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Modelling of California Bearing Ratio with Unconfined Compressive Strength for Cement Stabilized Laterite Soil.

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Abstract

This work focuses on the development of a model relating the unconfined compressive strength(UCS) with California bearing ratio (CBR) for soil cement stabilization. The modeling effort has been to create a platform for reducing time and money in the preliminary stage of cement stabilized laterite using the Box-Cox transformation regression analysis. The trend of data collected for this Box-Cox transformation regression exercise reveals that variations of UCS with CBR for cement stabilized laterite can be represented by a non-linear model.

Keywords: Cement stabilization, laterite soil, unconfined compressive strength, California bearing ratio, Box-Cox regression analysis.

1.0 Introduction:

Cement stabilized materials are usually accepted on the basis of meeting strength requirement which is often judged on the results of the unconfined compressive strength or California bearing ratio (Transport and Road Research Laboratory, 1996). These tests usually serve as base design as well as field control of constructional works of stabilized materials. They help to determine the success of the stabilized material to be used as road sub-base and base materials. Clause 6229 of the Nigeria General Specification(NGS) for Roads and Bridges [13], require that UCS and CBR tests be carried out for all soil cement mixture to determine the cement content required to adequately stabilize the soil to meet the recommended CBR value of 180% to be attained in site mix or 160% for plant mix. The principal manifestation of these in the construction of soil cement roads is high cost of construction (time and money cost). This work seeks to reduce these costs by developing appropriate regression model relating UCS and CBR for cement stabilized laterite.

There have been various models developed relating UCS and CBR for stabilized laterite. Adedimila and Usifo [1] model was developed based on sustained hypothesis imposed by the researcher before estimation of the model parameters according to their assumption concerning how the dependent and explanatory variables are related. This research works views the modeling relationship between UCS and CBR for stabilized laterite by accepting an entirely empirical approach to the choice of a relation, as there are no rules relating these parameters to each other,

The modeling procedure used for this work is the Box-Cox transformation regression. The Box-Cox transformation allows the data gathered to develop the most apposite form as against a sustained hypothesis basis in which the ultimate form of the model is imposed by the researcher before the estimation of the model parameters.

2.0 Modeling Procedure Adopted

Often, study data may be inappropriate for normality to be assumed or they may be such that the variance associated with the treatments are not constant and seem to vary with the magnitude of the treatment mean (Heteroscedasticity). Data re-scaling, through the application of a simple transformation, is particularly useful in these cases to enable the non-normality and non-constant variance to be corrected before implanting inferential data analysis, which may depend on these specifications being valid for the analyzed data. Re-scaling essentially changes the scale of the response data with the transformed metric becoming the basis of the inferential analysis.
A transformation was suggested by [3] and is most appropriate for single data sets. It represent a re-scaling
method whereby the response data as a whole generate the necessary function for transformation. This approach has been

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adopted for this study in order to allow the data evolve the most appropriate functional form as far as the limit of the modeling exercise carried out is concerned.

The Box-Cox transformation regression has been used very much in the field of science and Engineering. Examples are the works of [7, 11, 15, 18].
A linear model of the form

$$Y_i = \alpha + \beta X_i + \varepsilon_i \quad (1)$$

is written as a generalized [3] model as;

$$Y_i^{(\lambda)} = \alpha + \beta X_i + \varepsilon_i \quad (2)$$

where $= Y_i\ (\lambda)$ is transformed as;

$$Y_i^{(\lambda)} = \frac{Y_i^{(\lambda)}}{\lambda} \quad \lambda \neq 0 \quad (3)$$

$$Y_i^{(\lambda)} = \ln Y_i \quad \lambda = 0 \quad (4)$$

Extension of the Box-Cox model to include the transformation of the independent variable has been used by others, for example [9, 17] etc. The presentation from here on assumes this extension, but with different transformation applied to the dependent and independent variables, where $Y_i^{(\lambda)}$ and $X_i^{(\lambda)}$ are transformed to appear in the model as;

$$\frac{Y_i^{(\lambda_1)} - 1}{\lambda_1} = \alpha + \frac{X_i^{(\lambda_2)} - 1}{\lambda_2} + \varepsilon_i \quad (5)$$

