Journal of the Nigerian Association of Mathematical Physics
***Volume* 18 (May, 2011), pp 465 – 470**
© J. of NAMP

A Phase Plane Realization of a Class of Nonlinear Liquid Level Control Systems

E. J. Ogbebor and E. C. Obinabo

Department of Electrical and Electronic Engineering,
Ambrose Alli University, P. M. B, 14, Ekpoma, Edo State, Nigeria

Abstract

A realization of the state model of a liquid level control system is presented for establishing the structure of a class of nonlinear systems with feedback loops which can be employed for liquid level control in petroleum storage tanks. The results presented here are intended for application, among others, in determining optimum operating conditions of storage tanks of various types used in industry, and will thus open up a new and challenging area of study especially in selection and tuning of universal controllers for achieving the desired results. typical examples were considered to indicate the corresponding application.

1. Introduction:

The need for control of liquid level in chemical engineering systems to improve quality and to optimize production efficiency is becoming more pronounced. To deal with this problem, adaptive control algorithms have been developed [1 – 3] most of which use on-line parameter estimation techniques. Several methods have also been proposed to improve the numerical robustness of the estimator. These methods require a priori information concerning the process to be controlled to enable choice of specific parameters such as the response time, transportation lag and stability. Of all the variables that are of interest in measurement and control, liquid-level appears to be the most straightforward. It is the variable most easily measured directly, and it is unique in its simplicity of dimension. Whereas pressure is force- per-unit-area, flow is volume per-unit-time, and temperature is the measure of the activity of molecules, liquid-level is merely a measure of length. As a result of its simple character, liquid-level lends itself readily to numerous means of inferential measurement. This is particularly true in the petroleum industry, where the determination of liquid-level in many cases has a direct bearing on the amount of money that changes hands between customer and the producers. In such instance the accuracy of liquid-level measurement and control is required to be of highest practicable quality. In liquid level control devices, the forces needed to operate the control valves is provided by ON-OFF switching operation of the form associated by relay dynamics. In relay control applications, the magnitude of the corrective action is independent of the error, but the sign of this constant corrective action is directly dependent on the sign of the error signal.

The liquid level control devices considered in this study employs a thermostat to supply 24 volts to the coli of a relay, which switches 220 Volts to open or close the inlet valve of the liquid storage tank. The power is switched on when the liquid level in the storage tank is two low and off when the liquid level exceeds the desired value. Inherent within all relay elements is a certain amount of dead-band (or dead zone), and this is used to ensure that as the error magnitude is less than some defined value there is no corrective action. In electrical relays this dead-band arises because the coils require a finite amount of current to activate the relay contacts. In mechanical (i.e. hydraulic) systems, value overlap may be present to reduce fluid leakage of porting (i.e. fluid inlet/shut-off), and this creates a dead-band [4, 5]. Dead–band (or dead zones or threshold) is a kind of nonlinearity which exists in some control systems. It is typical in a relay of an ON-OFF servomechanism. In this system, there must be some lost motion of the armature in moving from one contact to the other. The extent of the movement over which neither contact is reached gives rise to a dead zone (or dead band) over which the output is zero. Graphically, the dead zone effect is illustrated in fig. 1.

This nonlinearity is single–valued, i.e., there is only one output value. There is yet a backlash nonlinearity effect which results in the occurrence of the phenomenon of hysteretic. When two values of output can occur for the same input, when the input is rising and the other when the input is falling, the phenomenon of hysteretic is said to occur. This class of nonlinearity is associated with inductive electrical circuits and is commonly met in mechanical system as a result of backlash in gears and linkages. The presence of dead-band may cause the system to exhibit self-sustained oscillations of

Corresponding authors: ***E. C. Obinabo***: E-mail: drobinaboec@yahoo.com; Tel.+2347034481461

constant amplitude and frequency, referred to as limit cycle [6 – 8]. Control over the size of the dead band to prevent limit cycles can only be exercised if the magnitudes of the signals within a system are known. A change in the set point of a system could, for example, be responsible for the onset of a limit cycle oscillation.

Analytical methods do exist which enable the plant operator to predict the dead band widths for given signal magnitudes which would cause limit cycle conditions. The limit cycle phenomenon [8] can be used to advantage in certain industrial situations to over come problems of value stiction which may otherwise lead to malfunction and component failure. When mechanical surface are operated in sliding contact, there exist at least three types of retarding force called friction [9, 10]. These are (a) coulomb friction which is a constant opposing force (independent of velocity), (b) stiction which is the force required to initiate relative motion when the surfaces are at rest. In general, surface at rest appear to stick and the force required to initiate motion is greater than the force required to maintain motion. (c) There is viscous friction which often predominates in control problems. This is force proportional to the relative velocity between the surfaces. Mathematically, the viscous friction force is proportional to the velocity of the motion and is expressed as $f\frac{dx}{dt}$, where f is the viscous frictional constant in Newton per meter/sec. The frictional force is represented graphically in fig. 2.

