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Sensitivity Analysis of a Horizontal Earth Electrode under Impulse Current Using a 2nd Order FDTD approach.

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Abstract

This paper presents the sensitivity analysis of an earthing conductor under the influence of impulse current arising from a lightning stroke. The approach is based on the 2nd order finite difference time domain (FDTD). The earthing conductor is regarded as a lossy transmission line where it is divided into series connected π −circiuts and the per-unit length parameters of the circiut are taken as non-uniform and computed using well known expressions. The sensitivity analysis carried out shows that there is limit in the length of the earthing conductor that lowers the transient potential and transient impedance values. This limit is called the effective length and a knowledge of this not only contributes to the design of an efficient and effective earthing system but also minimizes the cost of earthing conductors and labour.

1 Introduction:

The mathematical modelling of earthing systems provides a means of studying the transient behaviour of any electrical installation which could lead the proper and efficient design that minimizes cost but optimally provides protection. Transient analysis of earthing systems is of widespread interest in protection of personnel and equipment. Two of the most important parameters arising from the study of transients in earthing systems are the transient potential and transient impedance.

An effective earthing system with low transient potential and transient impedance to eletromagnetic diturbances such as lightning surges is strongly required. It is highly desirable to evaluate the transient impedance and resistance as a measure of performance of earthing systems in which lightning surge currents with fast rise time flow [1].

This work deals with the sensitivity analysis of some dominant parameters such as soil resistivity, length of earth conductor, impulse current and depth of burial as they affects the waveform of the resulting transient potential and transient impedance of a horizontally buried earth conductor.

The results of the analysis leads to the determination of the upper limit in the length of the earthing conductor that substantially affects the maximum transient potential and transient impedance values [2].

The mathematical approach used in this work is the 2nd order finite difference time domain (FDTD) where the earthing conductor is regarded as a lossy transmission line. The earthing conductor is treated as a series connected π −circiut tending to the open-ended transmission line when the number of circiuts increases. The earthing conductor model is constructed under the assumption that the soil is homogeneous and the per unit length inductance, capacitance, conductance are regarded as non-uniform [3].

2 Mathematical Formulations

The work reported in this paper refers to the sensitivity analysis of the behaviour of horizontal earthing system buried at a depth h in a soil of resistivity ρ that is been stroked directly by a lightning impulse current i. The model used for this analysis is the 2nd order finite difference time domain (FDTD) approach, where the earth conductor is regarded as a lossy transmission line. The lossy transmission line is mathematically described by the 2nd order telegrapher's equations as [4].

$$-\frac{\partial^2 v(x,t)}{\partial x^2} = \frac{R\partial i(x,t)}{\partial x} - L(x,t)G(x,t)\frac{\partial v(x,t)}{\partial t} - L(x,t)C(x,t)\frac{\partial^2 v(x,t)}{\partial t^2} \tag{1}$$

$$-\frac{\partial^2 i(x,t)}{\partial x^2} = \frac{G\partial v(x,t)}{\partial x} - C(x,t)R\frac{\partial i(x,t)}{\partial t} - L(x,t)C(x,t)\frac{\partial^2 i(x,t)}{\partial t^2} \tag{2}$$

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where R is the per-unit-length resistance and,C,L and$G$ are the non-uniform per-unit-length capacitance, inductance and conductance of the earth electrode respectively.

