Journal of the Nigerian Association of Mathematical Physics
***Volume* 18 (May, 2011), pp 563 – 568**

A Deterministic Approach to Noise Attenuation in Oil and Gas Seismic Data Acquisition

[1]Obinabo E. C. and [2]Anukwu E. N.

[1]Department of Electrical and Electronic Engineering,
Ambrose Alli University, P. M. B, 14, Ekpoma, Edo State, Nigeria.
[2] Department of Electrical and Electronic Engineering,
Petroleum Training Institute, P. M. B, 20, Effurun, Delta State, Nigeria.

Abstract

This paper presents an estimation of an oil and gas seismic data acquisition process which incorporates a priori knowledge of noise contamination in the measured data. A conceptual simplicity of parameter and state estimation by a least squares computational algorithm was developed and a filter was postulated to define the error covariance matrix which yielded unbiased estimates of the measured data.

Key words: Oil and gas seismic data acquisition, stochastic prediction, least squares estimates, linear time-invariant systems, measurement noise filtration, Kalman filter.

1 Introduction:

The conventional seismic exploration method in the oil and gas industry is by acoustic means whereby sound waves in the frequency range of $10-1000$ *cycles per* sec [1] are propagated into the earth's crust. The sound wave travels deep, up to 10 miles, in the earth and returns bringing back important information with it. In a related study involving acoustic liquid level surveys by data acquisition method [2], the complete acoustic signal record from the shot to the liquid level reflection employs digital filtering [3] and signal processing to establish a guaranteed interpretation of the liquid level position. Signal filtration in this application provides additional processing techniques, under operator control, to obtain accurate results in oil wells with shallow liquid levels and noisy well bores. This is enhanced by inclusion of an analogue filter [4] to reduce the interference from signals generated by the reflections at the tubing collars, and to ensure that the liquid level signal could be detected in the majority of the cases. In oil and gas seismic exploration [5, 6], the fundamental objective is markedly enhanced by increasing the signal-to-noise ratio in the recorded data. One method of doing this using the acoustic method [7, 2] was to filter measured values to reject undesirably high frequency components. This was achieved by passing the measurements through simple low-pass filters before connecting to the input. The cut-off frequency of the filter would then be selected in conjunction with the sampling and conversion rates in order to satisfy a given sampling theorem while preserving the signal components of interest. This method [5] was entrenched in the estimation of parameters or states from a set of correlated data, using least squares method [8, 9] and has a wide field of application which includes data smoothing. the problem is generally concerned with identifying the parameters of noisy dynamic processes.

In this study, we developed a least squares computational algorithm for application to noise attenuation in oil and gas seismic data acquisition using the filter design philosophy due to Hsiao and Wang [3]. A control system was modelled assuming all the noise processes were independent. This reduced the problem to one of optimal control whereby a filter was designed to give a minimum variance estimate using the measured values of the input and output of the system. Because measurements invariably contain errors [10 – 12], our approach to the problem utilized the concepts of probability and statistics in which state estimation was addressed because noise was known to be correlated with the measured data [13, 14].

Methodology

The seismic data acquisition was preceded with laying of geophones, fibre cables, remote data acquisition units and recorder takeout units. The geophone arrays were connected to takeout units via fibre optic strands which also connect the remote data acquisition units in cascade to form an unbroken chain of cable stations and remote data acquisition units that route the seismic

Corresponding authors: ***E. C. Obinabo:*** E-mail: drobinaboec@yahoo.com, Tel. +2347034481461

wavelets to and from the recording units. The acquired data was directed from the earth through seismometers for filtration and digitization. In digital signal processing [15, 16], the analogue-to-digital converter (ADC) gives high resolution of the data based on continuous integration and oversampling in combination with carefully designed low-pass filters (fig. 1) which eliminates gaps in the sampling process. The negative feedback from the output (fig. 1) along the $1-bit$ digital-to-analogue converter (DAC) and the sum amplifier at the input were performed at a high sample rate, transferring the quantization noise into the stop band of the digital low-pass filter, for decimation. In the process, the sampled analogue signal was fed to the sum amplifier along with the output of the $1-bit$ digital-to-analogue converter (DAC). The integrated difference signal was fed to the strobed comparator whose output samples the difference signal at the sampling frequency many times that of the analogue signal frequency. The output of the comparator provides the digital input for the $1-bit$ digital-to-analogue converter (DAC). Thus, the system functions on a negative feedback loop which minimizes the difference signal by tracking the input. The integrator was continuously fed with the differential signal and there was no gaps in the analogue input signal as introduced by sample and hold devices. The digital information representing the analogue input voltage was coded in the polarities of the pulse train appearing at the output of the comparator which can be retrieved as a parallel binary data word applying a digital filter operator.

