An EVI Model for Daily Rainfall Occurrence at Benin City, Nigeria.

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Abstract

By considering daily maximum rainfall values as random hydrologic variables, the Benin City rainfall occurrence from 1970 to 2004 (35 years) were analyzed in this study using Extreme value Type 1 (EVI) probability distribution in order to predict precipitation of desired return periods (T = 10, 100, 200, 500, 1000, 10,000yrs) which are to be used in watershed modeling to establish hydrologic design values for hydraulic structures in the catchment area.

Our results indicate that EVI probability distribution model predicted rainfall values compares favourably with observed values with a percentage deviation ranging from -13.53% to 9.7% and that for the desired return periods, the predicted point rainfall values are 141.29mm, 193.04mm, 208.12mm, 228.19mm, 243.48mm and 294mm respectively and also that the maximum probable precipitation in the catchment is 526.75mm and therefore suggesting that the EVI probability distribution model is adequate for analyzing rainfall events in the catchment area but the greater the number of available past records, the greater will be the accuracy of prediction.

Keywords: Extreme Value Type1, Probable maximum precipitation, random variable reduced variate, return period.

1.0 Introduction:

Precipitation frequency analysis is a useful tool in providing precipitation inputs in watershed models which provide design hydrographs used to size hydraulic structures such as dams, spillways, levees; channel improvements, storm sewers, detention basins, culverts, bridges and delineating flood plains in support of flood plain management programmes (Wurbs and James, 2009). Watershed models are especially important in developing countries where there are usually insufficient gauged stream flow measurements and where typically there are simply no stream gauges at the location of concern or where recently installed gauging stations may have insufficient length of record for hydrologic application. Even in cases where gauges have been operated for many years, observed flows may not be representative of current watershed conditions due to urbanization and other land use changes or construction of water control facilities. In such situations, precipitation-runoff modeling is advantageous as precipitation gauges are more abundant than stream flow gauges and watershed development does not affect the homogeneity of precipitation data like it does stream flow data hence precipitation runoff modeling facilitates predicting the effects of projected future developments.

Rainfall is the prevalent form of precipitation in most watershed modeling applications and extremes of floods may result from extreme rainfall events and it is rainfall of shorter durations that are closely watched with respect to formation of floods (Shaw, 1988) hence considerable attention is being given to the concept of probable maximum precipitation (PMP) to which no recurrence interval can be attached by water resources engineers. Estimates of extreme upper limit of precipitation runoff are required for design situations in which failures would result in catastrophic consequences like flows overtopping an embankment which could breach the dam and result in severe downstream flooding than if the dam did not exist.

PMP is the depth of precipitation which for a given area and duration can be reached but not exceeded under known meteorological conditions (Wiessner, 1970). It is the greatest rainfall that can occur in a given duration in a given location (Reddy, 2007) and if the probable maximum precipitation for a given catchment is estimated then it can be used to provide an estimate of the probable maximum flood (PMF) after appropriate adjustments for infiltration losses. Probable maximum storm (PMS) is the PMP with approximate temporal distribution and it is PMS that is provided as input to watershed models to develop the PMF hydrograph. When a value of probable maximum precipitation (PMP) is required for a specific project, it is usual to estimate it by two main methods and then make an engineering judgment of the value to be used (Shaw, 1988). One method uses statistical techniques applied to the measurement of extreme rainfall while the second group mainly used by meteorologists, studies the storm mechanisms causing heavy rain falls. As hydrologic phenomena such as stream flows, rainfalls or droughts are characterized by great variability, randomness and uncertainty, they are treated as random variables

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with associated measure of frequency that represent likelihood, percentage of time or probability (Wurbs and James, 2009) and are thus usually investigated by water resources engineers by analyzing their records of observations using probability and statistical methods (Ojha et al, 2008, Reddy, 2007). A number of probability distribution functions are available in the literature and some of them which are found to be useful in describing and analyzing random hydrologic variables include Normal, EVI, Pearson Type III and Log Pearson Type III (Ojha et al, 2008; Wurbs and James (2009).

However, the proper distribution to describe the random variable and estimation of the parameters of the distribution depends on the data of the random variable observed in the past and understanding of the underlying physical phenomenon.

