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Analysis of Axisymmetric Forging Operation Using the Bubnov-Galerkin Weighted Residual Finite Element Method

[1]Oviawe, C.I. and [2]Oviawe O.

**[1] Department of Mechanical Engineering,
Edo State Institute of Technology and Management,
Usen, Benin City, Nigeria**

**[2]Mathematics Department,
College of Education, Ekiadolor Benin City,
Edo State.**

Abstract

This paper reports on the analysis of axisymmetric forging operation using the Bubnov-Galerkin weighted residual finite element model to get the stresses and pressure fields set up at various cross-sections of a material during forging operation. The governing equation is a one dimensional differential equation which describes the pressures and stresses exerted on a forging. In conducting the analysis, we split the blank into a finite number of elements and apply the Bubnov-Galerkin, weighted residual scheme to obtain the weighted integral form. Four Lagrange quadratic elements were assembled to represent the blank. Using a numerical example, we show that the weighted residual finite element method is capable of accurately predicting the pressures and stresses in an axisymmetric forging operation.

Key words: Buba Ox-Galerkin Weighted Residual, Axisymmetric Operation, Finite Element Method, Lagrange, Quadratics.

Introduction:

Forging is one of the oldest and still remains one of the fastest method of shaping metal and other materials. The need therefore arises to critically and numerically analyze the drawing operation in order to predict the various stresses and pressures set up at a particular cross section of a given blank material. The estimated pressure and stresses can thus be compared with the strength of the material and this aids the determination of the smallest pressure required to cause the bulk plastic flow of the material. Consequent upon this, the fundamental and versatile forging process, a large number of research papers into metal forming process exist in the literature. Akpobi and Edobor [1] developed a model for analyzing forging process. Navarrete, et al [3] used a dimensional analysis approach to determine the die forging stress in open die forging. They proposed five dimensionless groups from the process variables in an attempt to simplify the forging stress determination. Alfozan and Gunasekera [2] proposed an upper bound element technique approach to the process design of axisymmetric forging by forward and backward simulation. Nye et al [4] carried out a real time process characterization of open die forging for adaptive control.

In this research, we employ the weighted residual finite element method as a numerical tool used in obtaining the distribution of the forging stresses and pressures on a material during forging. Due to symmetry, analysis is carried out on half the blank. The blank is represented by a mesh of finite elements and the Bubnov – Galerkin weighted residual scheme. Rao [5] is applied to get the value of pressure at nodal points. Four quadratic elements were used to ensure an accurate solution. A numerical analysis is done to compare the finite element results with the exact solution.

Corresponding authors: ***Oviawe, C.I.:*** E-mail: iyekowa@yahoo.co.uk. Tel. +2347080512776

