Journal of the Nigerian Association of Mathematical Physics
***Volume* 18 (May, 2011), pp 589 – 596**

The Determination of Stresses in Wire-Drawing Operation Using the Finite Element Model

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Abstract

In this work, the Bubnov-Galerkin finite element model is used to get the distribution of stresses and pressures set up at various cross-sections of a blank during metal forming process. Four Lagrange quadratic elements were assembled to represent the blank. The governing equation is a one dimensional differential equation describing the pressures and stresses exerted on the forming process. In conducting the analysis, the blank is divided into a finite number of elements and the Bubnov-Galerkin weighted residual scheme is applied to obtain the weighted integral form. The finite element model is obtained in a matrix form and then weighted residual boundary conditions are applied to obtained the pressure distribution across the cross section of the blank. Finite element results are obtained for a particular value of the coefficient of friction, die angle, length and blank radius and compared with the exact solution on a table.

Keywords: Metal forming process, Bubnov-Galerkin weighted residual, finite element method, Lagrange quadratic element.

1 Introduction:

Metal forming processes are based on permanent changes in the shape of the metal, that is, on the plastic deformation under the action of external forces. Metal forming processes includes; rolling, forging, extrusion, drawing and press working. But forging and extrusion still remain the fastest methods of shaping metals. Hence, the need arises to predict the various stresses and pressure fields set up at a particular cross-section of a given blank material. The estimated pressures and stresses can thus be compared with the strength of the material and this aids the determination of the smallest pressure needed to cause the bulk plastic flow of the material.

Consequent upon this the fundamental and versatile metal forming process, a large number of research papers on metal forming process exist in the literature. Akpobi and Edobor [1] developed a model for analyzing forging process. Navarrete, et al [4] used a dimensional analysis approach to determine the die forging stress in open die forging. They proposed five dimensionless groups from the process variables in an attempt to simplify the forging stress determination. Alfozan and Gunasekera [2] proposed an upper bound element technique approach to the process design of axisymmetric forging by forward and backward simulation. Nye et al [5] carried out a real time process characterization of open die forging for adaptive control. Johnson [3] The pressure for cold extrusion of lubricated rod through square dies of moderated reduction at slow speed.

In this work, the weighted residual method is used in obtaining the distribution of pressures and stresses on a material during drawing process. The blank is represented by a mesh of finite elements and the Galerkin (weighted residual) scheme is applied to obtain the value of pressures at nodal points. Four quadratic elements were used to ensure an accurate solution. It is assumed that there is no back pull on the metal during drawing. A numerical analysis is done to compare the finite element results with exact solutions.

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FORMULATION OF GOVERNING EQUATION

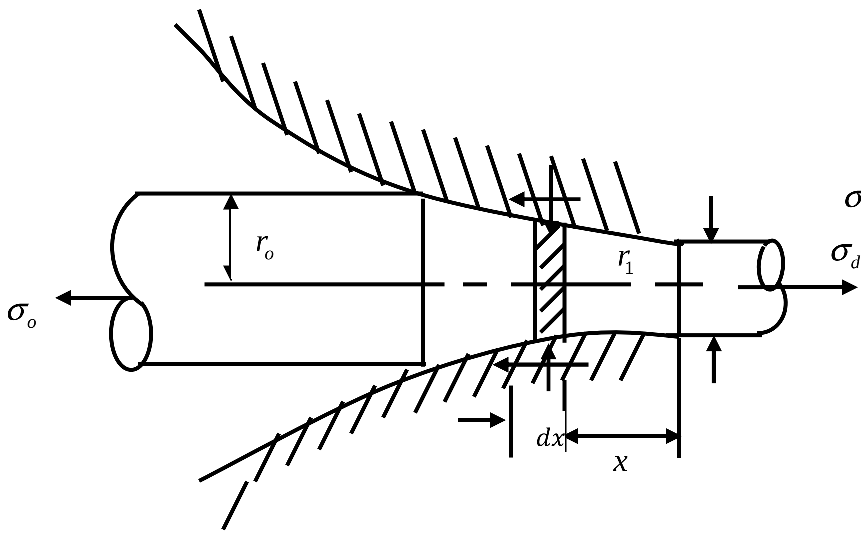

Fig. 1a. Stress equilibrium in wire drawing.