In this study, the extreme value Type I (Gumbel) distribution which may be used to model a variety of phenomena involving extreme events has been applied to Benin City rainfall data for the period 1970 - 2004 (35yrs). The specific objectives of the study include to:

- (i) Carryout precipitation modeling of Benin City catchment using daily rainfall records.
- (ii) Extract extreme value events from available daily rainfall records
- (iii) Fit extreme value type I (EVI) or Gumbel probability distribution to the rainfall data and hence determine probable catchment rainfall events of various return periods (i.e. T = 10, 100, 200, 500, 1000, 10,000)
- (iv) Predict or estimate the PMP for the catchment
- (v) Estimate the confidence intervals for the predicted return period precipitation (rainfall) and
- (vi) Based on (i), (ii), (iii), (iv), (v) above, make appropriate recommendations for application of EVI model to daily rainfall occurrence in Benin City

1.1 The Study Area

The study area is Benin City, the capital of Edo State in Nigeria. It is situated about 117km from where Benin river discharges to the Gulf of Guinea and some 160km due East of Lagos. It is located approximately within Latitudes 5^0 30'N and 5^0 45'N and Longitude $6^0 - 15$ ' E and $6^0 - 30$ ' E. It lies on a gently sloping coastal plain on a drainage divide between the headwaters of the sub catchment system of Ossiomo river and the Ikpoba and Ogba rivers (Rhief Taiwo, 2001). While the Ikpoba river basin drains approximately 520.3km², the Ogba basin drains approximately 340.1km² and both catchment areas are underlain by deeply weathered sedimentary rock often referred to as Benin formation.

The rainfall distribution within the study area is fairly uniform (Aziegbe, 2006) Benin City lies within the rain forest zone of Nigeria with a mean annual rainfall of 1996mm while the average mean monthly temperature varies from 23° C to 27° C. The climate is tropical moist. The tropical rainforest has now been removed by farming and urbanization is the natural vegetation of the watershed (Odemeho, 1992)

2.0 Theory of Extreme Value Type I (EVI) Probability Distribution for Rainfall Analysis

The Extreme Value Type I (EVI) or Gumbel probability distribution is based on the theory of extremes. The theory of extreme values considers the distribution of the largest or smallest observation occurring in each group of repeated samples. The actual rainfall or discharge distribution over a period of years is considered a continuous function because any value is possible at least within a broad range (Prasuhn, 1992) hence based on theory of extremes and treating each year as a sample, Gumbel (1958) applied extreme value Type I (EVI) to flood flows which may be used to model a variety of phenomena involving extreme events(Wurbs and James, 2009). The PDF and CDF of the distribution are given in Ojha et al (2008) as follows:

PDF:
$$f(x) = \frac{1}{\beta} \exp\left(-\frac{x-u}{\beta} - \exp\left\{-\frac{(x-u)}{\beta}\right\}\right)$$
 (2.1)

CDF:
$$F(x) = e^{-\exp\left[\frac{-(x-u)}{\beta}\right]}$$
 (2.2)

where u=location parameter, β =shape parameter And in terms of the reduced variate

$$y = \frac{(x-u)}{\beta} \tag{2.3}$$

PDF:
$$f(x) = e^{-y - (e^{-y})}$$
 (2.4)

CDF:
$$f(x) = e^{-e-y}$$
 (2.5)

The relationship between the parameters and statistical moments of the data are given by the following equations

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$$\beta = \frac{\sqrt{6\sigma}}{\pi}$$
(2.6)
$$u = \mu - 0.45\sigma$$
(2.7)

where μ =mean of the distribution and σ =standard deviation of the distribution.

The population statistics (μ , σ) are replaced by the sample statistics x (sample mean) and S (sample standard deviation). The extreme value distribution may be used to relate either the probability of exceedence or the recurrence interval to the magnitude of a design parameter such as discharge or precipitation.

