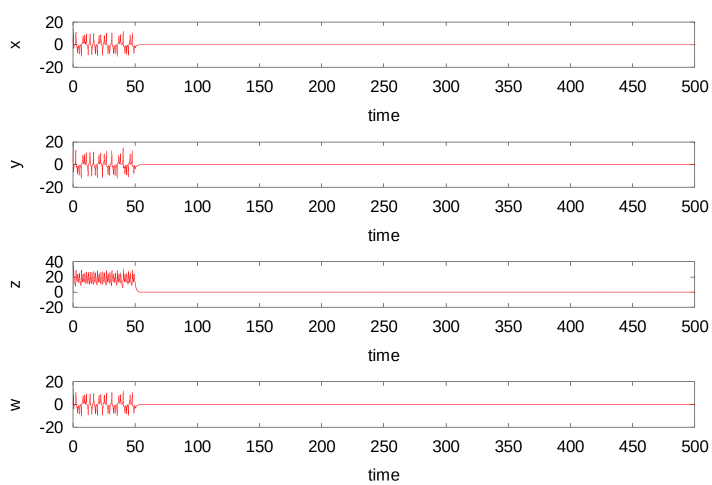

Figure 2: Controlled chaotic vibrations of the 4D GLS. Controls have been activated at $t \geq 50$.

4 Synchronization of Generalized Lorenz System

Here, we examine the synchronization dynamics of GLS via adaptive backstepping technique.

4.1 Design of backstepping controller

In order to achieve the synchronization behaviour between two GLS evolving from different initial conditions, the GLS in (1) is expressed as master:

$$\begin{aligned} \dot{x}_1 &= p(y_1 - w_1) \\ \dot{y}_1 &= rx_1 - y_1 - x_1 z_1 \\ \dot{z}_1 &= -z_1 + x_1 y_1 \\ \dot{w}_1 &= m[(x_1 - w_1) + pn(y_1 - w_1)] \end{aligned} \tag{2}$$

and slave

$$\begin{aligned} \dot{x}_2 &= p_1(y_2 - w_2) + \upsilon_1(t) \\ \dot{y}_2 &= r_1 x_2 - y_2 - x_2 z_2 + \upsilon_2(t) \\ \dot{z}_2 &= -z_2 + x_2 y_2 + \upsilon_3(t) \\ \dot{w}_2 &= m_1[(x_2 - w_2) + p_1 n_1 (y_2 - w_2)] + \upsilon_4(t) \end{aligned} \tag{3}$$

where υ_i (i=1,2,3,4) are control functions to be determined and p_1, r_1, m_1,n_1 are uncertain parameters to be estimated in the response system . In what follows, an effective adaptive controller is designed to achieve global chaos synchronization between two GLS with fully unknown parameters.

We define error vector as

$$\begin{aligned} & e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1, e_4 = w_2 - w_1 \\ & e_p = p_1 - p, e_r = r_1 - r, e_m = m_1 - m, e_n = n_1 - n \end{aligned} \tag{4}$$

By subtracting Eq. (2) from Eq. (3) and using the definition of error vector in Eq. (4), one readily obtains the following error system:

$$\begin{aligned} \dot{e}_1 &= e_p(y_2 - w_2) + a(e_2 - e_4) + \upsilon_1(t) \\ \dot{e}_2 &= x_2 e_r + re_1 - e_2 - e_1 e_3 - x_1 e_3 - z_1 e_1 + \upsilon_2(t) \\ \dot{e}_3 &= e_1 e_2 + y_1 e_1 + x_1 e_2 - e_3 + \upsilon_3(t) \\ \dot{e}_4 &= e_m[(e_m e_p + pe_n + ne_p + np)(y_2 - w_2) + (x_2 - w_2)] + m[(e_n e_p + pe_n + ne_p)(y_2 - w_2) \\ &\quad + npe_2 + e_1 - e_4(np+1)] + \upsilon_4(t) \end{aligned} \tag{5}$$

