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Another Look at Some Information Order Selection Criteria in Autoregressive Processes.

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Abstract

In this paper, the selection procedure of the order of an autoregressive process of order P, AR(p) using the information order selection criteria is considered. We compare the performance of three information criteria methods, the Akaike information criterion (AIC), Schwarz information criterion (BIC) and Hannan & Quinn information criterion (HQC). It is observed that the ability of a criterion to select the true order may depend on the coefficient(s) of the process. A special AR(2) process where these three criterion appears to select the true order with equal probability is also identified. A Monte-Carlo experiment is used to demonstrate the procedure.

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Keywords: AIC; SIC; HQC

1. Introduction:

In order to estimate the parameters of an AR(p) model, it is appropriate to estimate the maximum order, p of the process. If an order lower than the true order of the process is selected the estimate of the parameters will not be consistent, if higher order is selected the variance increases, [14]. The identification of the order of a stationary Box-Jenkins time series model has been crucial in the literature. Two main approaches have been considered. One is the use of the sample autocorrelation function (SACF) and the sample partial autocorrelation function (SPACF). Example references can be found in [2, 3, 7] and a host of others. The second approach is based on the use of information order selection criteria which involves the use of an order selection based on the minimisation of some given functions expressed in terms of the order p of the model. These information criteria has been studied independently by [1] which is called Akaike information criterion (AIC), [13] which is called Schwarz information criterion (SIC) and [6] which is called Hannan and Quinn information criterion (HQC). Other authors that have also studied and supported these criteria in one way or the other are [5, 8, 14, 15] and a host of others. Liew [9], Liew and Chong [10] show in a simulation study the consistencies of these order selection criteria even in presence of ARCH errors. A purpose of this study is to present an alternative way of assessing the performance of these order selection criteria by considering simple AR(p) processes which are commonly encountered in practice.

The rest of the paper is organized as follows: in Section 2, the methodology is described; Section 3, simulation experiments and results are given while Section 4 concludes the work.

2. Methodology

Consider a stationary time series $\{y_t\}$ which satisfies the following linear equation:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t \qquad (1.1)$$

for some integer $p \geq 1$; $\alpha_1, \alpha_2, \cdots, \alpha_p$ are real parameters such that the zeroes of the polynomial

$\phi(z) = 1 - \alpha_1 z - \cdots - \alpha_p z^p$ lie outside the unit disk and ε_t constitute a sequence of independent random variables with the same normal distribution $N(0, \sigma_\varepsilon^2)$.

As highlighted by [4], this simple model given in (1.1) often turns out to be very useful for descriptive and forecasting purposes. The order p of the process is very important and is usually unlikely to be large for many financial returns while it can take large value for volatility models. After the order of the process is identified, then the next stage is estimation of the parameters which is usually done either by the Yule-Walker or the least squares method and followed by test of

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model adequacy which is usually referred to as diagnostic checking such as in Ljung and Box (1978), McLeod and Li (1983) and a host of others.

Using the idea of [9], the following lag order selection criterion is considered in this study:

Akaike information criterion, $AIC(p) = n\log\hat{\sigma}_p^2 + 2p$ (1)

Schwarz information criterion, $SIC(p) = n\log\hat{\sigma}_p^2 + p\ln(n)$ (2)

Hannan-Quinn criterion , $HQC(p) = n\log\hat{\sigma}_p^2 + 2p\ln[\ln(n)]$ (3)

where in all cases, $\hat{\sigma}_p^2 = (n-p-1)^{-1}\sum_{t=p+1}^{n}\hat{\varepsilon}_t^2$ (4)

and the sequence $\{\hat{\varepsilon}_t\}$ is the residuals generated from the fitted model for a particular order p.