Since PDF is a continuous mathematical expression that determines the probability of occurrence of a particular event, if R₁, R₂, R₃, ---, Rn are annual extreme values of precipitation in a particular catchment area, from extreme value Type I method of analysis, the exceedence probability of a given precipitation (R_T) having a return period T – yrs being equaled or exceeded is given by Das and Saikia (2009), and Wurb and James (2009) as:

$$P = 1 - e^{-e^{-y}}$$
(2.8)

where y is the reduced variate. e= base of naperian logarithm.

$$Y = \frac{1}{0.78} \left(R_{T} - \overline{R} + 0.45\sigma \right)$$
(2.9)

where R is the magnitude of precipitation of probability P, \overline{R} is the mean precipitation. The probability P is related to the recurrence interval T by the relation (Garg (2008); Prasuhn (1992) :

$$\mathbf{P} = \frac{1}{T} \tag{2.10}$$

From equation (2.9) we have:

$$R_{\rm T} = \overline{R} + (0.78\,y + 0.45)\,\sigma \tag{2.11}$$

which is written as

$$R_{\rm T} = R + K\sigma \tag{2.12}$$

Where K is known as frequency factor

9

10

Equation 2.12 follows the form of the general frequency equation proposed by Chow (1964) as shown below

$$X_{\rm T} = X + K_{\rm T} S \tag{2.13}$$

Where X_T = magnitude of the variable at required return period T, K_T = frequency factor corresponding to X_T , while X and S are as previously defined.

For the extreme value Type 1 distribution, K_T is estimated by the following equation (Ojha et al, 2008):

$$K_{T} = \frac{\sqrt{6}}{\pi} \left\{ 0.5772 + \ln \left(\ln \left(\frac{T}{T - 1} \right) \right\}$$
(2.14)

where K_T is the frequency factor. Although it is dependent on the parameters of the probability distribution, K_T in equation (2.14) is a function of only the return period T and specifically for the EVI distribution; it is given in Shaw (1988) as shown in Table 2.1

| | | - (| | | |
|--------|----------------|--------|----------------|--------|--|
| T(yrs) | K _T | T(yrs) | K _T | T(yrs) | |
| 1 | _∞ | 15 | 1.64 | 100 | |
| 2 | -0.16 | 20 | 1.86 | 200 | |
| 3 | 0.25 | 25 | 2.04 | 400 | |
| 4 | 0.52 | 30 | 2.20 | 600 | |
| 5 | 0.72 | 40 | 2.40 | 800 | |
| 6 | 0.88 | 50 | 2.61 | 1000 | |
| 7 | 1.01 | 60 | 2.73 | | |
| 8 | 1 12 | 70 | 2.88 | | |

80

90

Table 2.1: The T - K_T relationship for EVI (Gumbel) distribution (Shaw, 1988)

Thus if an estimate of the annual maximum precipitation for a return period of 100 years is required, then T(x) = 100 yrs, $K_T = 3.14$ and thus

1 12

1.21

1.30

$$\mathbf{R}_{100} = \overline{\mathbf{R}} + 3.14\mathbf{S}_{\mathbf{R}}$$

With the mean and standard deviation of a sample of annual maximum precipitation and assuming extreme value type I distribution for the data, estimate of peak precipitation for any required return period may be obtained from equation 2.16 as follows:

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2.94

3.07

KT 3.14 3.68 4.08 4.52 4.76 4.94

(2.15)

$$\mathbf{R}_{\mathrm{T}} = \overline{\mathbf{R}} + \mathbf{K}_{\mathrm{T}} \mathbf{S}_{\mathrm{R}} \tag{2.16}$$

Using appropriate K_T values obtained from Table 2.1

 R_T = rainfall depth in the given duration of return period T. Estimate of probable maximum precipitate (PMP) can be obtained from equation 2.17 as follows

$$PMP = \overline{R} + Km\theta \tag{2.17}$$

It is suggested in the literature e.g (Hershfield, 1961) that Km = 15 gives an estimate of PMP

3.0 METHODOLOGY

The daily rainfall data for Benin City meteorological station are available at the Nigerian meteorological service. From the available records/ reports (Aim Consultants, 2006) the daily maximum rainfall series for the station for the period 1970 to 2004 (35 years) were extracted so as to fairly satisfy the assumption of independence and identical distribution by selecting maximum daily rainfall which is the largest rainfall occurring at anytime during the year (Chow et al, 1988) and thereby obtaining annual series data shown in Table 3.1.