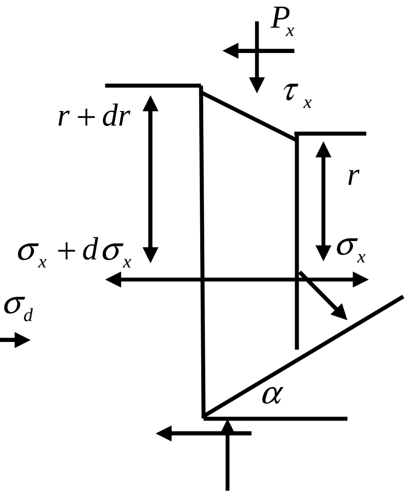

Fig 1b: Free body diagram in wire drawing operation

Considering the stress acting on an element in drawing of a wire fig 1b. The equilibrium equation in x – direction will be:

$$(\sigma_x + d\sigma_x)\Pi(r+dr)^2 - \sigma_x \Pi r^2 + \tau_x Cos\alpha \frac{(2\Pi r dx)}{Cos\theta} + P_x Sin\alpha \frac{(2\Pi r dx)}{Cos\theta} = 0 \qquad (1)$$

From here we get

$$\sigma_x 2rdr + d\sigma_x r^2 + d\sigma_x r^2 + 2r\tau_x + p_x 2rdx \tan\alpha = 0 \qquad (2)$$

Dividing by $r^2 dr$ and taking $\frac{dx}{dr} = Cot\alpha$, we get.

$$\frac{d\sigma_x}{dr} + \frac{d}{r}(\sigma_x + p_x) + \frac{2\tau_x}{r} Cot\ \alpha = 0 \qquad (3)$$

Equation (3) gives us the governing equation for the drawing operation.

1.1 METHODOLOGY

Governing equation

$$\frac{d\sigma_x}{dr} + \frac{2}{r}(\sigma_x + P_x) + \frac{2\tau_x}{r} Cot\alpha = 0$$

Using Tresca's yield criterion

$$\sigma_x + P_x = \sigma_o \qquad (4)$$

Where σ_x and P_x are principal stresses.

Taking,

$$\tau_x = \mu P_x$$

$$\because \tau_x = \mu(\sigma_o - \sigma_x) \qquad (5)$$

Substituting equations (4) and (5) into the equation (3) we get

$$\frac{d\sigma_x}{dr} + \frac{2\sigma_x}{r} + \frac{2\mu}{r}\frac{(\sigma_o - \sigma_x)}{r} Cot\alpha = 0 \qquad (6)$$

But, $P_x = \sigma_o - \sigma_x$

$$\because \frac{d\sigma_x}{dr} + \frac{2\sigma_o}{r} + \frac{2\mu}{r} P_x Cot\alpha = 0 \qquad (7)$$

Let, $\mu Cot\alpha = B$

$$\frac{d\sigma_x}{dr} + \frac{2\sigma_o}{r} + \frac{2BP_x}{r} = 0 \qquad (8)$$

From equation, (4), σ_o = constant

Differentiating equation (4)
w. r .t r, we get

$$\frac{d\sigma_x}{dr} + \frac{dP_x}{r} = 0$$

$$\because \frac{d\sigma_x}{dr} + \frac{dP_x}{r} \qquad (9)$$

putting equation (9) into equation (4), gives

$$-\frac{dP_x}{dr} + \frac{2\sigma_o}{r} + \frac{2BP_x}{r} = 0$$

$$\frac{dP_x}{dr} + \frac{2\sigma_o}{r} + \frac{2BP_x}{r} = 0 \qquad (10)$$

1.2 FINITE ELEMENT FORMULATION

To obtain the variation form of the wire drawing equation, we assume that the performance of the operation is affected by change in one of the variables. This gives

