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Clark and Reza-Latif-Shabgahi Algorithm for the Determination of the Minimal Cut-Sets of the Fault Tree of Pipeline Failure in the Niger Delta Region of Nigeria.

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Abstract

We apply the straight forward algorithm developed by Clark and Reza-Latif-Shabgahi bottom-up method in determining the minimal cut-sets of the modified fault tree of pipeline failure in the Niger Delta region of Nigeria occasioned by third party activity. It employs the bottom –up technique, producing a table containing the cut – sets for each event in the tree. In all, fifty-two basic events interconnected by six AND logic gates and twenty-one OR logic gates, were analyzed resulting in thirty minimal First-order cut sets, twelve minimal Third-order cut sets and sixteen Fourth-order minimal cut sets. Of importance is the twenty-eight minimal First cut sets since higher order cut sets have lower possibility of occurrences.

Keywords: Fault tree, event, codes, Minimal cut-set, Binary code.

1. Introduction:

Fault tree analysis describes an analytical technique whereby an undesired state of the system is specified and the system is then analyzed in the context of its environment and operation to find all realistic ways in which the undesired event can occur [8]. A typical fault tree consists of the top event, the basic events, and the logic gates. The top event represents an undesirable state of the system, the basic events represent the state of the systems components, and the logic gates describe the relationship between the basic events and the top event. In classic fault tree analysis the AND logic gate denotes that the output is in a failure state, if all the inputs are in failure state. The OR logic gate denotes that the output is in failure state, if at least one of the inputs is in failure state. An intermediate event represents an intermediate state of the system that is related directly or indirectly to the top event with a logic gate.

Generally, a fault tree has several cut sets prime implants. A *minimal cut set* (MCS) is the smallest combination of basic events which if they all occur will cause the top event to occur. Minimal cut-set's also represents the smallest combination of basic events whose failures are necessary and sufficient to cause the occurrence of the top event. If any event is removed from the set, the remaining events collectively are no longer a cut set.

2.0 Study Methodology

Yahua et al in 2005 [9] fault tree diagram for transmission pipeline failures is enlarged to include twelve identified third party activities contributory to leak, puncture or rupture of transmission pipeline in the Niger Delta region of Nigeria. [2]. The first part of fault tree [8],
(Figure 1) is quantitatively analyze to determine the minimal cut-sets or prime implants, which is a list of minimal combinations of events leading to the occurrence of the top event. [1,3]. Earlier fault tree techniques and methods are highlighted in reference [6] while computer-aided construction technique for fast and accurate generation of fault tree is highlighted in reference [7]. Indirect methods evaluate a fault tree by quantification of its minimal cut sets requiring approximation and truncation techniques to restrict the number of minimal cut-sets that are generated. Direct evaluation methods however, solve the fault tree exactly and avoid the need for approximations.

MOCUS algorithm, is based on the observation that AND gates increase number of elements in a CS and that OR gates increase the number of CSs. It uses two selection heuristics: the selection of the next product to process, and the selection of the gate variables to expand in the product under process. Clark and Reza-Latif-Shabgahi, in 2004 developed a straight forward approach which uses the bottom-up technique, producing table containing the cut-sets for each event in the tree.

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Table 1: Basic Events Responsible for Transmission Pipeline Failure

Events codes	**Events classification**
E1	Revenge
E2	Poverty
E3	Fishing
E4	Government neglect
E5	Get rich quickly
E6	Farming activities
E7	Militancy
E8	Population explosion
E9	Aging pipeline
E10	Company's operation
E11	Sabotage
E12	Poor engineering construction
E5	Stress concentration
E6	Residual stress
E7	Large internal stress
E8	Stress corrosion cracking with water
E9	Stress corrosion cracking with acid medium Hydrogen sulphide
E10	Stress corrosion cracking with acid medium Oxygen
E11	Stress corrosion cracking with acid medium Carbon dioxide
E12	Corrosion fatigue occasion by pressure surge
E13	Corrosion fatigue occasion by external load
E14	Quality of worker
E15	SCADA
E16	Equipment
E17	Apparatus
E18	Unreasonable strength
E19	Unsuitable material
E20	Earthquake
E21	Flood
E22	Subsidence
E23	Failure of Cathodic Protection
E24	Failure of coating
E25	ANII anti-corrosion
E26	High temperature induced soil corrosion
E27	Low resistance induced soil corrosion
E28	High water ratio induced soil corrosion
E29	High salt induced soil corrosion
E30	Electrical soil induced corrosion
E31	Bacteria induced soil corrosion
E32	Electrical interference
E33	Failure of inhibitors
E34	Failure of coating
E35	Bad clear pipe
E36	Debonding
E37	Construction defect
E38	Coarse grain
E39	Bad microstructure
E40	Mechanical inclusion
E41	Bad installation
E42	Bad weld
E43	Bad groove
E44	Mechanical damage

