Journal of the Nigerian Association of Mathematical Physics
***Volume* 18 (May, 2011), pp 617 – 620**

An Alternative to the Basket Weaving Method of Finding The Determinant of a Matrix

Abdulhadi Aminu[1], Abdulkadir Datti

Department of Mathematical Sciences,
Kano University of Science and Technology, Wudil,
P.M.B 3244, Kano, Nigeria

Abstract

Basket weaving method is a special trick of finding the determinant of a 3×3 matrix. But to use the method one must extend the matrix to 3×5. In this paper, we propose a method called triple cancellation method for finding the determinant of a 3×3 matrix which can be used directly without the need of extending the matrix to 3×5.

Keywords: matrix; determinant; basket weaving method; triple cancellation method**.**

1.0 Introduction:

The mathematical theory of matrix has its origin in the theory of determinant. Matrix plays an important role in many branches of Mathematics, Physics, Engineering, Statistics, and Economics. The origin of the determinants lies in the solution of linear equations. Most historians attributed the invention of determinant to Leibnitz. The theory of determinants is part of matrix theory, which in turn is a branch of algebra. Determinants are important in solving system of linear equations. They are also useful tools in areas of analytic geometric, calculus and differential equations. More on the determinant can be found in [1] and [2].
The Determinant of a matrix has several properties. We will state here the properties that are of interest in this paper and the reader is referred to [1] and [2] for more properties and applications of determinants.

Property1: Suppose $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then $\det(A)=a_{11}a_{22}-a_{12}a_{21}$

Property 2: If $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then

$$\det(A)=a_{11}a_{22}a_{33}+a_{21}a_{13}a_{32}+a_{31}a_{12}a_{23}-a_{31}a_{13}a_{22}-a_{21}a_{12}a_{33}-a_{11}a_{32}a_{23}$$

Property 3: The determinant is independent of the row used to evaluate it.

2.0 Existing methods

There are several methods of evaluating the determinant of a matrix. The method of interest here is the Basket weaving method (or Sarrus method); for more information the reader is referred to [1, 2] and [4]. Basket weaving method is used to compute the determinant of 3×3 matrices. It is considered as a classical 3×3 trick. In obtaining the determinant using this method, one proceeds by rewriting the first two columns of the matrix, say A, to the right of it and then looking at products of elements on the same ‘’slant’’ i.e. the elements on the same slope . The algebraic sign of each product of the elements which slant down to the right is taken to be positive. While for the product of those elements slanting down to the left is taken to be negative. For the slant which has less than three elements the product is considered to be zero, that is to say, slants with less than three elements are to be neglected.

[1] Corresponding author. Email: abdulaamin77@yahoo.com, Tel. +2348035185235

If $A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then

$$\det(A)=\left(\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right)$$

$$= a_{11}a_{22}a_{33} + a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} - a_{31}a_{13}a_{22} - a_{21}a_{12}a_{33} - a_{11}a_{32}a_{23}$$

3.0 The triple cancellation method

In this method the determinant is obtained by choosing each of the corner elements of the matrix, i.e. a_{11}, a_{13}, a_{31} and a_{33} and deleting those elements that are in the same row, column and diagonal with the selected element. This is illustrated in Figures 1, 2, 3 and 4 below:

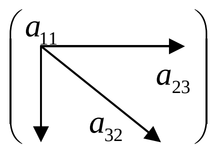

Fig 1: Triple cancellation of row, column and diagonal of a_{11}

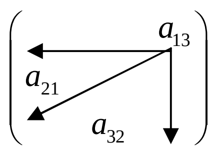

Fig 2: Triple cancellation of row and column and diagonal of a_{13}

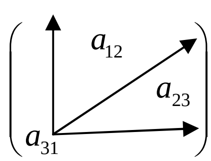

Fig 3: Triple cancellation of row and column and diagonal of a_{31}

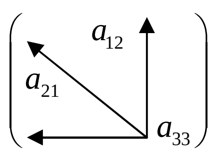

Fig 4: Triple cancellation of row and column and diagonal of a_{33}

The product of the chosen element together with the remaining elements is then computed. The algebraic sign of a_{13} and a_{31} are taken to be positive whereas that of a_{11} and a_{33} are taken to be negative and is illustrated in Fig 5 below:

$$\begin{pmatrix} - & . & + \\ . & . & . \\ + & . & - \end{pmatrix}$$

Fig 5: Algebraic signs of the diagonal elements

The product of both principal and secondary diagonal are then computed separately, in which the latter is taken with negative algebraic sign and the other with positive algebraic sign. The determinant is then obtained by summing up the resulting products.