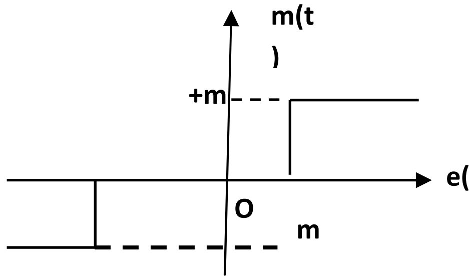

Fig. 1

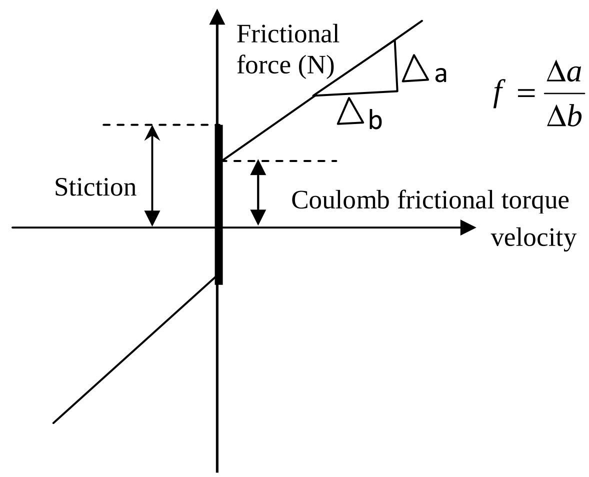

Fig. 2

In rotational system, these frictional forces (namely coulomb, stiction and viscous) give rise to frictional torques. Thus we have three basic types of frictional torques namely, (a) Coulombs frictional torques, (b) Stiction torque, and (c) Viscous frictional torque. These torques are represented graphically in fig. 3. Δa and Δb are the respective incremental increases in the frictional forces (and torques) and velocity.

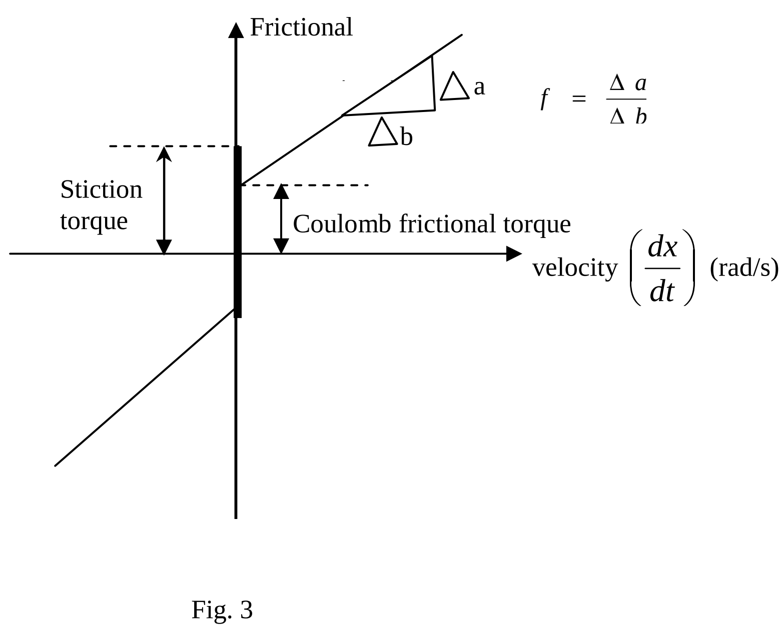

Fig. 3

The viscous component of the frictional torque often predominates in control system, and this may be described mathematically as Frictional torque = $f\,\frac{d\theta}{dt}$, where f is the viscous frictional torque constant having units of *Nm* per rad/s.
In the operation of the pilot valve air or other suitable gas under pressure is applied. Adjustable cam is attached to the rotary shaft and is therefore driven by the motion of the float element. A roller of the valve assembly rides against the cam and is linked to a pedal that actuates the pilot stem. Downward movement of the pilot stem acting through the steel closes exhaust port and opens inlet, port, allowing supply air to flow into the surrounding cavity and out to the control valve diaphragm. As the
pressure on the diaphragm and in the lower cavity of the pilot valve increases, bellows whose exteriors are under the same pressure because of the equalizing port, will be compressed against the spring. Eventually the spring will be compressed until, at equilibrium, the inlet valve is close, and pressure on the diaphragm actuator is maintained at a fixed valve. Should the cam move on the opposite direction the exhaust port will open, allowing pressure on the actuator to bleed off until equilibrium is again established.