1.0 FORMULATION OF GOVERNING EQUATION

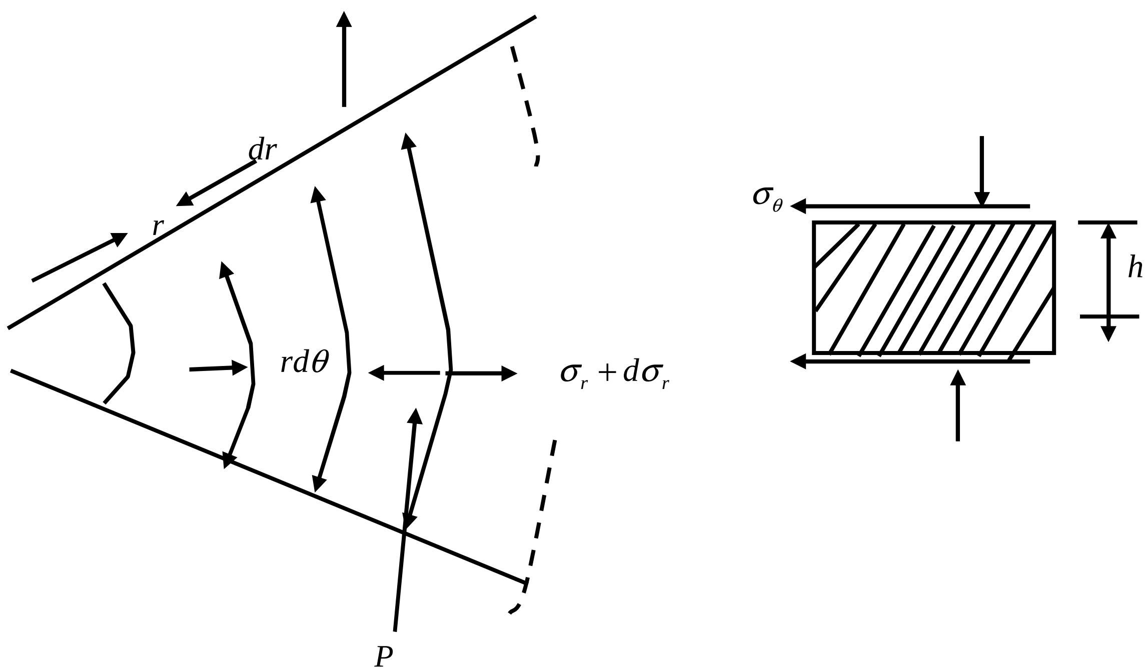

Fig. 1: Shows free body diagram of axisymmetric forging operation.

Considering the free body diagram in fig 1, we assume that σ_r and σ_θ are constant throughout the disk thickness. The radial equilibrium of the element requires that:

$$(\sigma_r + \sigma_r)h(r + \sigma_r)d\theta - \sigma_r hrd\theta - 2\sigma_e hdr \sin\frac{\sigma_e}{2} 2\mu prd\theta = 0 \qquad (1)$$

$$\sigma_r hdrd\theta + d\sigma_r hd\theta - \sigma_e hdrd\theta - 2\mu prd\theta dr = 0 \qquad (2)$$

Dividing through by $d\theta$, gives

$$\sigma_r hdr + d\sigma_r hr - \sigma_\theta hdr - 2\mu prdr = 0 \qquad (3)$$

$$\frac{\sigma_r}{r} + \frac{d\sigma_r}{dr} - \frac{\sigma_\theta}{r} - \frac{2\mu p}{h}$$

$$\because \frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} - \frac{\sigma_\theta}{r} - \frac{2\mu p}{h} = 0 \qquad (4)$$

Equation (4) gives us the governing equation for axisymmetric forging operation.

1.1 METHODOLOGY

Recall equation (4)

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} - \frac{\sigma_\theta}{r} - \frac{2\mu p}{h} = 0$$

Using Tresca's yield criterion. Rowe [6], we have:

$$\sigma_r + P = \sigma_\theta \qquad (5a)$$

$$d\sigma_r + dP = 0$$

$$d\sigma_r = -dP \qquad (5b)$$

Substituting equation (5) into equation (4) we have

$$-\frac{dP}{dr}+\frac{\sigma_r}{r}-\frac{\sigma_o}{r}=\frac{2\mu P}{h} \quad (6a)$$

Recall,

$$\sigma_r=\sigma_o \quad (6b)$$

Putting(6b) into equation (6a) gives

$$\frac{dP}{dr}=-\frac{2\mu P}{h}$$

$$\because \frac{dP}{dr}+\frac{2\mu P}{h}=0 \quad (7)$$

1.2 WEIGHTED INTEGRAL FORMULATION

The weighted integral form of equation (7) is obtained by multiplying it by the weight function w and integrating over the domain enclosing an element with respect to r, to get

$$\int_o^R W\left[\frac{dP}{dr}+\frac{2\mu P}{h}\right]dr=0 \quad (8)$$

where w = weight function. Expanding (8) gives

$$\int_o^R w\frac{dPdr}{dr}+\int_o^R W\frac{2\mu P}{h}dr=0 \quad (9)$$

An examination of equation (9) shows that the solution and hence the approximation function should be once differentiable with respect to (r). Thus, the Lagrange family of interpolation function can be used satisfactorily.

Let us assume that the solution P is approximated as follows:

$$P\approx P^e=\sum_{j=i}^{n}P_j^e\psi_j^e(r) \quad (10)$$

We adopt the Bubnov-Galerkin weighted residual method in which it is assumed that the weight function is equal to the interpolation function i.e.

$$W=\psi_j^e(r) \quad (11)$$

Substituting equations (10) and (11) into equation (9), we get

$$\int_o^R \psi_j\frac{d}{dr}\sum_{j=i}^{n}P_j^e\psi_j^e dr+\int_o^R \psi_j^e\frac{2\mu}{h}\sum_{j=i}^{n}P_j^e\psi_j^e dr=0$$

$$\sum_{j=i}^{n}\left\{\int_o^R\left(\psi_j^e\frac{d}{dr}\psi_j^e+\frac{2\mu}{h}\psi_j^e\psi_j^e\right)\right\}dr\left\{P_j^e\right\}=0 \quad (12)$$

Equation (12) can be recast in the form;

$$\left\{K_{ij}^e\right\}\left\{P_j^e\right\}=0 \quad (13)$$

Equation (13) is the weighted residual finite element model of equation (4)
where,

$$K_{ij}^e=\int_o^R\left(\psi_j^e\frac{d}{dr}\psi_j^e+\frac{2\mu}{h}\psi_j^e\psi_j^e\right)dr \quad (14)$$

where R is taken as outer diameter and r is the radial coordinate.