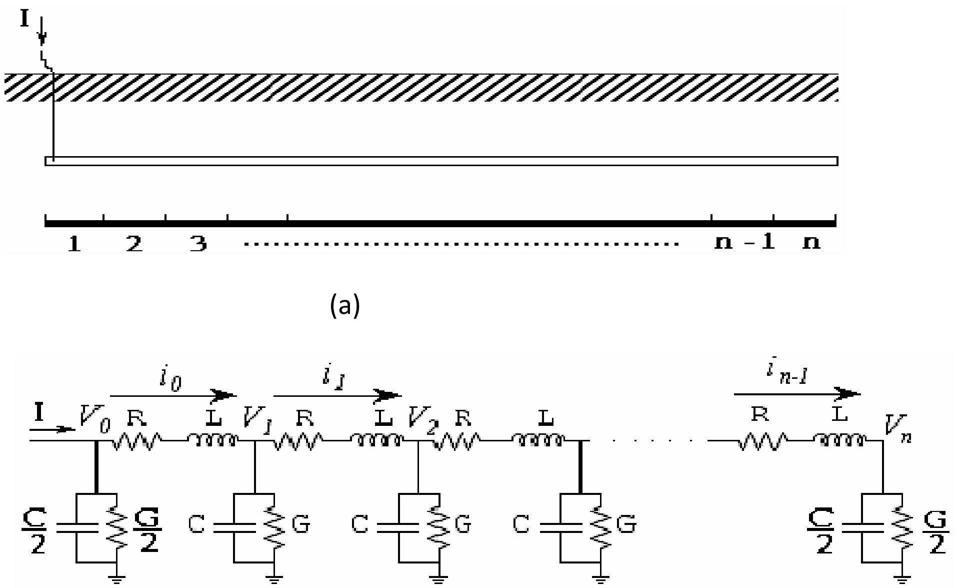
The FDTD approach is introduced by converting the derivates in (1) and (2) into their respective finite differences, dividing the earthing conductor into Δx pi-sections as shown in Fig. 1 and the total solution time into Δt segments. Thus the FDTD expressions for the 2nd order telegrapher's equations becomes

$$v_k^{n+1} = v_k^n - \frac{\Delta t}{C}\left[\left(\frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x}\right) + G v_k^n\right] +$$

$$+\frac{\Delta t^2}{2CL}\left[\left(\frac{v_{k+1}^n + v_{k-1}^n - 2v_k^n}{\Delta x^2}\right) + R\left(\frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x}\right) - LG\left(\frac{v_k^{n+1} - v_k^{n-1}}{2\Delta t}\right)\right] \tag{3}$$

$$i_k^{n+1} = i_k^n - \frac{\Delta t}{L}\left[\left(\frac{v_{k+1}^n - i_{k-1}^n}{2\Delta x}\right) + R i_k^n\right] +$$

$$+\frac{\Delta t^2}{2CL}\left[\left(\frac{i_{k+1}^n + i_{k-1}^n - 2i_k^n}{\Delta x^2}\right) + G\left(\frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x}\right) - CR\left(\frac{i_k^{n+1} - i_k^{n-1}}{2\Delta t}\right)\right] \tag{4}$$



(a)

(b)

Figure 1: (a) Segmented horizontal conductor(earth electrode) (b) pi-section circuit model of a Horizontal earth- electrode

The computation of the non-uniform per unit length parameters is done using the expressions given in [3].

$$R_{ji} = \frac{\rho_{soil}}{4\pi l_j}\left(\iint_{l_j l_i}\frac{1}{\acute{r}_{ji}} + k_{sigma}\iint_{l_j l\acute{i}}\frac{1}{\acute{r}_{j\acute{i}}}\right) dl dl \tag{5}$$

where R_{ji} is a earth electrode resistance matrix, R_{ji} is obtained by using the Method of Moment(MoM) on equation (19). When $i = j$ it is self earth electrode resistance of the segment l_{ii} and the segmental self earth electrode resistance is given as,

$$R_{ii} = \frac{\rho_{soil}}{2\pi l_{ii}} ln\left(\frac{2l_{ii}}{\sqrt{2rd}}\right) \tag{6}$$

When $i \neq j$,it is the mutaul earth electrode resistance between two segments i and j.

$k_{rho} = \frac{\rho_{air} - \rho_{soil}}{\rho_{air} + \rho_{soil}}$ is the reflection coeficient due to the difference resistivity of the air and the soil. ρ_{air} and ρ_{soil} are the air and soil resistivities respectively. The per unit length non-uniform conductance G is obtain from,

$$G = \frac{1}{\sum R_{ji}} \tag{7}$$

The elements of the susceptance or inductance matrix are computed using equation (5) by replacing the R_{ji} with P_{ji} or L_{ji} ,the k_{rho} with $k_{epsilon} = \frac{\varepsilon_{soil-}\varepsilon_{air}}{\varepsilon_{soil+}\varepsilon_{air}}$ or zero, and the ρ_{soil} with $\frac{1}{\varepsilon_{soil}}$ or μ_0 . The self susceptance P_{ii} and self inductance L_{ii} are computed using,

$$P_{ii} = \left(2\pi l_{ii}\varepsilon_{soil} ln\left(\frac{2l_{ii}}{\sqrt{2rd}}\right)\right)^{-1} \tag{8}$$

and

$$L_{ii} = \frac{\mu_0}{2\pi l_{ii}} ln\left(\frac{2l_{ii}}{\sqrt{2rd}}\right) \tag{9}$$