A class of pilot-operated liquid-level controllers is capable of giving excellent control and can be made to withstand considerable pressure [11 – 13]. Friction in the rotary shaft stuffing box can be source of error, but with modern Teflon packing and the use of thinner shafts, made possible by the decreased load, the error is apt to be insignificant. One other source of error is that arising from a possible change in specific gravity of the liquid under control. Obviously, a decrease in specific gravity of the liquid will allow the float to sink deeper, thus producing an error [14]. An increase in specific gravity will cause a change in the level indicator. This in turn will cause the density and specific gravity to change with temperature. This change for many liquids is significant and therefore must be taken into account. Perhaps more significant than changes due to temperature variations is the fact that the density of the fluid will vary with its quality. This is particularly significant in the case of petroleum and its products.

The study establishes the appropriate design data for level control in gasoline storage tanks characterized by nonlinear effects.

THE DESIGN OF THE LIQUID LEVEL CONTROL SYSTEM

The liquid level control system was modelled in this study by a spring-mass-damper interconnection shown schematically in fig. 4.

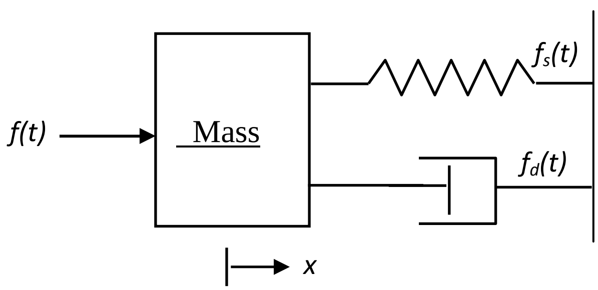

Fig. 4: A Model of Liquid Level Control System.

The equation of motion may be written as follows:

$$m\frac{d^2x(t)}{dt^2} + f\,\frac{dx(t)}{dt} + Kx(t) = 0 \quad (1)$$

which may be expressed as

$$m\frac{d}{dt}\left(\frac{dx}{dt}\right) + f\,\frac{d}{dt} + Kx = 0 \quad (2)$$

where $\frac{dx}{dt}$ is velocity and x position. Now if we write

$$\frac{dx}{dt} = velocity = y \quad (3)$$

then the equation of motion becomes

$$m\frac{dy}{dt}+fy+Kx=0 \quad (4)$$

Also $$\frac{dy}{dt}=\frac{dy}{dx}.\frac{dx}{dt}=\frac{dy}{dx}.y \quad (5)$$

Finally, this may be written as:

$$\frac{dy}{dx}=-\left(\frac{Kx+fy}{my}\right) \quad (6)$$

This equation contains only two variables namely, *x* and *y*, and may be investigated in an *x-y* or *phase-plane*. To sketch the phase-plane diagram for *v*, versus, *y*

$$\ddot{y}+2\dot{y}+4y=0$$

This is written as

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}4y=0 \quad (7)$$

Let $$v=\frac{dy}{dt} \quad (8)$$

then $$\frac{dv}{dt}=\frac{d}{dt}\left(\frac{dy}{dt}\right)=\frac{d^2y}{dt^2}=v\,\frac{dv}{dy} \quad (9)$$

Equation (1) then becomes

$$v\,\frac{dv}{dy}+2v+4y=0 \quad (10)$$

$$\frac{dv}{dy}=-\left(\frac{2v+4y}{v}\right) \quad (11)$$

Since the variables are not separable, method of Isoclines is used in drawing the trajectory

Let $\frac{dv}{dy}=K$, say, so that (11) becomes

Kv = – (2v + 4y), giving Kv + 2v = –4y, so that v = $-\frac{4y}{k+2}$

ie $$v=-\left(\frac{4}{k+2}\right)y \quad (12)$$

Equation (12) is a straight line relationship in *v* and *y*; it has the slope in = $-\left(\frac{4}{k+2}\right)$

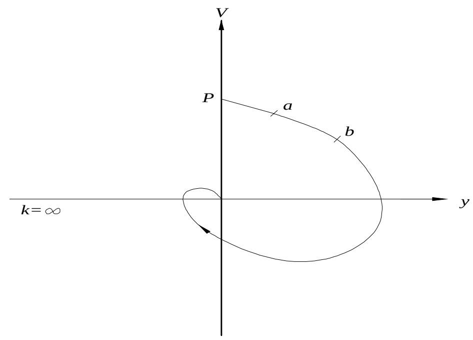

Fig.5

$$\text{Slope } m = -\left(\frac{4}{K+2}\right)$$

For various values of *K* we found the slope *m*, and drew in lines with these slopes. In order to plot the trajectory, we started form any convenient point, say, at a point on *K = –2*, say *p*. Now consider the mean of *K = –2* and *K = –3*

$$\text{ie} \quad \frac{-2-3}{2} = -2.5$$

Now we drew a line *Pa* form *K = –3* with slope –2.5 as shown above

For *K = –3* and *K = –4*, slope = $\dfrac{-3-4}{2} = -3.5$

Also we drew a line from *a* to *b* with slope *–3.5*, and so and for convenience the slope on two vertically opposite quadrants were drawn roughly, and the trajectory traced carefully in the end.