A gross correlation effect has been proved by [2] to attend the original Box - Cox transformation. To overcome this, [4] recommended using what was referred to in [3] as normalized transformation in the forms:

$$Y^{(\lambda)} = \frac{Y_i^{\lambda} - 1}{\lambda \bar{Y}^{\lambda - 1}} \quad \lambda \neq 0 \quad (6)$$

$$Y^{(\lambda)} = \bar{Y}\ \ln(Y) \quad \lambda = 0 \quad (7)$$

where Y is the geometric mean of the untransformed variable Y_i and is given by

$$\bar{Y} = exp\,[\textstyle\sum_i^n In(Y_i)/n] \quad (8)$$

According to [3] the maximum likelihood ($L_{max}(\lambda)$) corresponds to the value for which the sum of squares of residual SSR(λ) from the fitted model is a minimum with respect to λ. $L_{max}(\lambda)$ is a continuous function of λ. If λ differs from 0 or 1, then it can be obtained by establishing an approximate 100(1-α) % confidence interval from

$$L_{max}(\bar{\lambda}) - L_{max}(\lambda) < 0.5x^2(\alpha) \quad (9)$$

At 95% confidence level on which this work is based, for one independent component in λ, equation (8) translates to;

$$L_{max}(\bar{\lambda}) - L_{max}(\lambda) < 1.92 \quad (10)$$

The translation of this approximate confidence region test is that if the interval contains λ=1, there is essentially no transformation, since y = y – 1 (i.e a simple shift to the left by one unit). If the interval contains λ=0, it correspond to a logarithmic transformation. If nether 1 or 0 is contained or both are contained in the interval, the value of λ corresponding to the maximum value of λ is adopted.

3.0 Evaluation Criteria

The model estimation exercise is adjudged significant in terms of the parameters of the model analysis of variance (t-test and F-ratio) at 95% level of confidence and 1,5 (number of variable less 1 and number of observation less 2) degree of freedom. Thus from the statistical table [10], the t-test value of 2.571 and F-ratio of 6.61 are specified, while model selection is done using parameters of normality (kurtosis and skewness) and heteroscedasticity (Barlett's) tests. Based on the level of confidence and degree of freedom a Barlett's test value of 3.841 is specified [12].

4.0 Location of Sample

The laterite used for this work was obtained from an existing borrow pit at Obiaruku town, Ukuani Local Government Area of Delta State. It lies on Agbor – Abraka – Warri Highway in the Southern part of Nigeria.

5.0 Preparation of Sample and Specimen

The preliminary classification tests, as well as tests to determine the moisture-density relationship were performed for the soil sample in accordance with [5], Method of test for soils for civil engineering purposes, [6].
The unconfined compression strength and California bearing tests were performed on the soil-cement mixture in accordance with [5]. Stabilized materials for civil engineering purposes, [6], modified in line with the practice in Nigeria as specified in the [13].
Predetermined amounts of cement and soil were mixed thoroughly to achieve a uniform colour at each stage. The required amount (determined from moisture-density relationship for soil-cement mixtures) was added and the mixing continued. The various specimen used for the UCS and CBR tests were compacted within 20minutes of addition of water. In the stabilization

test, curing of the specimen were in line with [13]. The CBR (3No.) specimen were cured for six days by waxing the end caps of the mould and then soaked in water for one day. While the Unconfined compressive strength after de-moulded, 3 No. specimen were cured for seven days without soaking.

6.0 Results and Discussions

6.1 Soil Properties

The results of tests for the identification of the soil and determination of its properties are presented in Table 1. The grain size distribution of the soil is classified under A-2-4 subgroup using the American Association of State Highway Transportation Officials (AASHTO) classification.