Using the Lagrange quadratic interpolation function, we get

$$\psi_j^e=\left(1-\frac{r}{R}\right)\left(1-\frac{2r}{R}\right)$$

$$\psi_2^e=\frac{4r}{R}\left(1-\frac{r}{R}\right)$$

$$\psi_3^e=\frac{r}{R}\left(1-\frac{2r}{R}\right)$$

Hence, for one Lagrange quadratic element

$$K_{ij}^{e} = \frac{1}{60h} = \begin{bmatrix} -90h+4\mu R & 10h+2\mu R & -5h-2\mu R \\ -10h+2\mu R & 16\mu R & 15h+2\mu R \\ 5h+2\mu R & 10h+2\mu R & 90h+4\mu R \end{bmatrix} \quad (15)$$

In order to ensure high accuracy, we used a mesh of four quadratic elements (9 nodes).
Dividing the domain into four 1 – D quadratic finite elements then the finite elements model over an element is given as:

$$K_{ij}^{e} = \begin{bmatrix} K_{11}^{e} & K_{12}^{e} & K_{13}^{e} \\ K_{21}^{e} & K_{22}^{e} & K_{23}^{e} \\ K_{31}^{e} & K_{32}^{e} & K_{33}^{e} \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} Q_1 \\ 0 \\ Q_3 \end{pmatrix} \quad (16)$$

For a mesh of four 1 – D quadratic element the assembled equations are:

$$k_{ij}^{2} = \frac{1}{60} \begin{bmatrix} K_{11}^{1} & K_{12}^{1} & K_{13}^{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{21}^{1} & K_{22}^{1} & K_{23}^{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{31}^{1} & K_{32}^{1} & K_{33}^{1}+K_{11}^{2} & K_{12}^{2} & K_{13}^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{21}^{2} & K_{22}^{2} & K_{22}^{2} & K_{23}^{2} & 0 & 0 & 0 \\ 0 & 0 & K_{31}^{2} & K_{32}^{2} & K_{33}^{2}+K_{11}^{3} & K_{12}^{3} & K_{13}^{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{21}^{3} & & K_{22}^{3} & K_{23}^{3} & 0 \\ 0 & 0 & 0 & 0 & K_{31}^{3} & K_{32}^{3} & K_{33}^{3}+K_{11}^{4} & K_{12}^{4} & K_{13}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{21}^{4} & K_{22}^{4} & K_{23}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{31}^{4} & K_{32}^{4} & K_{33}^{4} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} \begin{bmatrix} Q_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_9 \end{bmatrix} = 0 \quad (17)$$

Substituting in values into the assembled equation yield.

$$\left[K_{ij}^{2} = \frac{1}{60}\right] = \begin{bmatrix} -90h+4\mu R & 10h+2\mu R & -5h-2\mu R & 0 & 0 & 0 & 0 & 0 & 0 \\ -10h+2\mu R & 16\mu R & 15h+2\mu R & 0 & 0 & 0 & 0 & 0 & 0 \\ -5h-2\mu R & 10h+2\mu R & 8\mu R & 10h+2\mu R & -5h-\mu R & 0 & 0 & 0 & 0 \\ 0 & 0 & 10h+2\mu R & 16\mu R & 15h+2\mu R & 0 & 0 & 0 & 0 \\ 0 & 0 & -5h+2\mu R & 8\mu R & 8\mu R & 10h+2\mu R & -5h-2\mu R & 0 & 0 \\ 0 & 0 & 0 & -10h+2\mu R & -10h+2\mu R & 16\mu R & 15h+2\mu R & 0 & 0 \\ 0 & 0 & 0 & -5h-2\mu R & -5h-2\mu R & 10h+2\mu R & 8\mu R & 10h+2\mu R & -5h-2\mu R \\ 0 & 0 & 0 & 0 & 0 & 0 & -10h+2\mu R & 16\mu R & 15+2\mu R \\ 0 & 0 & 0 & 0 & 0 & 0 & -5h-2\mu R & 10h+2\mu R & -90h+4\mu R \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{pmatrix} \ldots(18)$$

