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Selecting The Best Initial Method For A Transportation Problem

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Abstract

This paper is concerned with determining the best initial method for a transportation problem. Seven initial methods are considered and compared. One is a new method that has not been reported in the literature. Comparison is done on the basis of the number of iterations required to reach the final solution if the concerned methods were used as the initial methods. A C-program was developed to facilitate getting results for the number of iterations. Two statistical packages, (SPSS) and (SAS) were then used in determining the statistical performance between the methods based on the results obtained using the C- program.

Keywords: Transportation problems, Initial methods of solving transportation problems, Analysis of variance.

1.0 Introduction:

A transportation problem is one of the subclasses of linear programming problems where the objective is to transport various quantities of single homogeneous product that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum [4].

Transportation models are primarily concerned with the optimal way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses (called demand destinations). The objective in a transportation problem is to fully satisfy the destinations requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumer through a variety of channels of distribution (wholesalers, retailers, distributors, etc) there is need to minimise the cost of transportation so as to increase the profit on sales. Transportation problems arise in all such cases and they aim at providing assistance to the top management in ascertaining how many units of a particular product should be transported from each supply origin to each demand destination so that the total prevailing demand for the company's product is satisfied, and at the same time transportation cost is minimised [6].

To solve a transportation problem, one can use the simplex method for solving linear programming problems (LPP) or other techniques of solving LPP. This however is not a popular way of solving a transportation problem. The alternative way which is more popular is to start with an initial method which gives a feasible solution to the problem. The initial feasible solution may or may not be optimal. The next stage is to apply the Stepping stone method or Modified Distribution method (MODI) for instance to determine the optimal solution [1]. The initial methods for solving transportation problems in the literature are as follows: Northwest corner method, Least cost method, Vogel Approximation method, Column minimum method, Russell's approximation method and Row – minimum method [2]. We also come up with another initial method which we call Row-Column minimum method so called because it is a combination of row and column minimum methods.

This paper is a comparative study on the initial methods of solving transportation problems so as to determine the best initial method to apply in getting a feasible solution leading to optimality. Specifically, we determined the number of iterations required to reach the final solution if the initial methods were used to start the problem. Such comparative analysis has not been done as far as we know which is why we carry out this research.

We developed a C-program and applied it on 100 random problems. The program was used to solve any of the 100 problems first using an initial method and then used stepping stone algorithm to determine the optimal solution. The program then recorded the number of iterations required to reach optimality in respect of the chosen initial method for the chosen problem.

The random problems chosen have been between 3 and 10 destinations and also between 3 to 10 supply origins.

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2.0 THE TRANSPORTATION PROBLEM

Mathematically a transportation problem is a linear programming problem in which the objective function is to minimise the cost of transportation subject to the demand and supply constraints.

Let a_i *be the* quantity of the commodity available at the origin i

b_j be the quantity of the commodity demanded at the destination j

c_{ij} be the transportation cost from origin i to destination j and

x_{ij} be the quantity transported from origin i to destination j

m be the total number of units supplied at origin i

n be the total number of units demanded at destination j.

Then, the total cost of transportation is given by

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad (1.1)$$

The quantity transported from origin i *to destination* j is given as x_{ij} so that the quantity transported from origin i is $\sum_{j=1}^{m} x_{ij}$ and since the quantity at origin i is a_i, we must have

$$\sum_{j=1}^{m} x_{ij} = a_i \qquad (1.2)$$

The quantity transported to depot j is similarly given as

$$\sum_{i=1}^{n} x_{ij}$$

and since the quantity required there is b_j, we must have

$$\sum_{i=1}^{n} x_{ij} = b_j \qquad (1.3)$$

Our aim is to minimise equation (1.1) subject to equations (1.2) and (1.3).