Table 1: Properties of the Soil Sample

Characteristics	**Description**
Passing	21.45%
Liquid limit	25%
Plastic limit	17%
Plasticity Index	8%
Linear Shrinkage	4.3%
Group index	0
AASHTO classification	A-2-4
MDD (kg/m^2)	1946.86
OMC	9.92%
Specific gravity	2.78
Colour	Reddish Brown

Using the [8], plasticity/passing B.S. No. 200 sieve criteria for suitability for cement stabilization, this laterite soil is adjudged suitable for stabilization.

6.2 Stabilization Tests

The UCS and CBR test results are presented in Table 2. The cement increases the CBR and UCS of the soil sample.

Table 2: Strength Characteristics CBR (%) and UCS (N/mm^2)

Cement Content	0	2	4	6	8	10	12
Soaked CBR (%)	27	68	167	216	254	283	308
UCS unsoaked (N/mm^2)	0.214	0.408	1.371	1.681	1.871	2.084	2.131

7.0 Regression Modeling Exercise

The following cases were formulated for use in the Box-Cox transformation regression modeling;

Case 1: $UCS^{(\lambda)} = \alpha_1 + \beta_1 CBR$ (9)

Case 2: $UCS = \alpha_1 + \beta_1 CBR^{(\lambda)}$ (10)

Case 3: $UCS^{(\lambda)} = \alpha_1 + \beta_1 CBR^{(\lambda)}$ (11)

Where λ is an unknown parameter to be estimated from the data. It is usually chosen over a range of values ($-2.5< \lambda<2.5$) with intervals like 0.25. According to [3] the maximum likelihood ($L_{max}(\lambda)$) corresponds to the value for which the sum of squares of residual $SSR(\lambda)$ from the fitted model is a minimum with respect to λ.

In case 1, the UCS values was transformed using the values of **λ.** The transformed values was then used to calculate the maximum likelihood ($L_{max}(\lambda)$), coefficient of determination (R^2), F-ratio (F) and the t-test for both the constant (α_1) and coefficient (β_1) of the model.

The results of the model parameters for the Box-Cox transformation regression modeling exercise for the cases 1 and 2 considered for the relationship between UCS and CBR are shown in Tables 3 and 4 respectively. While the graph of L_{max} versus λ for cases 1 and 2 are also shown in Figs 1 and 2 respectively.

Table 3: Typical Box – Cox Regression Results (For UCS$^{(\lambda)}$= α_1 + β_1 CBR)

Λ	SSR	$L_{max}(\lambda)$	R^2	F	Constant(t-test)	CBR(t-test)
2.5	0.192	12.587	0.966	142.943	-0.829(-5.195)	0.009(12.162)
2.25	0.116	14.355	0.977	209.089	-0.815(-6.602)	0.008(14.460)
2.00	0.067	16.280	0.985	325.816	-0.815(-8.687)	0.008(18.050)
1.75	0.039	18.167	0.990	513.165	-0.829 (-11.571)	0.008(22.653)
1.50	0.028	19.275	0.993	663.758	- 0.859(-14.041)	0.007(25.744)
1.25	0.033	18.705	0.991	544.986	-0.906 (-13.662)	0.007(23.345)
1.00	0.055	16.979	0.985	332.487	-0.975 (-11.489)	0.007(18.234)
0.75	0.096	15.008	0.975	195.904	-1.070 (-9.510)	0.007(13.997)
0.50	0.166	13.091	0.961	121.876	-1.196 (-8.085)	0.008(11.040)
0.25	0.282	11.241	0.942	80.658	-1.362 (-7.071)	0.008(8.981)
0.00	0.478	9.398	0.918	55.912	- 1.579(-6.297)	0.009(7.477)
-0.25	0.808	7.569	0.891	40.857	-1.866 (-5.731)	0.010(6.392)
-0.50	1.384	5.673	0.860	30.712	-2.241 (-5.249)	0.011(5.542)
-0.75	2.424	3.711	0.826	23.763	-2.734 (-4.840)	0.013(4.875)
-1.00	4.333	1.679	0.791	18.871	-3.389 (-4.487)	0.015(4.344)