The boundary condition is

At $r = R, \sigma_r = 0$

But, $\sigma_r + P = \sigma_o = 2k$

Therefore, $r = R, P_9 = \sigma_o = 2k$

Hence, the only unknown pressures are P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8.
The assembled equation becomes

$$\left[K_{ij}^{2} = \frac{1}{60}\right] = \begin{bmatrix} --90h+4\mu R & 10h+2\mu R & -5h-2\mu R & 0 & 0 & 0 & 0 & 0 \\ -10h+2\mu R & 16\mu R & 15h+2\mu R & 0 & 0 & 0 & 0 & 0 \\ -5h-2\mu R & 10h+2\mu R & 8\mu R & 10h+2\mu R & 0 & 0 & 0 & 0 \\ 0 & -10h+2\mu R & 16\mu R & 10h+2\mu R & 5h-2R & 0 & 0 & \\ 0 & -5h-2\mu R & 10h+2\mu R & 8\mu R & 15h+2\mu R & 15h+2\mu R & 0 & 0 \\ 0 & 0 & 0-5h-2\mu R & 16\mu R & 8\mu R & 10h+2\mu R & 0 & 0 \\ 0 & 0 & 0 & -10h+2\mu R & 16\mu R & 16\mu R & -5h-2\mu R & \\ 0 & 0 & 0 & 0 & -10h+2\mu R & 10h+2\mu R & 15+2\mu R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -90h+4\mu R & \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{pmatrix} = \frac{1}{60h} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5h-2\mu R \\ 5h \end{bmatrix} \ldots(19)$$

Numerical Example

Consider the axi-symmetric forging of a circular disc of diameter 38mm and thickness 2mm with coefficient of friction μ = 0.25. Assume that no sticking occurs.

The stresses are obtained by substituting the values of the pressures into the equation:

$$\sigma_r = \sigma_o - P \qquad (20)$$

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} 115.8 \\ 70.2 \\ 42.4 \\ 25.6 \\ 15.7 \\ 9.4 \\ 5.7 \\ 3.5 \\ 2.0 \end{bmatrix} \qquad (21)$$

1.3 POST – PROCESSING OF SOLUTION

In order to get the pressure at any point of the blank, we make use of the following finite element solution.

$$P(r) = P_1\psi_1^1 + P_2\psi_2^1 + P_3\psi_3^1 \; For \;\; 0 \le r \le \frac{R}{4}$$

$$P(r) = P_3\psi_1^2 + P_4\psi_2^2 + P_5\psi_3^2 \; For \; \frac{R}{4} \le r\times \le \frac{R}{2}$$

$$P(r) = P_5\psi_1^3 + P_6\psi_2^3 + P_7\psi_3^3 \; For \; \frac{R}{2} \le r\times \le \frac{3R}{4}$$

$$P(r) = P_7\psi_1^4 + P_8\psi_2^4 + P_9\psi_3^4 \; For \; \frac{3R}{4} \le r\times \le R$$

Where P is the pressure at node 1 and ψ_i^e is the ith Lagrange radial interpolation function for the eth element.

1.4 EXACT SOLUTION

Recall the equation (7)

$$\frac{dP}{dr} + \frac{2\mu P}{h} = 0$$

Separating variables, gives

$$\frac{dP}{p} + \frac{2\mu dr}{h} = 0$$

Integrating both sides, gives

$$InP = -\frac{2\mu r}{h} + C \tag{22}$$

$$P = C.e^{\frac{2\mu P}{h}} \tag{23}$$

From boundary condition;

At $r = R, \sigma_r = 0$ (stress free surface) and we have:

Substituting into equation (22) gives

$$In\ \sigma_o = \frac{2\mu R}{h} = C$$

$$\because C = In\sigma_o + \frac{2\mu R}{h}$$

$$In\frac{P}{\sigma_o} = \frac{2\mu}{h}(R - r)$$

$$\because \frac{P}{\sigma_o} = e^{\frac{2\mu}{h}(R-r)} \tag{24}$$

Recall that

$$\sigma_r + P = \sigma_o = 2k$$

$$\because \frac{P}{2k} = \frac{P}{\sigma_o} = e^{\frac{2\mu}{h}(R-r)}$$

$$\because \sigma_r = \sigma_o - P = \sigma_o\left[I - e^{\frac{2\mu}{h}(R-r)}\right]$$

$$\because \sigma_r = \sigma_o\left[I - e^{\frac{2\mu}{h}(R-r)}\right]$$

Table 1 Distance from Center Of Blank(R) Against Forging Pressure

R(mm)	Pressure(N/mm^2)
0	115.6
2	70.1
4	42.3
6	25.8
8	15.6
10	9.5
12	5.8
14	3.4
16	2.1

Solution

Using the model developed, pressures at the nodes as solved by both the weighted residual finite element method and the exact method in equation (24) are shown in Fig. 2.