The non-uniform per unit length capacitance and inductance is thus computed using,

$$C = \frac{1}{\sum P_{ji}} \tag{10}$$

$$L = \sum L_{ji} \tag{11}$$

3 Sensitivity Results

The sensitivity analysis was performed using a round conductor made up of copper with a radius of 7.5 mm. The sensitivity parameters are: the depth of burial, the length of the conductor, the injected impulse current and the soil resistivity. The behaviour of transient voltage and the transient impedance due to the variations of the sensitivity parameters is the area of interest.

The waveform of the injected impulse current is given as [5].

$$i(t) = I_0 \cdot \left(e^{-\alpha t} - e^{-\beta t}\right) \tag{12}$$

The values of the parameters α and β of equation (12) are as presented in Table 1. The value of the coefficient I_0 is taken as 1.0 A.

Table 1. Values of parameters in Equation (12)

Case	$\alpha\ [s^{-1}]$	$\beta\ [s^{-1}]$	$T_{crest}\ [\mu s]$	$T_{half}\ [\mu s]$
1	16667	100000000	0.1	43
2	16667	25000000	0.3	43
3	16667	10000000	0.6	43
4	16667	2857143	1.8	43
5	16667	2127660	2.4	43
6	16667	1538462	3.0	43

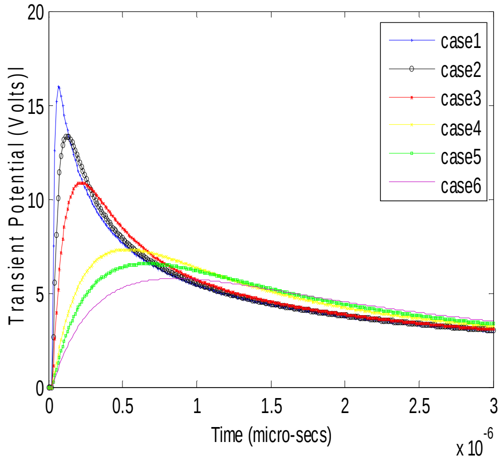


Fig.2: Transient potential versus time at injection point 0 metre of a 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100 Ohm-metre using cases 1-6 of Table 1.

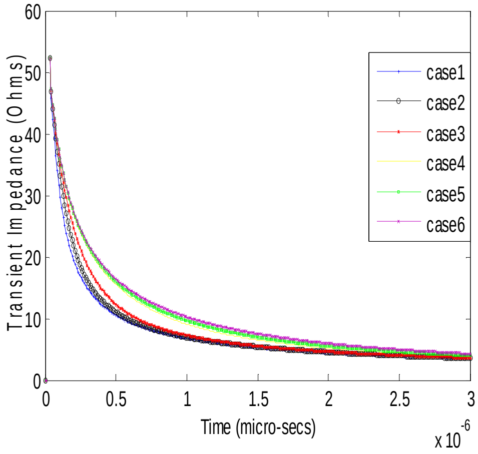


Fig.3: Transient impedance versus time at injection point 0 metre of a 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100 Ohm-metre using cases 1-6 of Table 1.

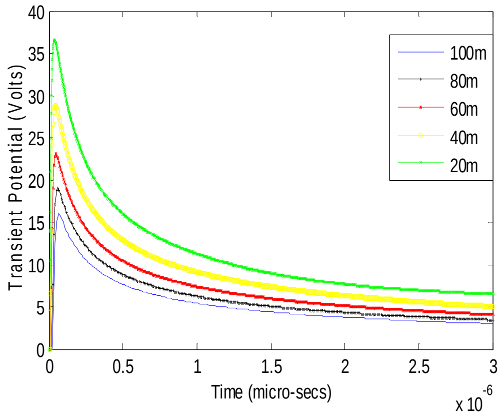


Fig.4: Transient potential versus time at injection point 0 metre of a 20, 40, 60, 80, 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100 Ohm-metre using case 1 of Table 1.