We denote by P_{U_L} the product of elements obtained by choosing the upper left hand corner element, P_{U_R} the product of elements obtained by choosing the upper right hand corner element, P_{L_L} and P_{L_R} the product of elements obtained by choosing the lower left hand and lower right hand corner elements respectively. Therefore we have

$$P_{U_L} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = -a_{11}a_{23}a_{32},$$

$$P_{U_R} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{13}a_{21}a_{32},$$

$$P_{L_L} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{31}a_{12}a_{23},$$

$$P_{L_R} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = -a_{33}a_{12}a_{21},$$

We also denote by P_P and P_S the product of principal diagonal and secondary diagonal elements respectively. That is

$$P_P = a_{11}a_{22}a_{33} \text{ and } P_S = -a_{13}a_{22}a_{31}$$

Theorem: *Let* $A = (a_{ij}) \in \mathbb{R}^{3\times 3}$ *then* $\det(A) = P_{U_L} + P_{U_R} + P_{L_L} + P_{L_R} + P_P + P_S$.

Proof The statement follows from the definitions of P_{U_L}, P_{U_R}, P_{L_L}, P_{L_R}, P_P, P_S and Property 2.

4.0 A numerical example

Consider the following 3×3 matrix

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & -1 \\ 6 & 4 & 2 \end{pmatrix}$$

The determinant of *A* is 24.

Using the triple cancellation method we have $\det(A) = P_{U_L} + P_{U_R} + P_{L_L} + P_{L_R} + P_P + P_S$. Now we determine the values of the products as follows:

$$P_{U_L} = \begin{pmatrix} 2 & . & . \\ . & . & -1 \\ . & 4 & . \end{pmatrix} = -(2\times -1\times 4) = 8\text{,}\quad P_{U_R} = \begin{pmatrix} . & . & -5 \\ 0 & . & . \\ . & 4 & . \end{pmatrix} = (-5\times 0\times 4) = 0\text{,}$$

$$P_{L_L} = \begin{pmatrix} . & 3 & . \\ . & . & -1 \\ 6 & . & . \end{pmatrix} = (6\times -1\times 3) = -18\text{,}\quad P_{L_L} = \begin{pmatrix} . & 3 & . \\ 0 & . & . \\ . & . & 2 \end{pmatrix} = (2\times 0\times 3) = 0\text{,}$$

$$P_P = \begin{pmatrix} 2 & . & . \\ . & 1 & . \\ . & . & 2 \end{pmatrix} = (2\times 1\times 2) = 4 \text{ and } P_S = \begin{pmatrix} . & . & -5 \\ . & 1 & . \\ 6 & . & . \end{pmatrix} = -(6\times 1\times -5) = 30\,.$$

Therefore,

$$\begin{aligned} \det(A) &= P_{U_L} + P_{U_R} + P_{L_L} + P_{L_R} + P_P + P_S \\ &= 8 + 0 - 18 + 0 + 4 + 30 \\ &= 24. \end{aligned}$$

Conclusion

In this paper we have developed a method called triple cancellation method which is an alternative to the well known basket weaving method for determining the determinant of a 3×3 matrix. The method is proved to be correct and is easier compared to the basket weaving method because there is no need to extend the matrix to 3×5. Further research may concentrate on how to extend the method to $n\times n$ matrices.

References

[1] S.I. Grossman (1984), *Elementary Linear Algebra*, Wadsworth, California.

[2] L. Hogben (ed.) (2007), *Handbook of Linear Algebra*, Discrete Mathematics and its Applications, Vol. 39, Chapman and Hall/CRC, Baton Rouge, L.A.

[3] D. C. Lay (2003). *Linear Algebra and Its Applications*, 3rd ed. Addison-Wesley, Boston.

[4] E. G. Goodaire (2003), *Linear Algebra a Pure and Applied First Course*. Prentice Hall, Upper Saddle River, New Jersey.