$$\frac{dP_x}{dr} + \frac{2\sigma_o}{r} + \frac{2BP_x}{r} = 0$$

Integrating over the whole wire, gives

$$\int_o^1 \int_o^{2\Pi} \int_{rA}^{rB} w\left[\frac{(dP_x)}{dr} - \frac{(2\sigma_o)}{r} - \frac{(2BP_x)}{r}\right] rdrd\theta dz = 0 \qquad (11)$$

where, w = weight function

$$2\Pi \int_{rA}^{rB} \left(w\frac{(dP_x)}{dr} - \frac{(2\sigma o)}{r} - \frac{(2BP)}{r}\right) rdr = 0$$

$$\int_{rA}^{rB} w\frac{rdPx}{dr} - \int_{rA}^{rB} w2\sigma_o dr - \int_{rA}^{rB} 2wBP_x dr = 0 \qquad (12)$$

where (r_A, r_B) is the domain of the element along the radial direction. Thus, the Lagrange family of interpolation functions can be used satisfactorily. Let us assume that the solution P is approximated as follows:

$$p \approx p^e = p_x(r)\sum_{j=1}^{n} p_j^e \psi_j^e(r) \qquad (13)$$

We adopt the Burnov-Galerkin weighted residual method in which it is assumed that the weight function is equal to the interpolation function i.e.

$$w = \psi_j^e(r) \qquad (14)$$

Substituting equations (13) and (14) into equation (12), we get

$$\int_{rA}^{rB} r\psi_j^e(r)\frac{d\sum_{i-1}^{n} P_j^e \psi_j^e(r)dr - 2}{dr}\int_{rA}^{rB} \psi_j^e(r)\sigma_o dr - 2B\int_{rA}^{rB} \psi_j^e(r)\sum_{j-1}^{n} P_j^e \psi_j^e(r)dr = 0$$

$$\sum_{j-1}^{n} P_j^e \int_{rA}^{rB} r\psi_j^e(r)\frac{d\psi_j^e}{dr}dr - 2B\int_{rA}^{rB} \psi_j^e(r)\psi_j^e(r)dr = 2\int_{rA}^{rB} \psi_j^e(r)\sigma_o dr$$

$$\sum_{j-1}^{n} P_j^e \left\{\int_{rA}^{rB} r\psi_j^e(r)\frac{d\psi_j^e}{dr}dr - 2B\int_{rA}^{rB} \psi_j^e(r)\psi_j^e(r)dr\right\} = 2\int_{rA}^{rB} \psi_j^e(r)\sigma_o dr \qquad (15)$$

The finite element model can therefore be represented as:

$$(K_{ij}{}^e)(P_j^e) = (F_i^e) \qquad (16)$$

Where

$$K_{ij}^{e}\int_{rA}^{rB}\left\{r\psi_{j}^{e}(r)\frac{d\psi_{j}^{e}}{dr}2B\psi_{j}^{e}(r)\psi_{j}^{e}(r)\right\}dr$$

$$\left(f_{i}^{e}\right)=2\sigma_{o}\int_{rA}^{rB}\psi_{j}^{e}dr$$

Using the radial I – D Lagrange quadratic interpolation function.

$$\psi_{1}^{e}(r)=\frac{1}{h^{2}}(r_{B}-r)(r_{B}-r_{A}-2r)$$

$$\psi_{2}^{e}(r)=\frac{4}{h^{2}}(r-r_{A})(r_{B}-r)$$

$$\psi_{3}^{e}(r)=\frac{1}{h^{2}}(r-r_{A})(r_{B}-r_{A}-2r)$$

Where $r_B = h + r_A$

Hence

$$K_{ij}^{e}=\frac{\sigma_{o}h}{30}\begin{bmatrix}-4h-15r_{A}-8B & 4h+20r_{A}-4B & 2h-5r_{A}+2B\\ -6h-20r_{A}--4B & -8h+32B & 14h+20r_{A}-4B\\ -3h-5r_{A}-B & -16-20r_{A}-4B & 13h+15r_{A}+8B\end{bmatrix} \quad (17)$$

$$K_{ij}^{e}=\frac{\sigma_{o}h}{30}\begin{pmatrix}1\\4\\1\end{pmatrix} \quad (18)$$

Using four quadratic element for first element, $r_A = 0$ for second elements $r_A = h$, for third element, $r_A = 2h$ A and for the fourth element, $r_A = 3h$.