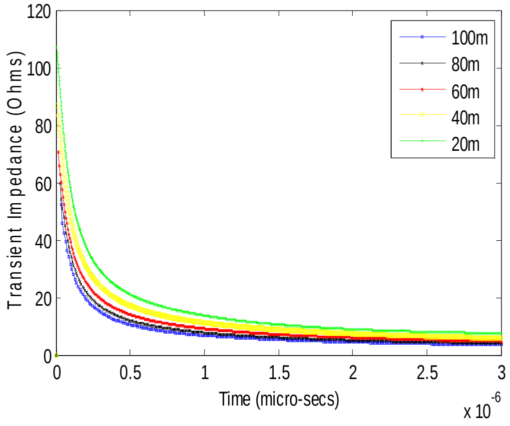


Fig.5: Transient impedance versus time at injection point 0 metre of a 20, 40, 60, 80, 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100 Ohm-metre using case 1 of Table 1.

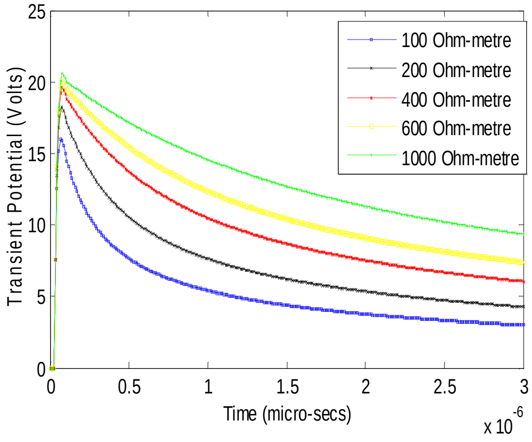


Fig.6: Transient potential versus time at injection point 0 metre of a 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100, 200, 400, 600, 1000 Ohm-metre using case 1 of Table 1.

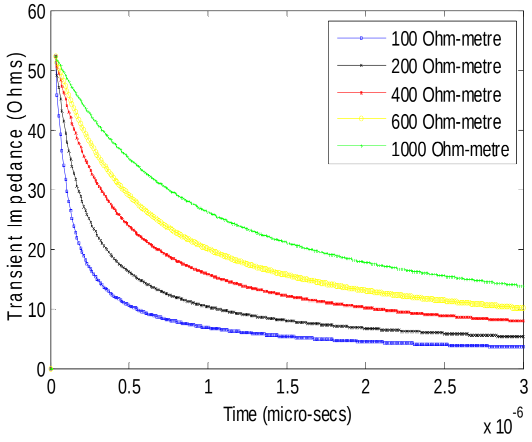


Fig.7: Transient impedance versus time at injection point 0 metre of a 100 metre conductor buried 0.5 metre deep in a soil of resistivity 100, 200, 400, 600, 1000 Ohm-metre using case 1 of Table 1.

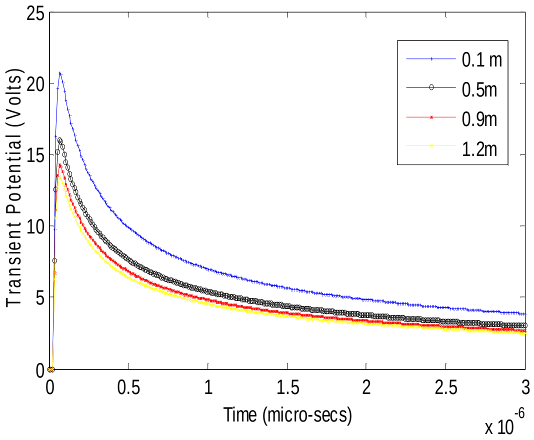


Fig.8: Transient potential versus time at injection point 0 metre of a 100 metre conductor buried 0.1, 0.5, 0.9, 1.2 metre deep in a soil of resistivity 100 Ohm-metre using case 1 of Table 1.