We compute the probability of each of these criteria in its ability to pick the true order of the AR(p) process and call it P(True order). Whenever it is unable to pick the true order, then it is either a lower order is picked or a higher order than its true order is picked. The probability associated with a lower order will be denoted by P(Under estimation) while that of a higher order will be denoted by P(Over estimation). These order estimation is carried out for a 1000 simulation. In this regard, if $\hat{p}$ is the true order of the process, then the three probabilities will be calculated thus;

(i) P(True order) = $\#(\hat{p} = p)/M$ (5)

(ii) P(Under estimation) = $\#(\hat{p} < p)/M$ (6)

(iii) P(Over estimation) = $\#(\hat{p} > p)/M$ (7)

where M denote the number of Monte-Carlo experiment and $\#(\bullet)$ denote the number of times event $(\bullet)$ happens.

3. Simulation experiments and results

For illustrative purposes, we consider the following Data generating process (DGP). Models I and II are examples of an AR(1) and AR(2) processes respectively.

Model (I) $y_t = \phi_1 y_{t-1} + \varepsilon_t\,,\quad y_0 = 0$ (8)

Model (II) $y_t = 0.75y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t\,,\quad y_0 = 0$ (9)

In Model I, the parameter ϕ_1 varies from -0.9 to 0.9 with an increment of 0.1 but ϕ_1 does not take the value 0 because the process will be a white noise. In Model II, the parameter α_2 takes on the values -0.5, -0.25, 0.25 and 0.5 respectively.

In order to demonstrate the ability of these information order selection criteria to correctly select the true order, we simulate artificial time series data using Model I and Model II for each of the parameter(s). The sequence of error $\{\varepsilon_t\}$ which is normally distributed with mean 0 and variance 1 was generated using the random number generator in MATLAB 7.5.0 which were in turn used in generating artificial time series data. We generated $(300+n)$ sample sizes. Only the last n observations are used for modelling while the first 300 are discarded to minimize initialization effect. Sample sizes of n = 25, 50 and 100 was considered in this study. The model identification and parameter estimation was written and executed in MATLAB 7.5.0, using simple MATLAB codes. The procedure is replicated 1000 times and the results are displayed in Tables 1, 2, 3 for Model I and Tables 4, 5, 6 and 7 for Model II. In addition, Figures 1, 2 and 3 give graphical representations of the p(True order) for the three criteria using Model II when parameter α_2 is varied for sample sizes n = 25, 50 and 100 respectively.

Table 1: Monte-Carlo study using Model I for sample size n = 25

ϕ_1	*AIC*	*SIC*	*HQC*	ϕ_1	*AIC*	*SIC*	*HQC*
-0.9	0.744 (0.256)	0.861 (0.139)	0.782 (0.218)	0.1	0.698 (0.302)	0.813 (0.187)	0.741 (0.259)
-0.8	0.748 (0.252)	0.847 (0.153)	0.785 (0.215)	0.2	0.723 (0.277)	0.838 (0.162)	0.766 (0.234)
-0.7	0.746 (0.254)	0.862 (0.138)	0.782 (0.218)	0.3	0.695 (0.305)	0.815 (0.185)	0.731 (0.269)
-0.6	0.733 (0.267)	0.849 (0.151)	0.772 (0.228)	0.4	0.729 (0.271)	0.827 (0.173)	0.752 (0.248)
-0.5	0.714	0.829	0.760	0.5	0.724	0.835	0.756

	(0.286)	(0.171)	(0.240)		(0.276)	(0.165)	(0.244)
-0.4	0.732 (0.268)	0.850 (0.150)	0.766 (0.234)	0.6	0.715 (0.285)	0.834 (0.166)	0.747 (0.253)
-0.3	0.731 (0.269)	0.846 (0.154)	0.771 (0.229)	0.7	0.731 (0.269)	0.830 (0.170)	0.758 (0.242)
-0.2	0.726 (0.274)	0.828 (0.172)	0.754 (0.246)	0.8	0.718 (0.282)	0.834 (0.166)	0.755 (0.245)
-0.1	0.715 (0.285)	0.836 (0.164)	0.753 (0.247)	0.9	0.722 (0.278)	0.841 (0.159)	0.763 (0.237)