Substituting first, second, third and fourth elements into equation (17), gives

For element $e_{1,}$ $r_A = 0$

$$K_{j}^{1}=\frac{\sigma_{o}h}{30}\begin{bmatrix}-2h-8B & 4h-4B & -7h+2B\\ -6h-4B & -8h-32B & 14h-4B\\ 3h+2B & -16h-4B & 13h-8B\end{bmatrix}$$

For element $e_2 r_A = h$

$$K^{2}=\frac{\sigma_{o}h}{30}\begin{bmatrix}-17h-8B & 24h-4B & -7h+2B\\ -26h-4B & -8h-32B & 34h-4B\\ 8h+2B & -36h-4B & 28h-8B\end{bmatrix}$$

For element $e_3 r_A = 2h$

$$K^{3}=\frac{\sigma_{o}h}{30}\begin{bmatrix}-32h-8B & 44h-4B & -12h+2B\\ -46h-4B & -8h-32B & 54h-4B\\ 13h+2B & -56h-4B & 43h-8B\end{bmatrix}$$

For element $e_4 r_A = 0$

$$K^{4}=\frac{\sigma_{o}h}{30}\begin{bmatrix}-47h-8B & 64h-4B & -17h+2B\\ -66h-4B & -8h-32B & 74h-4B\\ 18h+2B & -76h-4B & 58h-8B\end{bmatrix}$$

For a mesh of four 1 – D quadratic elements the assembled equation are:

$$k_{ij}^{e}=\frac{\sigma_O h}{30}\begin{bmatrix} K_{11}^{1} & K_{12}^{1} & K_{13}^{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{21}^{1} & K_{22}^{1} & K_{23}^{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ K_{31}^{1} & K_{32}^{1} & K_{33}^{1}+K_{11}^{2} & K_{12}^{2} & K_{13}^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{21}^{2} & K_{22}^{2} & K_{22}^{2} & K_{23}^{2} & 0 & 0 & 0 \\ 0 & 0 & K_{31}^{2} & K_{32}^{2} & K_{33}^{2}+K_{11}^{3} & K_{12}^{3} & K_{13}^{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{21}^{3} & & K_{22}^{3} & K_{23}^{3} & 0 \\ 0 & 0 & 0 & 0 & K_{31}^{3} & K_{32}^{3} & K_{33}^{3}+K_{11}^{4} & K_{12}^{4} & K_{13}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{21}^{4} & K_{22}^{4} & K_{23}^{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{31}^{4} & K_{32}^{4} & K_{33}^{4} \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix}\begin{bmatrix} Q_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_9 \end{bmatrix}$$

Substituting in values into the assembled equation, we have:

$$\left[K_{ij}^{e}\right]\left[P_j^{e}\right]=\frac{\sigma_o h}{30}\begin{bmatrix} -2h-8B & 4h-4B & -2h-2B & 0 & 0 & 0 & 0 & 0 \\ -6h-4B & -8h-32B & 24h-4B & 0 & 0 & 0 & 0 & 0 \\ 3h-2B & -16h-4B & -4h-16B & 24h-4B & -7h+2B & 0 & 0 & 0 \\ 0 & 0 & -26h-4B & -8h-32B & 34-4B & 0 & 0 & 0 \\ 0 & 0 & 8h-2B & -36h-4B & -4h-16B & 44h-4B & -12h+2B & 0 \\ 0 & 0 & 0 & 0 & -46h-4B & -8h-32B & 54h-4B & 0 \\ 0 & 0 & 0 & 0 & 13h+2B & -36h-4B & -4h-16B & 64h-4B \\ 0 & 0 & 0 & 0 & 0 & 0-66h-4B & -8h-32B & 74h-4B \\ 0 & 0 & 0 & 0 & 0 & 18h-2B & -76h-4B & -58h-4B \end{bmatrix}\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{pmatrix} \quad (19)$$