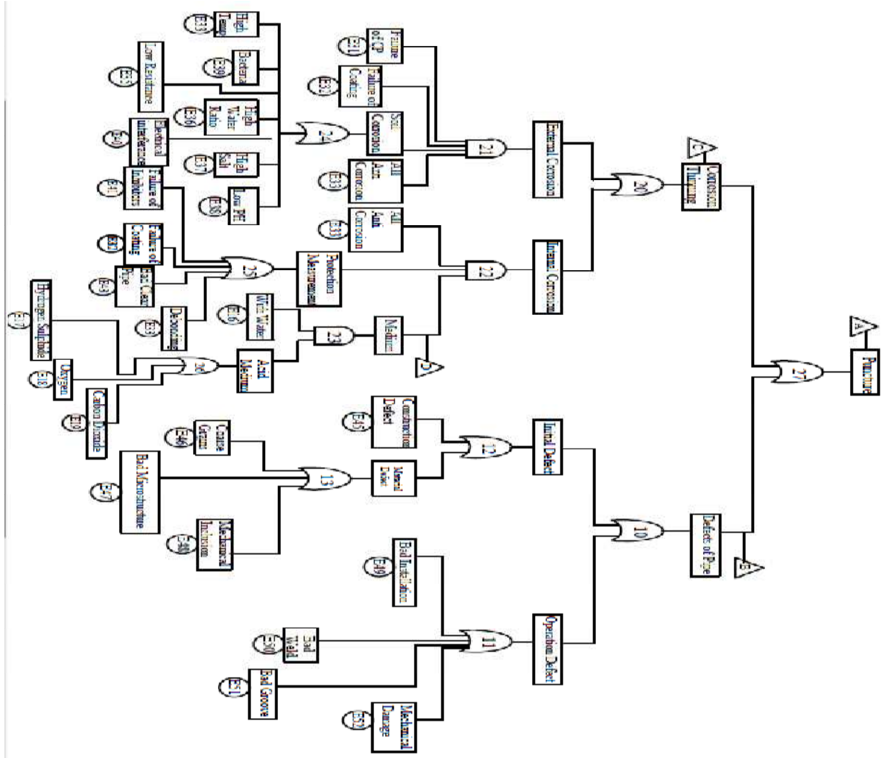

Figure 1 : Fault Tree diagram for Pipeline Failures

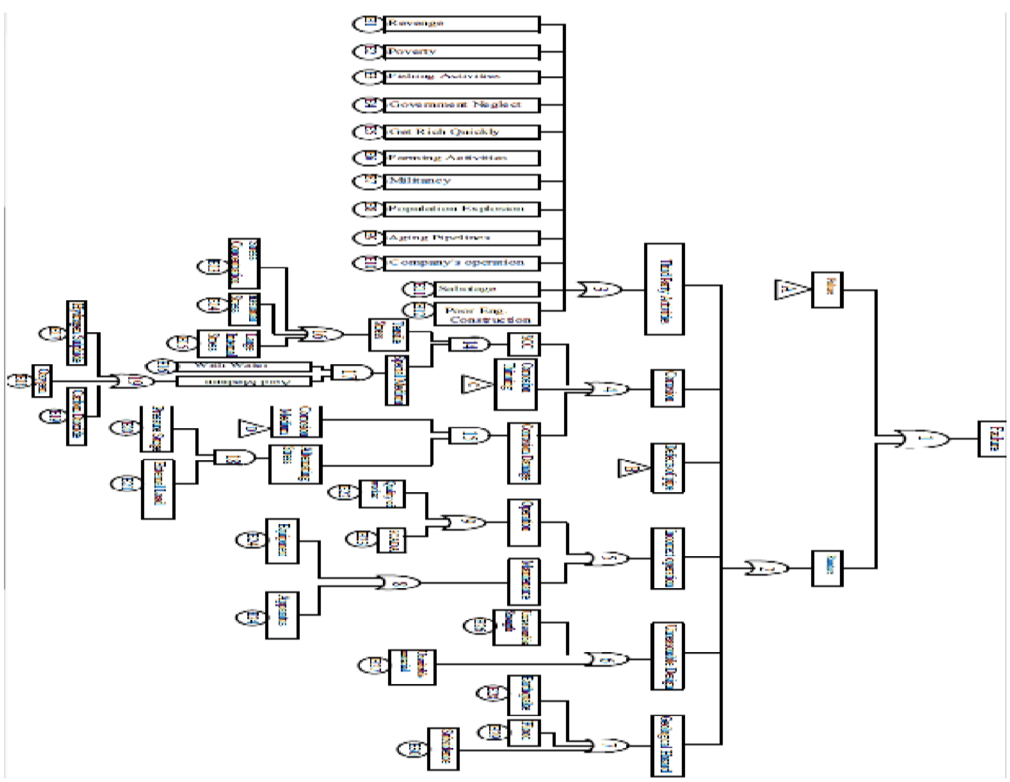

Figure 2: Fault Tree diagram for Pipeline Failures

3.0 Clark and Reza-Latif-Shabgahi Algorithm and Minimum Cut-Sets [5]

In this algorithm, Clark and Reza-Latif-Shabgahi assign a unique binary event code to each basic event that contains only one occurrence of the number 1 so that in a system with four basic system with four basic events, A, B, C D, the binary event code as shown in Table 2.

Table 2: Binary Events Code

Basic Events	Binary Code
A	1000
B	0100
C	0010
D	0001

Table 3: Gate output table for cut-sets *A, CD, BC*

A	B	C	D
1	0	0	0
0	0	1	1
0	1	1	0

The algorithm uses gate output tables to store the cut-sets of each event in the fault tree. The number of columns in the table is the number of bits in the binary event codes, and the number of rows is the number of cut-sets. The gate output table in Table 3 represents the cut-sets A, CD, BC for a fault tree with four basic events A, B, C, D, with A representing the msb of the binary event code. Gate output tables can be ANDed together and also ORed together. To AND gate output tables each row of the tables are ORed together as shown in table 4 for the event G3 = G1.G2 where the cut-sets of G1 are A and BC and the cut-sets of G2 are CD,AB. The resultant gate output table represents the cut-sets of G3, namely ACD, AB, BCD and ABC.

Table 4 Gate output tables showing the AND function

1	0	0	0	AND	0	0	1	1
0	1	1	0		1	1	0	0

gives

1	0	1	1
1	1	0	0
0	1	1	1
1	1	1	0

To OR gate output tables each row of the tables are copied into the resultant output table as shown in Table 5 for the event *G4 = G1 + G2* where the cut-sets of *G1* are *A* and *BC* and the cut-sets of *G2* are *CD* and *AB*. The resultant gate output table represents the cut-sets of *G4,* namely *A, BC, CD* and AB

Table 5 Gate output tables showing the OR function

1	0	0	0	OR	0	0	1	1
0	1	1	0		1	1	0	0

Gives

1	0	0	0
0	1	1	0
0	0	1	1
1	1	1	0

4.0 Algorithm for Transmission Pipeline Failure in the Niger Delta Region of Nigeria