From Table 3 , λ= 1.5 gives the best model for the transformation of the UCS data as it has the minimum sum of square residual (0.028), the maximum likelihood (19.275), highest coefficient of determination (0.993) and the highest F-ratio values (663.758).

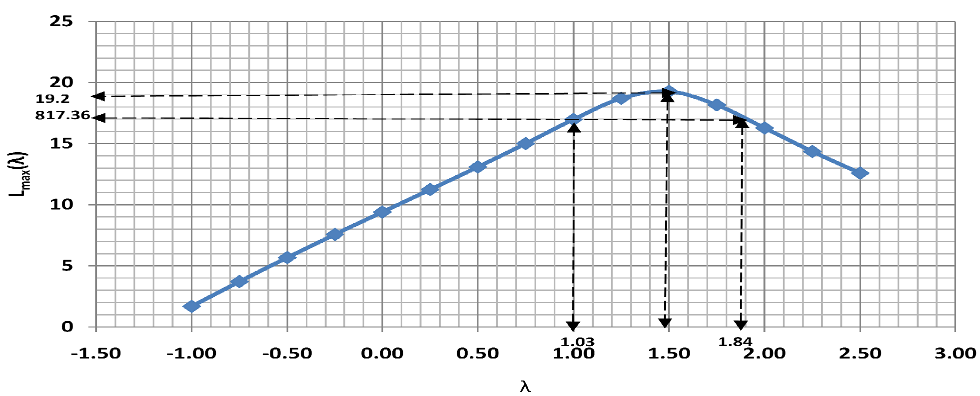

Fig. 1: L_{max} (λ) versus trends for the transformation regression of unconfined compressive strength

Fig. 1 shows L_{max} versus λ. The maximum likelihood occurs at λ= 1.5 with has lower and upper limits at 1.03 and 1.84 respectively. Since λ= 0 and λ= are outside the these limits, λ= 1.5 was adopted for further analysis.

Table 4: Typical Box – Cox Regression Results (For UCS = α_1 + β_1 CBR$^{(\lambda)}$)

λ	SSR	$L_{max}(\lambda)$	R^2	F	Constant(t-test)	CBR(t-test)
2.5	0.562	8.830	0.848	27.887	0.572(2.852)	0.005(5.281)
2.25	0.459	9.537	0.876	35.251	0.521(2.794)	0.006(5.937)
2.00	0.358	10.408	0.903	46.631	0.461(2.716)	0.006(6.829)
1.75	0.262	11.504	0.929	65.608	0.391 (2.590)	0.006(8.100)
1.50	0.175	12.912	0.953	100.570	0.304(2.348)	0.007(10.028)
1.25	0.104	14.744	0.972	173.180	0.191(1.796)	0.007(13.160)
1.00	0.055	16.979	0.985	332.487	0.032(0.379)	0.007(18.234)
0.75	0.036	18.446	0.990	508.194	0.212(-2.709)	0.007(22.543)
0.50	0.055	16.958	0.985	330.449	-0.620(-5.268)	0.007(18.178)
0.25	0.118	14.301	0.968	152.031	-1.362 (5.898)	0.006(12.330)

0.00	0.226	12.022	0.939	76.878	- 2.802(-5.774)	0.006(8.768)
-0.25	0.374	10.256	0.899	44.429	-5.719(-5.334)	0.005(6.666)
-0.50	0.553	8.887	0.850	28.426	-11.815(-4.763)	0.004(5.332)
-0.75	0.748	7.828	0.798	19.703	-24.780 (-4.201)	0.003(4439)
-1.00	0.944	7.044	0.744	14.563	-52.682 (-3.717)	0.003(3.816)

From Table 4 , λ= 0.75 gives the best model for the transformation of the CBR data as it has the minimum sum of square residual (0.036), the maximum likelihood (18.446), highest coefficient of determination (0.990) and the highest F-ratio values (508.194).