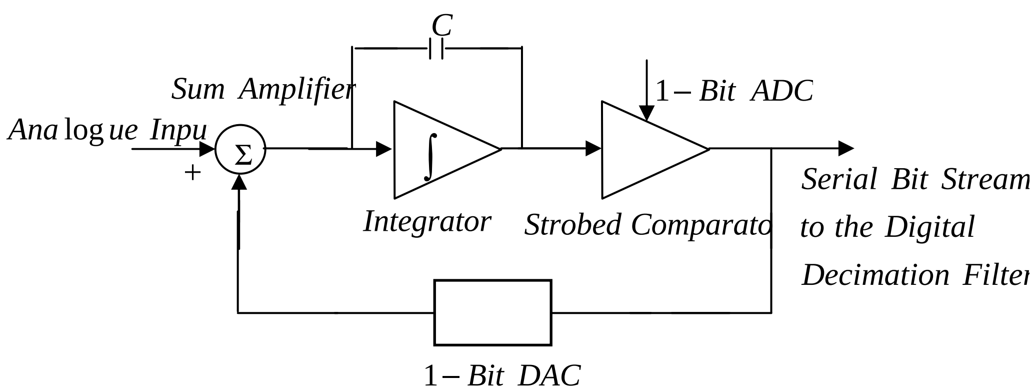

Fig. 1: A Continuous Sampling Signal Modulator

The State Estimation Problem:

The least squares estimation of the process was formulated on the basis of maximum likelihood and Bayesian techniques [17, 18] using statistical information in terms of joint probability distribution functions. The estimation problem was carried out for a sequence of the process and the noise model until the best and simplest possible model was obtained. Estimation of the process was based on the assumption that some or all of the parameters were unknown even though the structure of the differential equation characterizing the system as well as the initial and boundary conditions were available. This reduced the problem to one of optimal control whereby the best estimate using the measured values of the input and output the system were required. Because our measurements invariably contained errors, solution of the problem utilized concepts of probability and statistics in which the problem addressed state estimation because noise was known to be correlated with the measured data. This was obtained at the same time as the parameters during which filtration was mandatory in the processing of the data. Computation of the optimal estimates which relied on convergence of the iteration employed was accomplished through sequential filtration of the estimate.

The Computational Algorithm:

We modelled the process mathematically as follows:

$$x_{i+1} = a x_i + v_i \tag{1}$$

$$y_{i+1} = h x_{i+1} + w_{i+1} \tag{2}$$

and the predictor

$$x_{i+1}^{i} = a x_i^{\ i} \tag{3}$$

$$y_{i+1}^{i} = h x_{i+1}^{i} \tag{4}$$

Now, if we ignore the control input and assume that at time $t_i = 0$ a best estimate $x_i^{\ i}$ of the state is available, then the best prediction of x at time t_{i+1} is expressed as

$$x_{i+1}^{i} = ax_i^{\;i} \tag{5}$$

where $x_{i+1}^{i} \equiv x\left(\frac{i+1}{i}\right)$. At time $t = t_{i+1}$, a measurement is available as follows

$$y_{i+1} = hx_{i+1} + w_{i+1} \tag{6}$$

Now let

$$\begin{aligned}\delta y_{i+1} &= y_{i+1} - y_{i+1}^{i}\\ &= y_{i+1} - hax_i^{i}\end{aligned} \tag{7}$$