The goal of the control is to find an effective controller υ_i , i = 1, 2, 3, 4, with parameter update laws such that the response system (3) can globally and asymptotically synchronize with the drive system (2). Let us assume a Lyapunov function of the form:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_3^2 + e_4^2 + e_p^2 + e_r^2 + e_m^2 + e_n^2) \tag{6}$$

By inserting (5) into the time derivative of (6), we obtain the following

$$\begin{aligned}\dot{V} &= e_1\{a(e_2-e_4)+\upsilon_1(t)\}+e_2\{re_1-e_2-e_1e_3-x_1e_3-z_1e_1+\upsilon_2(t)\}+\\ &e_3\{e_1e_2+y_1e_1+x_1e_2-e_3+\upsilon_3(t)\}+e_4\{m[npe_2+e_1-(np+1)]+\\ &\upsilon_4(t)\}+e_p\{\dot{e}_p+(e_1+mne_4)(y_2-w_2)\}+e_r\{\dot{e}_r+x_2e_2\}+e_m\{\dot{e}_m+\\ &e_4[(e_ne_p+pe_n+ne_p+np)(y_2-w_2)+(x_2-w_2)]\}+e_n\{\dot{e}_n+me_4[e_p+p(y_2-w_2)]\}\end{aligned} \tag{7}$$

If the controllers υ_i , i=1, 2, 3, 4, are chosen as follows:

$$\begin{aligned}\upsilon_1 &= a(e_4-e_2)-e_1\\ \upsilon_2 &= e_2-re_1+e_1e_3+x_1e_3+z_1e_1-e_2\\ \upsilon_3 &= -e_1e_2+e_3-y_1e_1-x_1e_2-e_3\\ \upsilon_4 &= -mnpe_2-me_1+me_4(np+1)-e_4\end{aligned} \tag{8}$$

and the parameter estimation update law as follows

$$\begin{aligned}\dot{e}_p &= -e_p-(e_1+mne_4)(y_2-w_2)\\ \dot{e}_r &= -e_r-x_2e_2\\ \dot{e}_m &= -e_m-e_4[(e_ne_p+pe_n+ne_p+np)(y_2-w_2)-(x_2-w_2)]\\ \dot{e}_n &= -e_n-me_4[e_p+p(y_2-w_2)]\end{aligned} \tag{9}$$

then the derivative of the Lyapunov function in (7) is negative definite and according to Lassale-Yoshissawa theorem [31], the error dynamics will converge to zero as $t \to \infty$, while the equilibrium, (0,0,0,0), remains globally asymptotically stable. This theoretical result implies that the GLS system (1) would be driven to the equilibrium state as $t \to \infty$.

4.2 *Numerical Results*

Utilizing the fourth-order Runge-Kutta routine with time step of 0.001, r = 20, p = 10, m = 0.1, n = 0.1 and the initial conditions of the drive and response systems are respectively set as x_1 = 0.1, y_1 = 0. 2, z_1= 0.4, w1 = 0.6 and x_2=2.0, y_2= 4.0, z_2=0.8, w_2 = 0.6 respectively. When the controllers $\upsilon_i(t)$ (i = 1,2,3,4) are deactivated, a plot of average error ($e=\sqrt{e_1^2+e_2^2+e_3^2+e_4^2}$) against time (see Fig. 3) reveals irregular pattern that is comparable to the size of the attractor.