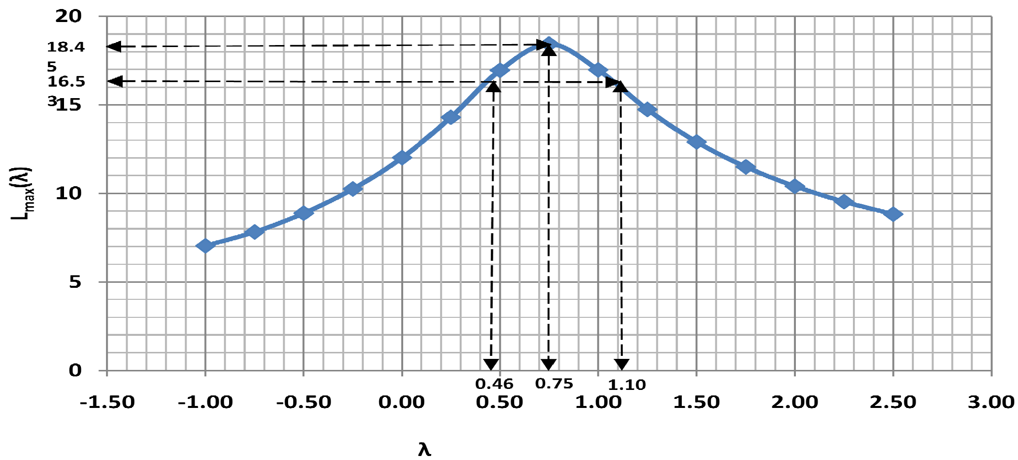

Fig. 2: L_{max} (λ) versus trends for the transformation regression of California bearing ratio

1.10

respectively. These limits excludes λ= 0 but includes λ= 1. Hence λ= 1 was adopted for further analysis.
In case 3 (i.e equal transformation or $\lambda_1=\lambda_2=\lambda$) suitable results were not achieved i.e no concurrence of the values of L_{max} (λ), minimum sum of squares of residual and maximum coefficient of determination. This is interpreted to mean that the collected data do not support the same transformation for the dependent and independent variables. The results were therefore discarded.
The results of the Box-Cox transformation regression modeling exercise for two cases considered for the relationship between CBR and UCS is given in Table 5.

Table 5: Summary of Results of Models Estimation

Model	Λ	SSR	L_{max} (λ)	R^2	F	Constant	Coefficient
1	1.5	0.028	19.275	0.993	662.758	-0.859(-14.041)	0.007(25.744)
2	0.75	0.036	18.446	0.990	508.194	-0.212(-2.709)	0.007(22.543)

A study of the information in table 5, shows that of the two models, model 1 has the lower sum of squares of residual and hence a higher maximum likelihood value and a lower standard error of estimate. Also there is a greater correlation between the UCS and CBR and hence a higher predictable value as indicated by R^2 and F value of the model
Re-estimating the actual data using the adopted λ values (i.e λ_1= 1.5 and λ_2=1 for models 1 and 2 respectively) is shown in Table 6.

Table 6: Results of Model Evaluation Tests

Model	Constant(t-test)	Coefficient(t-test)	Skewness	Kurtosis	Barlett's test
1	-0.336(-3.539)	0.011(25.806)	-1.150	0.680	0.339
2	0.025(0.292)	0.007(18.234)	-1.190	-0.800	0.591

The result of the Barlett's test shows that both models are homoscedastic. Model 1 has the smaller value of skewness and kurtosis value closer to zero than the other model. Also the constant and coefficient values are significant at 5% as indicated by the t-test values in bracket. Thus this model can be regarded as the better one.