To improve on the estimate x_{i+1}^{i} we required an addition of a proportion of δy_{i+1}, that is,

$$\begin{aligned}x_{i+1}^{i+1} &= x_{i+1}^{i} + k_{i+1}(\delta y_{i+1})\\ &= ax_i^{i} + k_{i+1}\left(y_{i+1} - hax_i^{i}\right)\end{aligned} \tag{8}$$

where ax_i^{i} are the prediction and $k_{i+1}\left(y_{i+1} - hax_i^{i}\right)$ the desired correction. We now define δx_{i+1} as follows

$$\delta x_{i+1} = x_{i+1} - x_{i+1}^{i+1} \tag{9}$$

and $$p_{i+1}^{i+1} = E\left[\delta x_{i+1}^{2}\right] \tag{10}$$

Then $$\delta x_{i+1} = (I + k_{i+1}h)a\delta x_i + (I - k_{i+1}h)v_i + k_{i+1}w_{i+1} \tag{11}$$

$$\begin{aligned}E\left[\delta x_{i+1}\delta x_{i+1}^{T}\right] &= p_{i+1}^{i+1}\\ &= (I - k_{i+1}h)ap_i^{\,i}a^{T}(I - k_{i+1}h)^{T} + (I - k_{i+1}h)GQG^{\;T}(I - k_{i+1}h)^{T} + k_{i+1}Rk_{i+1}^{\,T}\end{aligned} \tag{12}$$

We required k_{i+1} such that p_{i+1}^{i+1} is a minimum. We did this by evaluating the differential

$$\frac{\partial}{\partial k_{i+1}} p_{i+1}^{i+1} = 0 \tag{13}$$

and then substituting

$$p_{i+1}^{i} = ap_i^{i}a^{T} + GQG^{T} \tag{14}$$

to obtain

$$k_{i+1} = p_{i+1}^{i}h^{T}\left(hp_{i+1}^{i}h^{T} + R\right)^{-1} \tag{15}$$

Now with the process given by

$$x_{i+1} = ax_i + v_i \tag{16}$$

$$y_{i+1} = hx_{i+1} + w_{i+1} \tag{17}$$

with the predictor

$$x_{i+1}^{i} = ax_i^{i} \tag{18}$$

$$y_{i+1}^{i} = hx_{i+1}^{i} \tag{19}$$

the time-varying gain k_{i+1} was evaluated by defining the estimation error

$$\delta x_{i+1} = x_{i+1} - x_{i+1}^{i+1} \tag{20}$$

The mean value of $\delta x_{i+1} = 0$ and the variance of δx_{i+1} was obtained as

$$E\left[\left(\delta x_{i+1}^{2}\right)\right] = p_{i+1}^{i+1} \tag{21}$$

which is a function of k_{i+1}. From the process and filter equations, we obtained

$$\begin{aligned}\delta x_{i+1} &= x_{i+1} - x_{i+1}^{i+1}\\ &= ax_i + v_i - \left[ax_i^{i} + k_{i+1}\left(y_{i+1} - hx_{i+1}^{i}\right)\right]\\ &= ax_i + v_i - ax_i^{i} - k_{i+1}\left[h(ax_i + v_i) + w_{i+1} - h\left(ax_i^{i}\right)\right]\\ &= a\left(x_i - x_i^{i}\right) - k_{i+1}ha\left(x_i - x_i^{i}\right) + v_i + k_{i+1}hv_i + kw_{i+1}\\ &= (1 - k_{i+1}h)a\delta x_i + (1 - k_{i+1}h)v_i + k_{i+1}w_{i+1}\end{aligned} \tag{22}$$

Therefore

$$E\left[\delta_{i+1}^2\right] = p_{i+1}^{i+1}$$

$$= \left(1 - k_{i+1}h\right)^2 a^2 p_i^i + \left(1 - k_{i+1}h\right)^2 Q_i + k_{i+1}^2 R_{i+1} \qquad (23)$$

since

$$E\left[\delta x_i w_{i+1}\right] = E\left[v_i w_{i+1}\right] = E\left[\delta x_i v_i\right] = 0 \qquad (24)$$