For the EVI distribution being investigated, alternative estimates of recurrence interval (T) were obtained by using Weibull and Gringorten plotting position formulae following the recommendations of Ojha et al (2008) summarized in Table 3.2.

To apply the Weibull and Gringorten plotting position formulae, the observed data were arranged or ranked in descending order of magnitude and computation for return period made according to the formular shown in Table 3.2. The reduced variate corresponding to each return period was computed using equation (3.1).

Table 3.1: Maximum Daily Rainfall for Benin City*

| Year | Year Rainfall | | Rainfall (mm) |
|------|---------------|------|---------------|
| | (mm) | | |
| 1970 | 149.3 | 1988 | 88.50 |
| 1971 | 92.20 | 1989 | 133.40 |
| 1972 | 160.80 | 1990 | 114.40 |
| 1973 | 87.10 | 1991 | 110.40 |
| 1974 | 129.30 | 1992 | 120.8 |
| 1975 | 94.70 | 1993 | 79.90 |
| 1976 | 134.40 | 1994 | 156.10 |
| 1977 | 76.50 | 1995 | 91.90 |
| 1978 | 99.90 | 1996 | 105.80 |
| 1979 | 110.70 | 1997 | 122.60 |
| 1980 | 83.30 | 1998 | 142.2 |
| 1981 | 82.00 | 1999 | 103.00 |
| 1982 | 164.30 | 2000 | 97.40 |
| 1983 | 88.70 | 2001 | 76.90 |
| 1984 | 54.70 | 2002 | 95.80 |
| 1985 | 61.60 | 2003 | 102.5 |
| 1986 | 59.60 | 2004 | 97.3 |
| 1987 | 93.60 | | |
| | | | |

*Source: Aim Consultants (2006)

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 Table 3.2: Commonly used unbiased plotting position formulae (Ojha et al, 2008)

| Distribution | Recommended plotting position formula | Form of the position (T) |
|---|---------------------------------------|---|
| Normal/Log Normal | Blom | $\frac{\mathrm{N} + 0.25}{\mathrm{m} - 0.35}$ |
| Gumbel (EVI) | Gringorten | $\frac{N + 0.12}{m - 0.4}$ |
| Any Distribution | Weibull | $\frac{N+1}{m}$ |
| Pearson Type III or Log Pearson Type III | Cunnane | $\frac{N + 0.2}{m - 0.4}$ |

Where m is the rank number obtained by arranging the annual maximum series in descending order of magnitude with the highest value being 1 and N is the number of years of record of data.

$$Y_{T} = -\ln\left(\ln\left(\frac{T}{T-1}\right)\right)$$
(3.1)

A plot of the precipitation magnitude against reduced variate on ordinary graph paper was made to check whether a linear relationship exist in order to determine the appropriateness of using extreme value Type I distribution fit for the precipitation data. This is shown in Figure 4.1

From the assembled annual series data in Table 3.1, the mean (\overline{X}) and standard deviation (S) of the precipitation series were calculated using the following equations:

$$\overline{\mathbf{X}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{X}_{i}$$
(3.2)

and

$$S = \left[\frac{1}{(n-1)}\sum_{i=1}^{n} (X_{i} - X)^{2}\right]^{0.5}$$
(3.3)

in order to provide estimates for the population parameters (mean and standard deviation), i.e. μ and σ respectively.

The EVI probability distribution shape and location parameters β and μ were computed using equation (2.6) and (2.7) respectively

For the selected return periods (T = 10,100,200,500,1000,10,000 yrs), the EVI reduced variates (y_T) were computed using the relation given by equation (3.1) Estimates of T-year recurrence interval precipitation using EVI distribution for the selected return periods were obtained using:

$$\mathbf{R}_{\mathrm{T}} = \mathbf{u} + \beta \mathbf{y}_{\mathrm{T}} \tag{3.4}$$

Confidence intervals for the predicted return period precipitation were estimated using the following steps suggested in Ojha et al (2008);