Table 2: Monte-Carlo study using Model I for sample size n = 50

ϕ_1	*AIC*	*SIC*	*HQC*	ϕ_1	*AIC*	*SIC*	*HQC*
-0.9	0.791 (0.209)	0.928 (0.072)	0.865 (0.135)	0.1	0.732 (0.268)	0.889 (0.111)	0.808 (0.192)
-0.8	0.755 (0.245)	0.892 (0.108)	0.824 (0.176)	0.2	0.744 (0.256)	0.888 (0.112)	0.808 (0.192)
-0.7	0.739 (0.261)	0.890 (0.110)	0.812 (0.188)	0.3	0.772 (0.228)	0.916 (0.084)	0.838 (0.162)
-0.6	0.760 (0.240)	0.888 (0.112)	0.831 (0.169)	0.4	0.741 (0.259)	0.893 (0.107)	0.819 (0.181)
-0.5	0.782 (0.218)	0.911 (0.089)	0.846 (0.154)	0.5	0.744 (0.256)	0.896 (0.104)	0.798 (0.202)
-0.4	0.740 (0.260)	0.890 (0.110)	0.812 (0.188)	0.6	0.749 (0.251)	0.892 (0.108)	0.816 (0.184)
-0.3	0.741 (0.259)	0.899 (0.101)	0.823 (0.177)	0.7	0.789 (0.211)	0.920 (0.080)	0.855 (0.145)
-0.2	0.756 (0.244)	0.886 (0.114)	0.823 (0.177)	0.8	0.742 (0.258)	0.895 (0.105)	0.813 (0.187)
-0.1	0.765 (0.235)	0.910 (0.090)	0.833 (0.167)	0.9	0.768 (0.232)	0.910 (0.090)	0.838 (0.162)

Table 3: Monte-Carlo study using Model I for sample size n = 100

ϕ_1	*AIC*	*SIC*	*HQC*	ϕ_1	*AIC*	*SIC*	*HQC*
-0.9	0.749 (0.251)	0.930 (0.070)	0.851 (0.149)	0.1	0.778 (0.222)	0.938 (0.062)	0.863 (0.137)
-0.8	0.769 (0.231)	0.920 (0.080)	0.852 (0.148)	0.2	0.759 (0.241)	0.922 (0.078)	0.848 (0.152)
-0.7	0.782 (0.218)	0.930 (0.070)	0.872 (0.128)	0.3	0.764 (0.236)	0.919 (0.081)	0.863 (0.137)
-0.6	0.781 (0.219)	0.951 (0.049)	0.868 (0.132)	0.4	0.769 (0.231)	0.936 (0.064)	0.856 (0.144)
-0.5	0.741 (0.259)	0.917 (0.083)	0.827 (0.173)	0.5	0.734 (0.266)	0.929 (0.071)	0.830 (0.170)
-0.4	0.749 (0.251)	0.921 (0.079)	0.855 (0.145)	0.6	0.766 (0.234)	0.929 (0.071)	0.853 (0.147)
-0.3	0.765 (0.235)	0.936 (0.064)	0.853 (0.147)	0.7	0.781 (0.219)	0.934 (0.066)	0.871 (0.129)
-0.2	0.771 (0.229)	0.937 (0.063)	0.859 (0.141)	0.8	0.779 (0.221)	0.940 (0.060)	0.866 (0.134)
-0.1	0.730 (0.270)	0.923 (0.077)	0.841 (0.159)	0.9	0.773 (0.227)	0.937 (0.063)	0.861 (0.139)

Tables 1, 2 and 3 represents simulation studies using the three information criteria in identifying the order of AR(1) process for sample sizes n = 25, 50 and 100 respectively for Model I using the parameter value ϕ_1 ranging from -0.9 to 0.9. The upper values represent P(True order) while the values in bracket represent P(Over estimation).

Table 1 reveals that the AIC is able to identify the true order in about 72% of the experiment, the SIC is about 83% while HQC is about 75%. Then in Table 2, the AIC account for more than 75% in its ability to pick the true order, the SIC account for about 90% while the HQC is about 82%.

In Table 3, the AIC manages to pick the true order in about 75%, the SIC is about 93% while that of HQC is about 85%.