$$\left[F_j^{e}\right]=\frac{\sigma_o h}{3}\begin{pmatrix} F_1^{1} \\ F_2^{1} \\ F_3^{1}+F_1^{2} \\ F_2^{2} \\ F_3^{2}+F_1^{3} \\ F_3^{2} \\ F_3^{3}+F_1^{4} \\ F_2^{4} \\ F_3^{4} \end{pmatrix}=\frac{\sigma_o h}{3}\begin{pmatrix} 1 \\ 4 \\ 1+1 \\ 4 \\ 1+1 \\ 4 \\ 1+1 \\ 4 \\ 1 \end{pmatrix}=\frac{\sigma_o h}{3}\begin{pmatrix} 1 \\ 4 \\ 2 \\ 4 \\ 2 \\ 4 \\ 2 \\ 4 \\ 1 \end{pmatrix}$$

Boundary condition at $r = r_o, \sigma_x = \sigma_d$

Where σ_d = backing stress

So therefore, from the Tresca's yield criterion

$P_x = \sigma_o - \sigma_x$

Assuming zero backing stress,

$P_x = \sigma_o$ This implies that

$P_9 = \sigma_o$

In equation (19), the right hand side becomes $\left((F_i)-(P_9)\right)$

$$[Fi]-[P_g]=\frac{\sigma_o h}{3}\begin{pmatrix}1\\4\\2\\4\\2\\4\\2\\4\end{pmatrix}\begin{pmatrix}0\\0\\0\\0\\0\\0\\-17+2B\\74-2B\end{pmatrix}\begin{pmatrix}\sigma_o h/3\\4\sigma_o h/3\\2\sigma_o h/3\\4\sigma_o h/3\\2\sigma_o h/3\\4\sigma_o h/3\frac{\sigma_o h}{30}(-17+2B)\\2\sigma_o h/3-\frac{\sigma_o h}{30}(74-43)\end{pmatrix}$$

The only unknown pressures are $P_1, P_2, P_3, P_4, P_5, P_6, P_7, \text{ and } P_8$.

So therefore

$[Kij]$ becomes:

$$[K_{ij}^e][P_j^e]=\frac{\sigma_o h}{30}\begin{bmatrix}-2h-8B & 4h-4B & -2h+2B & 0 & 0 & 0 & 0 & 0\\ -6h-4B & -8h-32B & 24h-4B & 0 & 0 & 0 & 0 & 0\\ 3h+2B & -16h-4B & -4h-16B & 24h-4B & -7h+2B & 0 & 0 & 0\\ 0 & 0 & -26h-4B & -8h-32B & 34-4B & 0 & 0 & 0\\ 0 & 0 & 8h+2B & -36h-4B & -4h-16B & 44h-4B & -12h+2B & 0\\ 0 & 0 & 0 & 0 & -46h-4B & -8h-32B & 54h-4B & 0\\ 0 & 0 & 0 & 0 & 13h+2B & -36h-4B & -4h-16B & 64h-4B\\ 0 & 0 & 0 & 0 & 0 & 0 & -66h-4B & -8h-32B\end{bmatrix}\begin{pmatrix}P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\\P_8\end{pmatrix}$$

From equation (19)

$$[K_{ij}][P_j]=[F_i] \tag{20}$$

Since we now have eight unknowns, $P_1, P_2, P_3, P_4, P_5, P_6, P_7, \& P_8$, equation (20) becomes

$$[K_{ij}][P_j]=[F_i]-[P_9]$$

$$[P_j]=[K_{ij}]^{-1}[\{F_i\}-\{P_9\}]$$

Numerical example

Consider a wire drawing operation in which

$$r_o = 5.5mm,\ r = 5mm,\ h = 15mm,\ \sigma_o = 240N/mm^2,\ \alpha = 16^o \ \ and \ \ \mu = 0.1$$