(i) Compute standard error (SE) for the EVI distribution using:

$$SE = \frac{\alpha}{\sqrt{n}} \left[1 + 1.396K_{\rm T} + 1.1(K_{\rm T})^2 \right]^{0.5}$$
(3.5)

(ii) The precipitation values for a particular confidence limits was computed using :

$$\mathbf{R}_{\rm conf} = \mathbf{R}_{\rm T} \pm \mathbf{f}_{\rm c} \mathbf{S}_{\rm E}$$

where f_c is the function of the confidence probability given in Table 3.4

Table 3.4: Determination of f_c for Particular Confidence Interval (Ojha, et al, 2008)

| C(%) | 50% | 68% | 80% | 90% | 95% | 99% |
|------|-------|-----|-------|-------|------|------|
| f(c) | 0.674 | 1.0 | 1.282 | 1.645 | 1.96 | 2.58 |

Estimate of PMP was obtained by applying equation (2.17)

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(3.6)

4.0 PRESENTATION, ANALYSIS AND DISCUSSION OF RESULTS

Estimates of return periods obtained by application of the Weibull and Gringorten plotting position formulae to the observed data and their corresponding reduced variates (\mathbf{Y}_T) are shown in Table 4.1

Table 4.1: Return periods computations using Weibull & Gringorten formulae and their corresponding reduced variates

| Rank | Rainfall | Weibull | | Gringorten | | |
|------|---------------|------------------|----------------|----------------|----------------|--|
| | (mm) | | | | | |
| | | Tr. (Weibull) | Y _T | T _r | Y _T | |
| 1 | 164.3 | 36 | 3.67 | 62.71 | 4.13 | |
| 2 | 160.80 | 18 | 2.86 | 22.5 | 3.09 | |
| 3 | 156.10 | 12 | 2.45 | 13.7 | 2.58 | |
| 4 | 149.30 | 9 | 2.14 | 9.86 | 2.24 | |
| 5 | 142.20 | 7.2 | 1.90 | 7.70 | 1.97 | |
| 6 | 134.40 | 6 | 1.701 | 6.32 | 1.76 | |
| 7 | 133.40 | 5.142 | 1.53 | 5.35 | 1.58 | |
| 8 | 129.30 | 4.5 | 1.38 | 4.77 | 1.45 | |
| 9 | 122.60 | 4 | 1.25 | 4.1 | 1.27 | |
| 10 | 120.80 | 3.6 | 1.12 | 3.67 | 1.15 | |
| 11 | 114.40 | 3.27 | 1.00 | 3.33 | 1.03 | |
| 12 | 110.70 | 3 | 0.90 | 3.03 | 0.92 | |
| 13 | 110.40 | 2.76 | 0.80 | 2.79 | 0.81 | |
| 14 | 105.80 | 2.57 | 0.71 | 2.59 | 0.72 | |
| 15 | 103.00 | 2.40 | 0.62 | 2.41 | 0.62 | |
| 16 | 102.50 | 2.25 | 0.53 | 2.25 | 0.53 | |
| 17 | 99.90 | 2.11 | 0.44 | 2.12 | 0.45 | |
| 18 | 97.40 | 2.0 | 0.37 | 2.0 | 0.37 | |
| 19 | 97.30 | 1.90 | 0.29 | 1.89 | 0.28 | |
| 20 | 95.80 | 1.80 | 0.21 | 1.79 | 0.20 | |
| 21 | 97.70 | 1.71 | 0.13 | 1.70 | 0.12 | |
| 22 | 93.60 | 1.64 | 0.06 | 1.63 | 0.05 | |
| 23 | 92.20 | 1.56 | -0.024 | 1.56 | -0.022 | |
| 24 | 91.90 | 1.50 | -1.09 | 1.49 | -0.106 | |