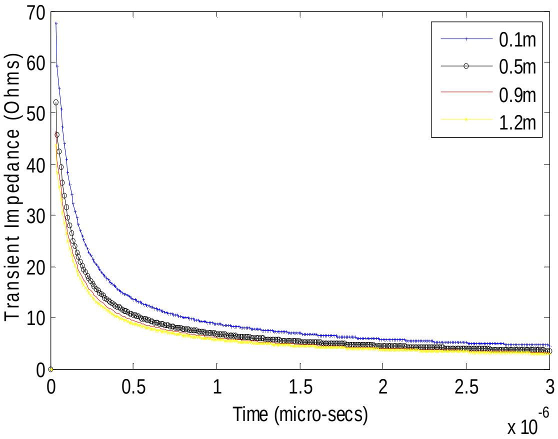


Fig.9: Transient impedance versus time at injection point 0 metre of a 100 metre conductor buried 0.1, 0.5, 0.9, 1.2 metre deep in a soil of resistivity 100 Ohm-metre using case 1 of Table 1.

4. Analysis of the Results

The waveform of the transient potentials and impedances due to the variation in the time to crest are as plotted in Figs 2 and 3. The magnitude of the transient potential is a function of the variation of time to crest as presented in cases 1-6 in Table 1. The faster, the time to crest of the injected impulse current, the higher the magnitude of the transient potential. The transient impedance in these cases has almost the same steady state level response, though with case 1 faster decreasing exponentially to the steady-state level up to the time of 2 micro-seconds.

Fig. 4 and 5 shows the plot of the transient potential and impedance due to the variation in the length of the earth conductor using case 1 as the injected impulse current and a soil resistivity of 100 ohm-metre at a burial depth of 0.5 metres. As can be seen the magnitude of the transient potential as well as that of the steady state level of the transient impedance decreases as the length of the earth conductor decreases. The deduction from these behaviour shows a capacitive nature as the length of the conductor increases.

The transient behaviour of a 100 metre earth conductor buried at a depth of 0.5 metre in soils of various soil resistivities is shown in Fig. 6 and 7. In Fig. 6 and 7 the magnitude of both the transient potential and the steady state level of the transient impedance increases as the soil resistivity increases respectively. This is due to the increasing resistive component of both the transient potential and transient impedance.

Fig 8 and 9 shows both the transient potential and impedance of various depth of burial of a 100 metre earth conductor buried in a soil of resistivity 100 ohms-metre. The depth of burial shows a significant change from 0.1 metre to 1.5metre and there after the change is highly in significant, thus a burial depth of 0.5 metre is an acceptable maximum limit.

The earth electrode resistance and the transient impedance of earthing system are necessary for the determining earthing performance. The earthing system behaviour must tend towards a low transient impedance as well as low earth electrode resistance. The best way out so far from the analysis presented is by increasing the length of the earthing conductor. If an impulse current is applied at the energization end of an earth electrode, when its length increases, the maximum transient potential decreases almost continuously, but as the length increases an upper limit is reached beyond which no serious decrement in the amplitude of the transient potential as observed in Fig.4. The length of the conductor value beyond which no serious decrement is observed is called the effective length. Another definition is that, the effective length of a single horizontal earthing wire is the length above which no further reduction of the transient impedance is observed. This can be seen in Fig 4 and 5 as the maximum transient potential and transient impedance values for an 80 metre and 100 metre earthing conductor buried in the same soil shows no appreciable decrement, thus an 80 metre conductor is regarded in this case as the effective length. The values of the effective length of earthing conductor on various soil resistivities and impulse currents can be estimated using the same procedure. Knowing the effective length of a proposed earthing system does not only contribute to efficient earthing system design but also minimize the cost of construction of the earthing system.

A comparison of the results obtained from the 2nd order FDTD analysis with that obtained from a 1st order FDTD as presented in [3] and in [6] shows that the 2nd order FDTD is more stable than the 1st order FDTD but uses more memory space thus resulting in a longer simulation time.

5. Conclusions

In this paper a 2^{nd} order FDTD approach was used to perform a sensitivity analysis on a horizontally buried earthing conductor stroked by a lightning impulse current. The analysis was based on the determination of the transient potential and transient impedance of the earthing conductor by varying the value of some dominant earthing parameters. The results obtained was used in the determination of the effective length that limits the performance of earthing electrode in which lightning impulse current with fast rise time and high frequency flows.

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