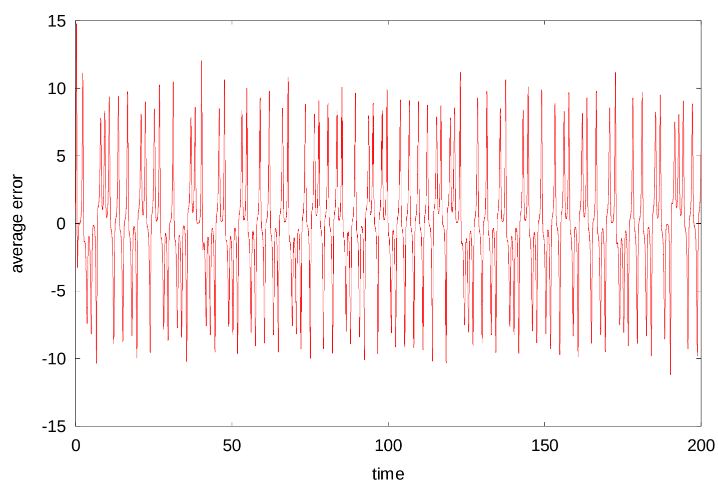

Fig.3: Time series of the generalized Lorenz system when the controllers, $\upsilon_i(t)$ (i = 1,2,3,4) were deactivated.

When the controllers are activated at t = 50 as shown in Fig. 4. The simulations results show that the error variables e_1, e_2, e_3, e_4 tend to zero, respectively. Fig.5 reveals that the estimated values of the unknown parameters converge to p=10, r=0.3, m=0.1, n=0.1 as $t \rightarrow \infty$ respectively.

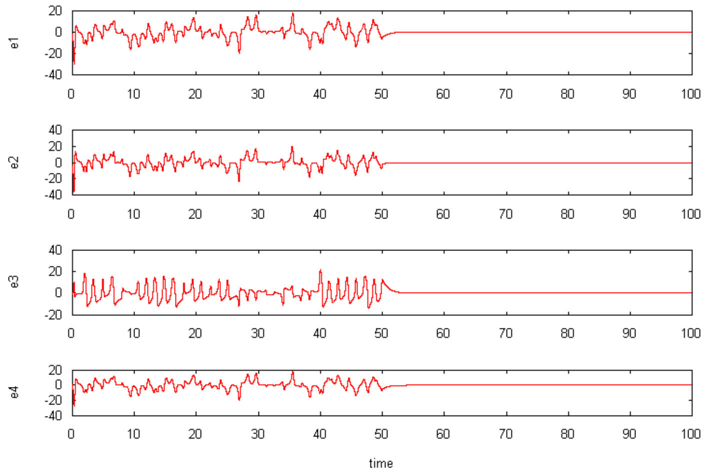

Fig. 4: Error dynamics between two GLS with controllers activated at $t \geq 50$

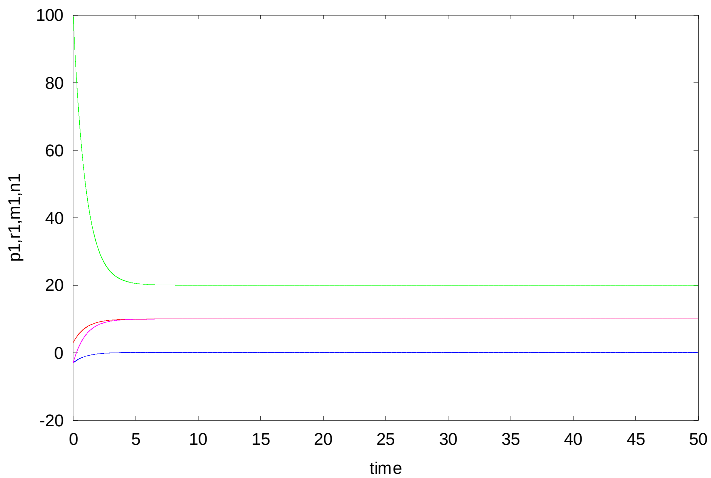

Fig. 5: Estimate values of parameters p_1, r_1, m_1, n_1 for the response system (2)

5. Conclusion

In this paper, control functions have been designed via backstepping nonlinear control techniques for the stabilization, control and synchronization of the GLS of Oldroyd-B fluids. The designed recursive nonlinear controllers were capable of stabilizing the GLS at any chosen position. Based on Lyapunov stability theory, we designed two novel adaptive synchronization controllers with corresponding parameter update laws to globally synchronize two GLS evolving from different initial conditions. The performance of the theoretically designed controllers were verified by numerical simulations which confirmed the effectiveness of the proposed controllers.

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