| 25 | 88.70 | 1.44 | -0.170 | 1.43 | -0.184 |
|----|-------|------|----------|------|--------|
| 26 | 88.50 | 1.38 | - 0.2540 | 1.37 | -0.27 |
| 27 | 87.10 | 1 33 | 0.2340 | 1 32 | -0.35 |
| 27 | 07.10 | 1.55 | 0.3320 | 1.52 | 0.55 |
| 28 | 83.30 | 1.29 | - | 1.27 | -0.44 |
| | | | 0.4004 | | |
| 29 | 82.00 | 1.24 | -0.496 | 1.23 | -0.52 |
| 30 | 79.90 | 1.20 | -0.583 | 1.18 | -0.63 |
| 31 | 76.90 | 1.16 | - | 1.15 | -0.71 |
| | | | 0.6836 | | |
| 32 | 76.50 | 1.13 | -0.771 | 1.12 | -0.80 |
| 33 | 61.60 | 1.09 | 9139 | 1.07 | -1.00 |
| 34 | 59.60 | 1.05 | -1.13 | 1.05 | -1.10 |
| 35 | 54.70 | 1.03 | -1.263 | 1.02 | -1.37 |

This was achieved by programming the formulae on spread sheet in MS EXCEL environment. It can be seen from Table 4.1 that the return periods and EVI reduced variates obtained by applying the two plotting position formulae to observed data are comparable and this is expected as suggested in Ojha et al (2008). From the plots shown in Figure 4.1, it can be seen that a linear relationship exists between the observed data and their corresponding reduced variates with both Gingorten and Weibull plotting position formulae suggesting that the EVI probability distribution is a satisfactory and appropriate fit for the observed data and hence it can adequately make predictions for the selected return periods. However, as seen in Figure 4.1, the Gringorten formula gave a higher coefficient of correlation value of 96.8% as against 95.25% value obtained with Weibull formula suggesting that Gringorten formula is a better plotting position formula for the data and hence was the one used for further analysis.



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The maximum daily rainfall data and their corresponding computed return periods using Gringorten plotting position equation, the KT values for EVI distribution obtained by using equation (2.14), the predicted rainfall depths and the percentage deviation of predicted rainfall using EVI analysis from observed maximum daily rainfall are presented in Table 4.2.

Table 4.2: Comparison of Predicted rainfall values with Observed values

| Ran | Max.Rainfal | T(Gringorten | KT | Predicted | % |
|-----|-------------|--------------|-----------|---------------|----------|
| k | l depth(mm) |) | (Computed | Rainfall | Deviatio |
| | | (yrs) |) | depth(mm) | n |
| 1 | 164.3 | 62.71 | 2.770 | 182.57 | -1.1 |
| 2 | 160.8 | 22.5 | 1.960 | 159.72 | 0.67 |
| 3 | 156.1 | 13.7 | 1.560 | 148.52 | 4.85 |
| 4 | 149.3 | 9.86 | 1.290 | 140.92 | 5.6 |
| 5 | 149.2 | 7.7 | 1.088 | 135.24 | 9.35 |
| 6 | 134.4 | 6.32 | 0.9201 | 130.51 | 2.89 |
| 7 | 133.4 | 5.35 | 0.7781 | 126.51 | 5.16 |
| 8 | 129.3 | 4.77 | 0.6781 | 123.70 | 4.33 |
| 9 | 122.6 | 4.1 | 0.5438 | 119.92 | 2.18 |
| 10 | 120.8 | 3.67 | 0.4431 | 117.08 | 9.7 |
| 11 | 114.4 | 3.33 | 0.3531 | 114.55 | -0.13 |
| 12 | 110.7 | 3.03 | 0.2641 | 112.05 | -1.22 |
| 13 | 110.4 | 2.79 | 0.184 | 109.80 | 0.54 |
| 14 | 105.8 | 2.59 | 0.1091 | 107.69 | -1.74 |
| 15 | 103 | 2.41 | 0.0361 | 105.64 | -2.56 |
| 16 | 102.5 | 2.25 | -0.0349 | 103.63 | -1.1 |
| 17 | 99.9 | 2.12 | -0.0994 | 104.62 | -4.72 |
| 18 | 97.4 | 2 | -0.1641 | 100 | -2.67 |
| 19 | 97.3 | 1.89 | -0.2289 | 98.18 | -0.9 |
| 20 | 95.8 | 1.79 | -0.2929 | 96.37 | -0.6 |
| 21 | 94.7 | 1.7 | -0.3569 | 94.58 | 0.13 |
| 22 | 93.6 | 1.63 | -0.4109 | 93.05 | 0.58 |
| 23 | 92.2 | 1.56 | -0.4687 | 91.43 | 0.84 |
| 24 | 91.9 | 1.49 | -0.5327 | 89.48 | 2.63 |
| 25 | 88.7 | 1.43 | -0.5931 | 87.93 | 0.87 |