Using the above Algorithm procedure, we developed an algorithm for transmission pipeline failure in the Niger Delta region of Nigeria. The algorithm uses gate output to store cut – sets of each event in the fault tree. The number of columns in the table is the number of bits in the binary codes, and the number of rows is the number of cut – sets. Thus, we would use fifty – two bit code with each basic event of the fault tree assigned a code containing only one occurrence of the number 1 as indicated in for the first thirteen events in Table 2.

Table 2 Basic Event Codes for Fault Tree of Pipeline Failure

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
E1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
E2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
E3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
E4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
E5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
E6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
E7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
E8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
E9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
E10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
E11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
E12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0

To AND gate output tables each row of the tables are ORed (copied) into the resultant output table. For example, the OR

gate output number G19, comprising of events E17 for Stress corrosion cracking with acid medium Hydrogen sulphide , E18 for Stress corrosion cracking with acid medium Oxygen and E19 for Stress corrosion cracking with acid medium Carbon dioxide,, therefore, OR gate output G19=E17+E18+E19 as shown in Table 3

Table 3 Gate output for G19=E17+E18+E19

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
G19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

. To AND gate output tables each row of the tables. For example, the AND gate output number G17, comprising events E16 and OR gate G19, G17=E16.G19 as shown in Table 4.

Table 4: Gate output for G17 = E16 ANDed G19 = E16 ANDed (E17 ORed E18 ORed E19)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
G 17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

The complete gate output table (algorithm) for the transmission pipeline failure fault tree in figure 2 and figure 3 for the generation of the bottom-up algorithm to determine the minimum cut-sets is shown in steps 1 through 27 below.