Table 4: Monte-Carlo study using Model II for parameter α_2 ***= -0.5***

Criteria	P(True order)	P(Under estimation)	P(Over estimation)
	Sample size n = 25		
AIC	0.879	0.111	0.010
SIC	0.827	0.168	0.005
HQC	0.864	0.129	0.007
	Sample size n = 50		
AIC	0.985	0.015	0
SIC	0.955	0.045	0
HQC	0.974	0.026	0
	Sample size n = 100		
AIC	1.000	0	0
SIC	0.999	0.001	0
HQC	1.000	0	0

Table 5: Monte-Carlo study using Model II for parameter α_2 ***= -0.25***

Criteria	P(True order)	P(Under estimation)	P(Over estimation)
	Sample size n = 25		
AIC	0.542	0.443	0.015
SIC	0.401	0.592	0.007
HQC	0.501	0.486	0.013
	Sample size n = 50		
AIC	0.708	0.287	0.005
SIC	0.501	0.499	0
HQC	0.633	0.364	0.003
	Sample size n = 100		
AIC	0.895	0.105	0
SIC	0.717	0.283	0
HQC	0.829	0.171	0

Table 6: Monte-Carlo study using Model II for parameter α_2 ***= 0.25***

Criteria	P(True order)	P(Under estimation)	P(Over estimation)
	Sample size n = 25		
AIC	0.237	0.702	0.061
SIC	0.137	0.836	0.027
HQC	0.212	0.739	0.049
	Sample size n = 50		
AIC	0.434	0.525	0.041
SIC	0.243	0.747	0.010
HQC	0.345	0.631	0.024
	Sample size n = 100		
AIC	0.733	0.250	0.017
SIC	0.508	0.488	0.004
HQC	0.645	0.345	0.010

Table 7: Monte-Carlo study using Model II for parameter α_2 = 0.5

Criteria	P(True order)	P(Under estimation)	P(Over estimation)
	Sample size n = 25		
AIC	1.000	0.000	0.000
SIC	1.000	0.000	0.000
HQC	1.000	0.000	0.000
	Sample size n = 50		
AIC	1.000	0.000	0.000
SIC	1.000	0.000	0.000
HQC	1.000	0.000	0.000
	Sample size n = 100		
AIC	1.000	0.000	0.000
SIC	1.000	0.000	0.000
HQC	1.000	0.000	0.000

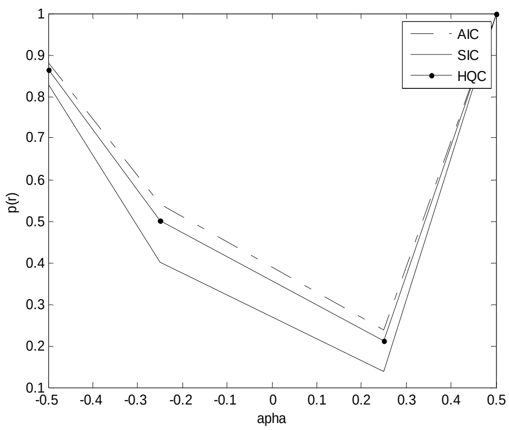

Figure 1: The (p(r)= p(true order)) for (apha = α_2 = -0.5, -0.25, 0.25, 0.5) when sample n = 25.

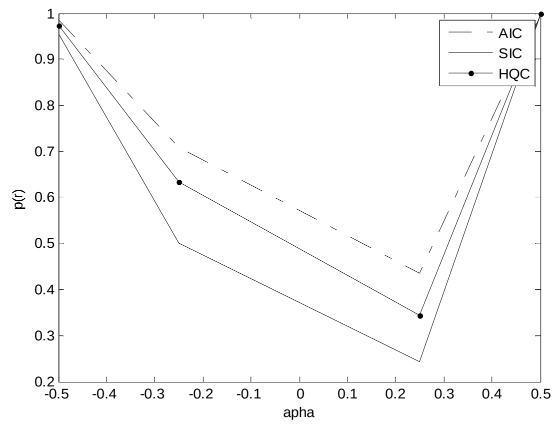

Figure 2: The (p(r)= p(true order)) for (apha = α_2 = -0.5, -0.25, 0.25, 0.5) when sample n = 50.