| 26 | 88.5 | 1.37 | -0.6598 | 86.05 | 2.77 |
|----|------|------|---------|-------|--------|
| 27 | 87.1 | 1.32 | -0.7217 | 84.31 | 3.2 |
| 28 | 83.3 | 1.27 | -0.7909 | 82.36 | 1.12 |
| 29 | 82 | 1.23 | -0.8529 | 80.62 | 1.68 |
| 30 | 79.9 | 1.18 | -0.9423 | 78.10 | 2.37 |
| 31 | 76.9 | 1.15 | -1.005 | 76.34 | 0.73 |
| 32 | 76.5 | 1.12 | -1.0731 | 74.42 | -2.72 |
| 33 | 61.6 | 1.07 | -1.232 | 69.94 | -13.53 |
| 34 | 59.6 | 1.05 | -1.318 | 67.53 | -9.94 |
| 35 | 54.7 | 1.02 | -1.518 | 61.90 | -13.16 |

From Table 4.2, it is seen that the predicted rainfall depths compares favourably with observed rainfall values and that the percentage deviation ranges from -13.53 % to 9.7%. The plots of observed and predicted rainfall values against the reduced variates are shown in Figure 4.2



For return periods of 10 years, 100 yrs, 200 yrs, 500yrs, 1000 and 10,000yrs whose predicted precipitation are to be obtained, the EVI reduced variates (y_T) obtained by application of equation (3.1) is presented in Table 4.3.

| | p # | | | |
|---|------------|-----------------|---------------------------------|--------------------|
| Т | (T - 1) | $\frac{T}{T-1}$ | $\ln\left(\frac{T}{T-1}\right)$ | $y_T = - \ln (\ln$ |

| | | | | $\left(T_{T-1}\right)$ |
|--------|------|--------|----------|------------------------|
| 10 | 9 | 1.1111 | 0.1053 | 2.25 |
| 100 | 99 | 1.010 | 9.9503 | 4.610 |
| 200 | 199 | 1.0050 | 5.0125 | 5.295 |
| 500 | 499 | 1.002 | 1.998 | 6.21 |
| 1000 | 999 | 1.0010 | 0.000999 | 6.907 |
| 10,000 | 9999 | 1.0001 | -9.999 | 9.21 |

The mean (X) and standard deviation (S) of the observed rainfall data shown in Table 3.1 are 104.62mm and 28.142mm respectively.

Using equations (2.6) and (2.7) the relationship between the parameters and statistical moments of the observed data were obtained as

 $\beta = 21.939$ and $\mu = 91.956$

From equation (3.4) that is, $R_T = \mu + \beta y_T$ and applying appropriate y_T as given in Table 4.3, the predicted rainfall depths at the desired return periods are obtained and results are presented in Table 4.4.

| RT (1) | μ (2) | у _т (3) | β _{yT} (4) | $R_{T} = \frac{\mu + \beta yT}{RT}$ $RT = (2) + (4)$ |
|--|---|---|---|--|
| $\begin{array}{c} R_{10} \\ R_{100} \\ R_{200} \\ R_{500} \\ R_{1000} \\ R_{10,000} \end{array}$ | 91.95 91.95 91.95 91.95 91.95 91.95 91.95 | 2.25 4.610 5.295 6.21 6.907 9.21 | 49.34 101.09 116.167 136.24 151.53 202.058 | 141.29 193.04 208.12 228.19 243.48 294.00 |

Table 4.4: Predicted rainfall depths at desired return periods.

From Table 4.4, the predicted precipitation for the selected return periods of 10yrs, 100yrs, 200yrs, 500yrs, 1000yrs and 10,000yrs are 141.29mm, 193.04mm, 208.12mm, 243.48mm and 294mm respectively.