Step 1	G11 = E49 ORed E50 ORed E51 ORed E52
Step 2	G13 = E46 ORed E47 ORedE48
Step 3	G12 = E45 ORed G13 = E45 ORed (E46 ORed E47 ORedE48)
Step 4	G10 = G12 ORed G11 = (E45 ORed (E46 ORed E47 ORedE48)) ORed (E49 ORed E50 ORed E51 ORed E52)
Step 5	G26 = E17 ORed E18 ORed E19
Step 6	G23 = E16 ANDed G26 = E16 ANDed (E17 ORed E18 ORed E19)
Step 7	G25 = E41 ORed E42 ORed E43 ORed E44
Step 8	G22 =E33 ANDed G25 ANDed G23 = E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19))
Step 9	G24 = E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40
Step 10	G21 = E31 ANDed E32 ANDed E33 ANDed G24 = E31 ANDed E32 ANDed E33 ANDed (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)
Step 11	G20 = G21 ORed G22 = (E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19))
Step 12	G27 = G20 ORed G10 = ((E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19)) ORed (E45 ORed (E46 ORed E47 ORedE48)) ORed (E49 ORed E50 ORed E51 ORed E52)
Step 13	G7 = E28 ORed E29 ORed E30
Step 14	G6 = E26 ORed E27 ORed E28
Step 15	G8 = E24 ORed E25
Step 16	G9 = E22 ORed E23
Step 17	G18 = E20 ORed E21
Step 18	G15 = G23 ANDed G18 = (E16 ANDed (E17 ORed E18 ORed E19) ANDed (E20 ORed E21)
Step 19	G19 = E17 ORed E18 ORed E19

Step 20	G17 = E16 ANDed G19 = E16 ANDed (E17 ORed E18 ORed E19)
Step 21	G16 = E13 ORed E14 ORed E15
Step 22	G14 = G16 ANDed G17 = (E13 ORed E14 ORed E15) ANDed (E16 ANDed (E17 ORed E18 ORed E19)
Step 23	G5 = G9 ORed G8 = (E22 ORed E23) ORed (E24 ORed E25)
Step 24	G4 = G14 ORed G20 ORed G15 = (E13 ORed E14 ORed E15) ANDed (E16 ANDed (E17 ORed E18 ORed E19) ORed (E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19)) ORed (E16 ANDed (E17 ORed E18 ORed E19) ANDed (E20 ORed E21)
Step 25	G3 = E1 ORed E2 ORed E3 ORed E4 ORed E5 ORed E6 ORed E7 ORed E8 ORed E9 ORed E10 ORed E11 ORed E12
Step 26	G2 = G3 ORed G4 ORed G10 ORed G5 ORed G6 ORed G7 = E1 ORed E2 ORed E3 ORed E4 ORed E5 ORed E6 ORed E7 ORed E8 ORed E9 ORed E10 ORed E11 ORed E12) ORed (E13 ORed E14 ORed E15) ANDed (E16 ANDed (E17 ORed E18 ORed E19) ORed (E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19)) ORed (E16 ANDed (E17 ORed E18 ORed E19) ANDed (E20 ORed E21) ORed (E45 ORed (E46 ORed E47 ORedE48)) ORed (E49 ORed E50 ORed E51 ORed E52)ORed (E26 ORed E27 ORed E28) ORed (E28 ORed E29 ORed E30)
Step 27	G1= G27 ORed G2 = ((E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19)) ORed (E45 ORed (E46 ORed E47ORedE48)) ORed (E49 ORed E50 ORed E51 ORed E52) ORed E1 ORed E2 ORed E3 ORed E4 ORed E5 ORed E6 ORed E7 ORed E8 ORed E9 ORed E10 ORed E11 ORed E12) ORed (E13 ORed E14 ORed E15) ANDed (E16 ANDed (E17 ORed E18 ORed E19) ORed (E31 ANDed E32 ANDed E33 (E34 ORed E35 ORed E36 ORed E37 ORed E38 ORed E39 ORed E40)) ORed (E33 ANDed (E41 ORed E42 ORed E43 ORed E44) ANDed (E16 ANDed (E17 ORed E18 ORed E19)) ORed (E16 ANDed (E17 ORed E18

	ORed E19) ANDed (E20 ORed E21) ORed (E45 ORed (E46 ORed E47 ORedE48)) ORed (E49 ORed E50 ORed E51 ORed E52)ORed (E26 ORed E27 ORed E28) ORed (E28 ORed E29 ORed E30)

Discussion

Applying the principle of culling and reductions to the resultant , the above algorithm produced thirty, first- order minimal cut-sets, twelve third-order minimal cut sets, sixteen forth – order minimal cut-sets, and three-order minimal cut sets. Of importance are the first-order cut-sets because the higher the cut-sets order, the least possibility of its occurrence.

Conclusion

The simple straight forward Clark and Reza-Latif-Shabgahi Algorithm has been succeefully used to determine the minimal cut – sets of the modified fault tree diagram of pipeline failure in the Niger Delta Region of Nigeria as occasioned by third party activities. The fast execution attained as a result of the binary nature of the data representation and data manipulations enables the simple application of Boolean algebra to progress from the bottom to the top of the fault tree.

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