8.0 Conclusion

The Box-Cox transformation regression modeling exercise yielded a non-linear functional form with 1.5 power transformation applied to the dependent variable, the UCS. Statistical inference suggests that the non-linear form developed in this work depicts a significant correlation between UCS and CBR for cement stabilized laterite. This model can be said to be an improved one compared to [1] model as it has a higher predicable value as indicated by R^2 value of the model. The implication of the result of this work for practice is that substantial savings (in terms of money and time) can be made in preliminary design stage of soil cement stabilization, in which it will be required to estimate the UCS value from a knowledge of the CBR value using the developed model without having to undergo the process of estimating the UCS through laboratory measurement.

References

[1] Adedimila, A.S., and Usifo, J.E.F., (1994). *A Comparative Study of California Bearing Ratio and Unconfined Compressive Strength for Stabilized Laterite*. NSE Technical Transactions Vol. 29 No. 3 PP 66 – 74

[2] Bickel, P.J and Doksum, K.A., (1981). *An Analysis of Transformation Revisited*. J.Am. Stat. Assoc. 76:296-311

[3] Box, G.E.P. and Cox, D.R., (1964). *An Analysis of Transformation. Journal of the Royal Statistical Society*, B26, 211 – 234.

[4] Box, G.E.P. and Cox, D.R., (1982). *An Analysis of Transformation Revisited, Rebutted. Journal of the Royal Statistical Society*, 77: 209-210.

[5] B.S. 1377 (1990). Method of Testing for Civil Engineering Purpose. British Standard Institution, London, England.

[6] B.S. 1924 (1990). Method of Test for Stabilized Soils. British Standard Institution, London, England.

[7] Hensher, D.A., and Johnson, L.W., (1979). *External Structure of Variable in Individual Choice Models of Travel Demand*. Inter. J. Transp. Econs 6(1): 51-61

[8] Highway Research Board (1943). Use of Soil Cement Mixture for Base Courses. Wartime Road Problems No. 7 Res. Council Div. Eng. Indust. Res. Washington, D.C

[9] Judge, G.G., Hill, C.R., Griffiths, W.E., Lutkepohl, H., and Lee, T. (1998). Introduction to the Theory and Practice of Econometrics. John Wiley, New York.

[10] Kmenta, J. (1986). Elements of Econometrics. Macmillan, New York.

[11] Mainly, B.F.J., (1997). Randomization and Monte-Carlo Methods in Biology. Second Edition Chapman and Hall NY.

[12] Montgomery, D. and Peck, E.A. (1992). Introduction to Regression Analysis. John Wiley, New York.

[13] Nigeria General Specification (1997). Federal Ministry of Works Specification for Roads. Federal Government of Nigeria, Lagos, Nigeria.

[14] Obaji, N.O. (1986). Correlation between the Unconfined Compressive Strength, the Indirect Tensile Strength and the California Bearing Ratio of Lime stabilized laterite. B.Sc. Project Report, Department of Civil Engineering. University of Lagos.

[15] Osula, D.O.A and Adebisi, O., (2001). *Testing the Stability of Travel Expenditure in Nigeria*. Transportation Research Part A 35 (2001) : 269-287

[16] Transport and Road Reasearch Laboratory (1996). A Guide to the Structural Design of Bitumen-Surfaced Roads in Tropical and Sub-Tropical Countries. H.M. Stationery Office, London.

[17] Wang, G.H.K., Maling, W. and McCarthy, E., (1981). *Functional forms and Aggregate, U.S. Domestic Air Cargo Demand* : 1950-1977. Transpn. Res. 15A: 249 – 256.

[18] Zarembka, P. (1968). Functional form in the demand for money. J. Am. Stat. Assoc. 63: 502 – 511.