The PMP for the catchment was obtained by using equation (2.17) and adopting a km value of 15 as suggested by Hershfield (1961). The PMP value was obtained to be 526.75mm as shown below.

 $PMP = \overline{R} + K_m \sigma, \text{ for } \sigma = 28.142, \overline{R} = 104.62, Km = 15$ $= 104.62 + 15 \times 28.142$ = 104.62 + 422.13= 526.75mm.

The standard error (S_E) for the EVI distribution for the selected return periods was estimated using equation 3.5. The results are presented in Table 4.5.

| Table 4.5: | Computation | of Standard Error | r (S _E) |
|------------|-------------|-------------------|---------------------|
|------------|-------------|-------------------|---------------------|

| Т | 10 | 100 | 200 | 500 | 1000 | 10,000 |
|----------------|-------|-------|------|------|-------|--------|
| K _T | 1.3 | 3.14 | 3.68 | 4.31 | 4.94 | 6.59 |
| S _E | 10.28 | 19.18 | 21.8 | 24.9 | 28.03 | 36.22 |

Calculation for the 95% confidence limits for the predicted data set out in Table 4.3 are shown in Table 4.6 **Table 4.6: Computation of 95% Confidence Limits (EVI distribution),**

 $f_{c} = 1.96$

| 10 - 11) | 0 | | | | |
|----------|----|-------|----------------|---------|----------------------|
| T(yrs) | SE | Fc SE | R _T | RT+fcSE | R _T -fcSE |
| | | = | | = Upper | = |

| | | 1.96SE | | R _T (mm) | Lower RT(mm) |
|--------|-------|--------|--------|---------------------|-----------------|
| | | | | | |
| 10 | 10.28 | 20.05 | 141.29 | 161.34 | 121.24 |
| 100 | 19.18 | 37.59 | 193.04 | 230.63 | 155.45 |
| 200 | 21.80 | 42.73 | 208.12 | 250.85 | 165.38 |
| 500 | 24.9 | 48.80 | 228.19 | 276.99 | 179.39 |
| 1000 | 28.03 | 54.93 | 243.48 | 298.41 | 188.55 |
| 10,000 | 36.22 | 70.99 | 294.00 | 364.99 | 223.01 |

Also confidence limits (95%) about the fitted straight line relationship between the predicted annual maxima and reduced variate for the EVI probability fit is constructed in the figure for the range of the selected T values.



EVI distribution may be used to model a variety of extreme events. It is used in Europe to model flood flows and has been applied by the US weather service (WWS) in analyzing precipitation

5.0 CONCLUSIONS

From the extreme value type 1 probability distribution fit to the Benin City rainfall data the following conclusions are made

- (i) From the daily rainfall records the extreme value rainfall events have been extracted as given in Table 3.1
 (ii) Precipitation modeling of the Benin City catchment area has been undertaken using the Extreme value Type 1
- (ii) Probability distribution by considering maximum annual rainfall as extreme events.
 (iii) Dr. films EVU and chility distribution to the chemical manual rainfall data the extreme node. Tame 1 (Combal)
- (iii) By fitting EVI probability distribution to the observed rainfall data, the extreme value Type 1 (Gumbel) probability distribution has been used to model the Benin City rainfall data and for the desired return periods of T = 10, 100, 200, 500, 1000 and 10,000 years, the predicted point rainfalls are 141.29mm, 193.04mm, 208.12mm, 228.19mm, 243.48mm and 294mm respectively
- (iv) The probable maximum precipitation(PMP) is estimated as 526.75mm
- (v) The 95% confidence limits for the predicted precipitation values at desired return periods using EVI probability distribution is provided in Table 4.6 and amplified in Figure 4.3.

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(vi) Based on the findings of this study, it is concluded that EVI model is satisfactory in predicting rainfall events in the catchment area. But for longer return periods say, T=1000yrs and 10,000 years, caution should be exercised in adopting the predicted values obtained as the available past records are for only 35 years and the greater the number of available past records, the greater will be the accuracy of predictions.

It should be noted that, though probability and statistical techniques are helpful in the analysis and interpretation of hydrologic data and in the prediction or establishment of design values, engineering judgment is needed always in the application of such techniques as they are not yet an exact science.

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