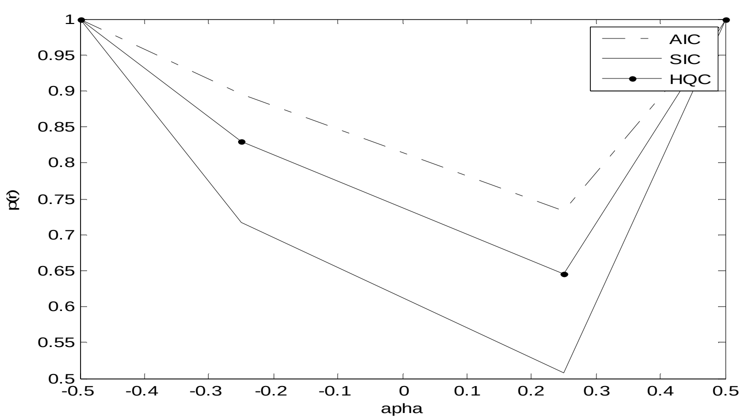

Figure 3: The (p(r)= p(true order)) for (apha = α_2 = -0.5, -0.25, 0.25, 0.5) when sample size n = 100.

Tables 4, 5, 6 and 7 represents simulation studies using the three information criteria in identifying the order of AR(2) process for sample sizes n = 25, 50 and 100 in each cases for Model II with lag 1 coefficient α_1 fixed at 0.75 while the lag 2 coefficient α_2 takes on the values -0.5, -0.25, 0.25 and 0.5.

In Table 4, the AIC is able to identify the true order in about 88%, SIC 83% and HQC 87% while the probability of under estimation and over estimation are very low for the three criteria when the sample size is n = 25. When the sample size is n = 50 and 100 the ability of the three criteria to identify the true order are approximately very high and equal. The probability of over estimation stands at zero while that of under estimation is very low and close to zero.

In Table 5, for sample size n =25, the AIC is able to identify the true order in about 54%, SIC 40% and HQC 50%. For under estimation, the AIC is 44%, SIC 59% and HQC is about 49%. Then for over estimation, the associated probabilities for the three criteria are very close to zero. Now when the sample size n = 50 and 100, the three criteria increases in its ability to identify the true order while that of under estimation also decreases correspondingly. The probability of over estimation remains zero.

In Table 6, the ability of the three criteria to identify the true order increases steadily as the sample size increases while that of under estimation decreases. For the over estimation, the associated probabilities for the three criteria are also very close to zero throughout the sample sizes.

In Table 7, the ability of the three criteria to identify the true order remains equal with probability 1, irrespective of the sample size. Probabilities of their under estimation and over estimation are respectively zero.

Figures 1, 2 and 3 exhibit the same structures. These Figures reveal that irrespective of the sample size, the three criteria are able to identify the true order with high probability when the parameter α_2 is -0.5 and decreases at α_2 = -0.25, 0.25 and increases again at 0.5. Furthermore, it is pertinent to mention that when the sample size is small, the AIC and HQC are close in its ability to identify the true order. However, for moderate and large sample sizes, the AIC performs better than the others, followed by the HQC and then the SIC in its ability to identify the true order for the special AR(2) process considered using equation (9).

4 Conclusion

The study highlights the ability of three popular criteria AIC, SIC and HQC to identity the true order of an AR(1) process irrespective of its lag coefficient. It also reveals the superiority of the SIC irrespective of the sample size for the AR(1) case. For an AR(2) model, it is observed that when lag 1 coefficient is 0.75 and lag 2 coefficient is -0.5, the three criteria is able to identify the true order of the process with equal probability. When the lag 2 coefficient is -0.25 and 0.25, the three criteria performs poorly in its ability to identify the true order. However, when the lag 2 coefficient is 0.5, the three criteria perform excellently.

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