$$B = \mu Cot\alpha = 0.1 \times Cot16^o$$

$$= \frac{0.1}{\tan 16_o} = \frac{0.1}{0.2867} = 0.3488$$

Using MathCAD software to solve the numerical example, we get

$$[F]-[P_9]=\begin{pmatrix}300\\1200\\600\\1200\\600\\1200\\1089\\-978\end{pmatrix}$$

$$[P_j]=[K_{ij}]^{-1}[\{F_i\}-\{P_9\}]$$

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{pmatrix} = \begin{pmatrix} -1.481 \\ -0.653 \\ -0.534 \\ -0.200 \\ -0.168 \\ 0.212 \\ 0.097 \\ 0.207 \\ 1.000 \end{pmatrix} \qquad (21)$$

The negative sign in equation (21) indicate compression.

1.3 Exact Solution

Recall equation (10)

$$\frac{dp_x}{dr} - \frac{2\sigma_o}{r} - \frac{2BP_x}{r} = 0$$

$$\frac{dp_x}{dr} = \frac{2}{r}\left(BP_x + \sigma_o\right)$$

Separating the variables, we get

$$\frac{drp_x}{\left(BP_x + \sigma_o\right)} = \frac{2dr}{r}$$

and hence,

$$\text{In } \left(BP_x + \sigma_o\right) = In\left(rC\right)^{2B} \qquad (22)$$

where C is the constant of integration

$$\text{In } \left(BP_x + \sigma_o\right) = In\left(rC\right)^{2B}$$

$$.: \left(BP_x + \sigma_o\right) = In\left(rC\right)^{2B} \qquad (23)$$

From the boundary condition, at $r = r_o, \sigma_x = \sigma_b$ but it is assumed that there is no backing stress hence substituting this into the Tresca's yield criterion.

$$P_x = \sigma_o$$

Substituting into equation (23)

$$B\sigma_o + \sigma_o = \left(r_o C\right)^{2B}$$

$$\sigma_o\left(1 + B\right) = \left(r_o C\right)^{2B}$$

$$r_o C = \sigma_o\left(1 + B\right)^{1/2B}$$

$$\because C = 1/r\left[\sigma_o\left(1 + B\right)^{1/2B}\right] \qquad (24)$$

Substituting equation (24) into equation (23) gives,

$$BP_x + \sigma_o = \left[\frac{r}{r_o}\right]^{2B} \left[\sigma_o\left(1 + B\right)^{1/2B}\right]$$

$$BPx = -\left[\frac{r}{r_o}\right]^{2B} \left[\sigma_o\left(1 + B\right)\right] - \sigma_o$$

$$\because P_x = \frac{\sigma_o}{B}\left[\left(1 + B\right)\left[\frac{r}{r_o}\right]^{2B} - 1\right] \qquad (25)$$

Result from exact solution are shown below

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \end{pmatrix} = \begin{pmatrix} -1.500 \\ -0.621 \\ -0.538 \\ -0.199 \\ -0.165 \\ 0.212 \\ 0.100 \\ 0.201 \\ 1.000 \end{pmatrix}$$

Due to symmetry, the pressures would be similar on the other half of the wire hence we can compare the exact solutions and the finite element solution on table 1.

Table 1: Finite element and exact solution with differences

Nodal Point	FEM (N/mm^2)	EXACT (N/mm^2)	Differences
1	- 1.481	-1.500	- 0.019
2	0.653	-0.621	- 0.032
3	- 0.534	- 0.538	- 0.004
4	- 0.200	- 0.199	- 0.001
5	- 0.168	- 0.165	- 0.003
6	0.212	0.212	0.000
7	0.097	0.100	- 0.003
8	0.207	0.201	0.006
9	1.000	1.000	0.000

Discussion and Results

Table 1 shows that as the nodal points increases, the finite element solution was closed to exact value solution and the negative sign in table 1 indicates compression during the wire drawing process. Also, careful look at the difference between the finite element and exact value solutions indicated little or no difference in value.

Conclusion

Looking at the analysis, it can therefore be concluded that the Bubnov-Galerkin weighted residual finite element method is a better engineering tool that is capable of adequately and accurately predicting the stresses pressures set up in wire drawing operation.

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