All noise processes were assumed to give a minimum variance estimate of x_{i+1}^{i+1} as follows

$$\frac{\partial}{\partial x_{i+}}\left(p_{i+1}^{i+1}\right) = -2\left(1 - k_{i+1}h\right)ha^2 p_i^i - 2\left(1 - k_{i+1}h\right)hQ_i + 2k_{i+1}R_{i+1} = 0 \qquad (25)$$

Let $a^2 p_i^i = p_{i+1}^i$ (26)

then the predicted variance becomes

$$k_{i+1} = p_{i+1}^i h\left(h^2 p_{i+1}^i + R_{i+1}\right)^{-1} \qquad (27)$$

and $p_{i+1}^{i+1} = \left(1 - k_{i+1}h\right)p_{i+1}^i$ (28)

$$\frac{d}{dk_{i+1}}\left(p_{i+1}^{i+1}\right) = -2\left(1 - k_{i+1}h\right)ha^2 p_i^i - 2\left(1 - k_{i+1}h\right)hQ_i + 2k_{i+1}R_{i+1}$$

$$= -2\left(1 - k_{i+1}h\right)hp_{i+1}^i + 2k_{i+1}R_{i+1} \qquad (29)$$

where $p_{i+1}^i \triangleq a^2 p_i^i + Q_i = E\left[\delta x_{i+1}^{i\ 2}\right]$ (30)

and

$$E\left[v_i^{\ 2}\right] = Q_i \qquad (31)$$

RESULTS AND DISCUSSION

A set of oil and gas seismic data was defined for a single-input single-output system as follows:

Time (i)	0	1	2	3	4
Input $u(i)$	1	-1	-1	1	1
Output $y(i+1)$	-0.1	0.56	-0.34	-0.63	0.42

The system was identified using a discrete weighting sequence

$$y(i+1) = \sum_{j=0}^{1} h(j)u(i-1) + v(i-1)$$

Here, we formulated a sequential least squares procedure for $\hat{h}(0)$ and $\hat{h}(1)$ as follows

$$\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} u(0) & 0 \\ u(1) & u(0) \\ u(2) & u(1) \\ u(3) & u(2) \\ u(4) & u(3) \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \end{bmatrix} + \begin{bmatrix} V(1) \\ V(2) \\ V(3) \\ V(4) \\ V(5) \end{bmatrix}$$

$$\hat{h}_{k+1} = \hat{h}_k + P_{k+1}H(k+1)^T\left[y(k+1) - H(k+1)\hat{h}_k\right]$$

$$\hat{h}_o = \begin{bmatrix} u(0) & 0 \\ u(1) & u(0) \end{bmatrix}^{-1} \begin{bmatrix} y(1) \\ y(2) \end{bmatrix} = -\begin{bmatrix} -0.1 \\ 0.46 \end{bmatrix}$$

$$P_o = \left(H_o^{\ T} H_o\right)^{-1} = \begin{bmatrix} 11 \\ 12 \end{bmatrix},\ P_1 = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix},\ \hat{h}_1 = \begin{bmatrix} -0.1067 \\ 0.45 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} \frac{4}{5} & 0.1 \\ 0.1 & \frac{7}{20} \end{bmatrix},\ \hat{h}_2 = \begin{bmatrix} -0.1189 \\ 0.4683 \end{bmatrix},\ \hat{h}_3 = \begin{bmatrix} -0.106 \\ 0.488 \end{bmatrix},\ P_3 = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

This result gives an unbiased estimate of the sampled data.

Conclusion

A stochastic time-invariant model has been developed for linear filtering and prediction of state in oil and gas seismic data acquisition. The model provides a processing technique under operator control for eliminating the interference from signals generated by seismic wave reflections further down in the earth's crust. The convergence approach described in this work operates on the basis of data filtration. The plant itself is represented by a stable, linear, parameter-dependent state model. The filtration algorithm guarantees stable generation of the residuals.

References

[1] Bouvier, J. D., C. H. Kaars-Sijpesteijn, D. F. Kluesner, C. C. Onyejekwe and R. C. Van Der Pal, 1989, Three-dimensional Seismic Interpretation and Fault Sealing Investigations, Nun River Field, Nigeria, The American Association of Petroleum Geologists Bulletin, Vol. 73, No. 11, 1397 – 1414.