Results and Discussion

Often level control depends on numerous parameters, which must be constant or allowed to vary in very tight intervals. The solution of installing control loops independent of each other, one for each variable, is sometimes impossible and, in any case, does not ensure satisfactory results. This is due to that fact that the variables are often physically linked to each other. Moreover there are some variables which cannot be controlled and thus cause disturbance to the control loops. For these reasons it is necessary to establish the physical principles on which the process is based. This will enhance selection of adequate control methods.

The phase plane analysis is only successfully applicable to systems the linear part of which can be described by a second order differential equation. The method is yet inapplicable to system with time-varying parameters. The form of the input signal *r(t)* is limited to initial conditions only. This in fact allows steps or ramp inputs which can be incorporated as constant, but sinusoidal or random input is not allowable. The Phase Plane Plot is a plot of $\frac{dx}{dt}$ vertically against *x*, where *x* is a function of time (*t*) and is the system output or the error. The general equation which can be studied under the above rules is the equation of an unexpected second order system:

$$a\ddot{x} + b\dot{x} + cx = K \quad \text{(a constant)} \tag{13}$$

where *a, b,* and *c* need not be constant coefficient but can be dependent upon the magnitude of x, or the first derivative of magnitude $\dot{x}$, thus covering a wide range of non-linear system.

Conclusions

A representation for nonlinear multivariable systems similar to that of (1) has been used by Cook [5], but his analysis was rather limited in scope and his viewpoint was quite different from that indicated in this paper. The representation (1) and the approach adopted in this study are consistent with the existing literature [2, 7, 8] involving an interconnection of sub units. A considerable effort has been devoted in this study to the development of conditions which guarantees stability of the nonlinear liquid level control systems. Our efforts are also verifiable for application to frequency response analysis nonlinearities in the interconnection of the sub units. However, these results have not yet proved completely satisfactory in practice since the associated conditions are exceedingly difficult. Nevertheless, recent efforts have been made to derive simpler conditions, which should be easier to verify. In particular, we have required that each linear subsystem in our approach satisfy a certain scalar conditions; such conditions are easily verifiable and serve as the main point of difference between the results of this paper and those obtained previously. In addition, our conditions indicate rather explicit relationships between the variables, which guarantee stability. Consequently, our results should be suitable as a basis for designing nonlinear multivariable systems which are stable. We have been able to obtain conditions for stability in this form as a consequence of the structure of the representation (1).

References

[1] M. Araki, Input-output stability of composite feedback systems. No. 75/1. Department of Computing and Control, Imperial College, London.

[2] J.D. Blight, Scalar stability criteria for nonlinear multivariable feedbacks. University of Michigan, Ph.D. Thesis, 1973.

[3] J.A. Blight and N.H. McClamroch, Graphical stability criteria for large scale nonlinear multiloop systems. 1975 IFAC Congress, Boston.

[4] P.A. Cook, Nyquist-plot methods of investigating multivariable feedback systems. IFAC Symposium on Multivariable Technological Systems, Manchester, 1974.

[5] P.A. Cook, On the stability of interconnected systems. Int. J. Control 20, 407-415 (1974).

[6] C.A. Desoer and M. Vidyasagar, Feedback Systems: Input-output Properties. Academic Press, New York (1975).

[7] N.H. McClamroch, A representation for multivariable feedback systems and its use in stability analysis. Part 1: linear systems. Department of Engineering, Cambridge University (1975).

[8] N.H. McClamoch and G.D. Lancelescu, Global stability of two linearly interconnected nonlinear systems. To appear in IEEE Trans. AC.

[9] K.S. Marendra and J.H. Taylor, Frequency Domain Criteria for Absolute Stability. Academic Press, New York (1973).

[10] S. Partovi and N.E. Nahi, Absolute stability of dynamic systems containing nonlinear functions of several state variables. Automatic 5, 465-473 (1969).

[11] V.M. Popov, Hyper stability of Control Systems. Springer-Verlag, New York (1973).

[12] D.W. Porter and A.N. Michel, Input-output stability of time varying nonlinear multiloop feedback systems. IEEE Trans. AC-19, 422-427 (1974).

[13] H.H. Rosenbrock, Multivariable circle theorems. Recent Mathematical Developments in Control, ed. D.J. Bell, Academic Press, New York, 345-365 (1973).

[14] D. Siljak, Stability of large-scale systems under structural perturbations. IEEE Trans. SMC-2, 657-663 (1972).