Forging pressure against distance from centre of blank r

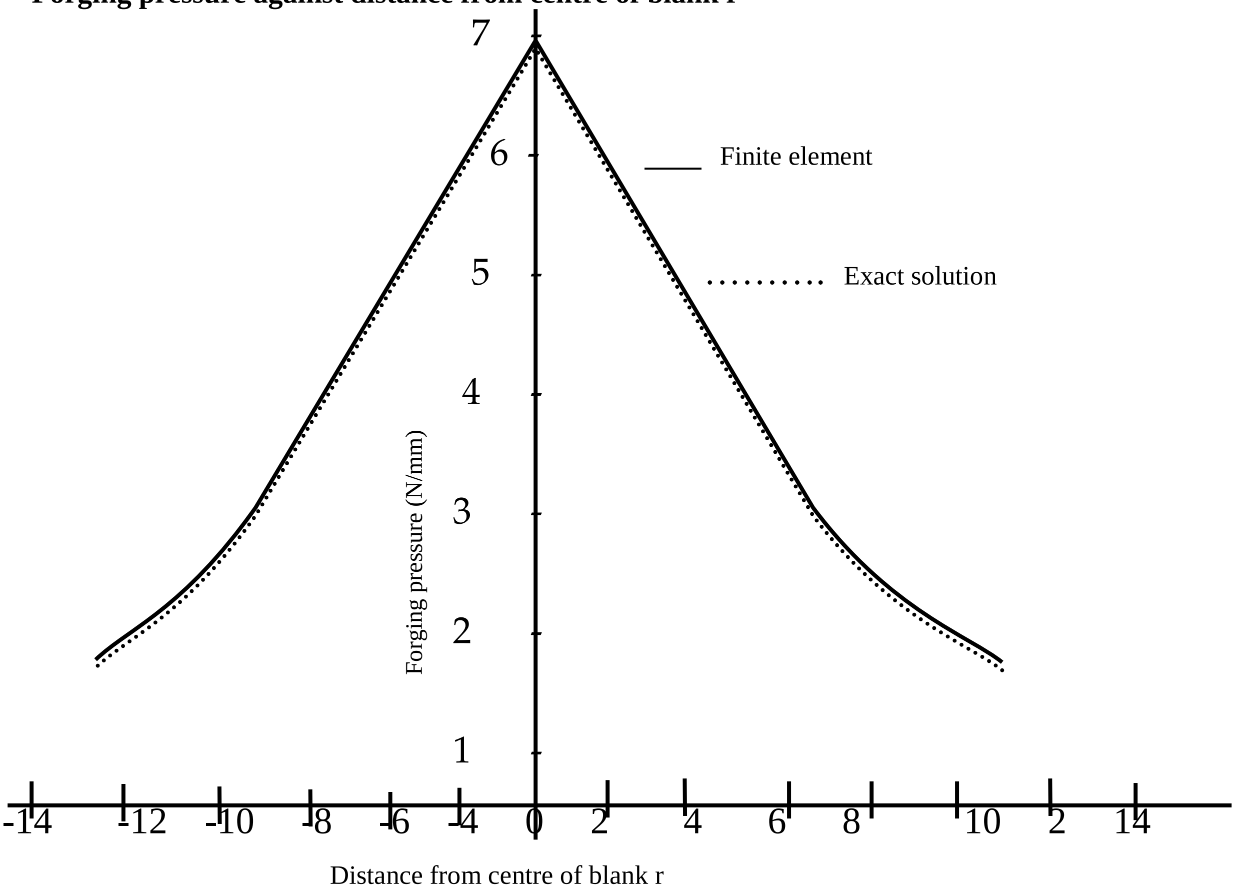

Fig. 2 Graphical comparison of friction hill of the exact solution and finite element solution

Discussion and Results

Table 1 shows that as the length of the blank increases the value of the finite element tends to be exact, and a negative sign in Table 1 indicates compression during the forging process. Hence, from the observable solutions of the analyzed problem the difference between finite elements solution and the exact solution is insignificant and negligible.

Conclusion

From the observation of the analysis it can therefore be concluded the finite element model is capable of adequately and accurately predicting the stresses set up in a particular metal forming process.

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