[2] Kochhar, A. K., 1978, Comparison of Stochastic Identification Techniques for Dynamic Modelling of Plastic Extrusion Processes, Proc. Instn. Mech. Engrs., Vol. 192, No. 28, 299 – 309.

[3] Hsiao C. H. and W. J. Wang, 2000, State Analysis and Parameter Estimation of Bilinear Systems Via Haar Wavelets, IEEE Trans. on Circuits and Systems 1: Fundamental Theory and Applications, vol.47, no. 2, 246 – 250.

[4] Hansen, P. D., 2000, Robust Adaptive PID Tuning for Unmeasured-load rejection, Proc. 2000 IFAC Workshop on Digital Control, 487 – 494.

[5] Ahmed, J. O., F. Chang, I. P. J. M. Stulemeijer, X. Y. A. Le Varlet, P. J. F. Verbeek and H. Williams, 1993, From 3-D Seismic Interpretation to Volumetric Estimate Using a Quantitative Reservoir Model, The EA Field, OML 79, Nigeria, Society of Petroleum Engineers, Nigeria Council, 47 – 57.

[6] Nwaezeapu, A. U., 1992, Hydrocarbon Exploration in Frontier Basins: The Nigerian Chad Basin Experience, Presented at the 28th Annual Conference of the Nigerian Mining and Geosciences Society held in Port Harcourt, Nigeria,1st to 5th March,

[7] Bacher R. A., W. Gray and D. J. Murray–Smith, 1981, Time Domain System Identification Applied to Non–invasive Estimation Cardiopulmonary Quantities ibid, Vol. 128 No. 2, pp. 56 – 64.

[8] Obinabo, E. C. and C. E. Ojieabu, 2007, On the Estimation of Billet Re-heating Furnace Temperatures by Least Squares Method, Journal of Applies Sciences, 10 (1): 6854 – 6866.

[9] Karimi H. R., B. Lohmann, P. J. Maralani and B. Moshiri, 2004, A Computational Method for Solving Optimal Control and Parameter Estimation of Linear Systems using by Haar Wavelets, Trans. ASME J. Basic Eng., vol. 81, no. 9, 1121 – 1132

[10] Ekejiuba, A. O., M., 1986, Locating By-passed Hydrocarbon Accumulation in Old Reservoirs, Nigerian Association of Petroleum Explorationists, Fourth Annual Conference.

[11] Ahonsi, B. and J. Gdula, 1993, A Case History of By-passed Oil in the Niger Delta, Society of Petroleum Engineers, Nigeria Council, 35 – 46.

[12] Weber, K. J., 1971, Sedimentological Aspects of Oil Fields in the Niger Delta, Goelogie en Mijnbouw Sc., 559 – 576.

[13] Dalley, R. M., E. C. A Gevers, G. M. Stamfli, D. J. Davies, C. N. Gastaldi, P. R. Ruijtenberg and G. I. O. Vermeer, 1989, DAZIL (Dip Azimuth, Illumination), First Break, Vol. 7, No. 3, 86 – 100.

[14] Esho, K. M., 1993, Prediction of Remaining Oil in the Imo River D2.0 Sand Using 3D Seismic Data, Society of Petroleum Engineers, Nigeria Council, 23 – 28.

[15] Weber, K. J., 1986, Hydrocarbon Distribution Patterns in Nigeria Growth Fault Structures Controlled by Structural Style and Stratigraphy, Journal of Petroleum Science and Engineering, Vol. 1, 91 – 104.

[16] Van Wagoner, J. C., R. M. Mitchum, K. M. Campion and V. D. Rahmanian, 1990, Siliciclastic Sequence Stratigraphy in Well Logs, Core and Outcrops: Concept for High-Resolution Correlation of Time and Facies, AAPC Methods in Exploration Series, No. 7.

[17] Athans, M., 1971, The role and use of the stochastic linear quadratic Gaussian problem in control system design, IEEE Trans. Automatic Control AC-16, 529 – 552.

[18] Grimble M. J. (1981), Adaptive Kalman Filter for Control Systems with Unknown Disturbance IEE Proc. Vol. 128, pt. D, No. 6, 263 – 267.