3.0 METHODS OF RESEARCH

We started from the work by Taghrid et al [5], who developed an object oriented model as a decision support tool to solve five initial methods of the transportation problem using C++ programming language. In this paper however we used a C-program to solve the problems and we are interested in the number of iterations needed to reach optimality. This was done for all the 100 problems and using all the seven initial methods. We then used two statistical packages, SPSS version 17.0 and SAS version 9.0 to run an analysis of variance (ANOVA) on the number of iterations for each method. The final solutions show us which method if any, is the best and whether there is any significant difference between the initial methods in respect of the number of iterations leading to optimality.

4.0 METHODS FOR FINDING INITIAL SOLUTION USED IN THIS STUDY

As already indicated, seven initial solution methods were used in the comparative analysis and a brief explanation on each of the methods is as follows:

(i) North-West Corner Method (NWC): This method starts at the northwest corner cell or upper left corner cell of a transportation table. It allocates as much as possible to the selected cell and adjust the associated amount of supply and demand by subtracting the allocated amount. This results in crossing out a row or column.

If exactly one row or column is left uncrossed out, we stop, otherwise, we move to the cell which is to the right if a column has just been crossed out or below if a row has been crossed out [5]. We denote this method as M1 for the purpose of this research.

(ii) Least Cost Method:

The least cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satisfied row or column is crossed out and the supply and demand are adjusted accordingly. We next look for the uncrossed out cell with smallest unit cost and repeat the process until exactly one row or column is left uncrossed [5]. We denote this method as M2 for the research purpose.

(iii) Vogel's Approximation Method (Vam)

Vogel's approximation method, denoted as M3 in this research, is an improved version of the least cost method that generally has been obtained to produce better starting solutions. It has been claimed to be the best initial method [2]. The complete steps in implementing the methods are as follows:

Step I: For each row (column) determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step II: Identify the row or column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the identified row or column, adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, one of the two is crossed out and the remaining row (column) is assigned zero supply (demand).

Step III: (a) If exactly one row or column with zero supply or demand (column) remains uncrossed out stop.
(b) If one row (column) with positive supply remains uncrossed out, determine the basic variables in the row (column) by the least cost method. Stop.
(c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least-cost method. Stop.
(d) Otherwise go to step (I) [5].

(iv) Row-Column Minimum Method

Row-Column minimum method is a combination of both row and column minimum methods. It is developed in this research with the intention of taking the advantages of the two methods. It starts with allocation in the first row and first column simultaneously choosing the lowest cost cells one after the other of the first row or column so that the capacities of the first supply and first demand are both satisfied. Next the satisfied row and column are both crossed out, and the amount of supply and demand are adjusted accordingly. The method next considers allocation in the second row and second column, choosing the lowest cost cells and repeating the process until exactly both the capacities of the columns and the rows are exhausted. This method is denoted as M4.

(V) Column Minimum Method

Column minimum method denoted as M5 in the research, starts with first column and chooses the lowest cost cell of first column so that either the demand of the first distribution centre is satisfied or the capacity of the i^{th} supply is satisfied or both. Three cases arise as follows:

i. If the demand of the first distribution centre is satisfied cross out the first column and move right to the second column.
ii. If the capacity of i^{th} supply is satisfied, cross out the i^{th} row and reconsider the first column with the remaining demand.
iii. If the demands of the first distribution centre as well as the capacity of the i^{th} supply are completely satisfied, cross out the column as well as the i^{th} row and move right to the second column.

Continue the process for the resulting reduced transportation table until the last column [3].

(vi) Russell's Approximation Method

Russell's approximation method, denoted as M6, is a more recently proposed method that seems very promising. Computation takes a longer time because each cell is involved or considered. The complete steps followed in implementing the method are as follows:

Step I: For each source i remaining under consideration determine its u_i which is the largest cost (c_{ij}) still remaining in that row.
Step II: For each destination column j remaining under consideration, determine its v_j which is the largest unit cost (c_{ij}) still remaining in that column.
Step III: For each variable x_{ij} not previously selected in these rows and columns, calculate $\Delta_{ij} = c_{ij} - u_i - v_j$. Allocation is then made to the cell having the largest negative value of $\Delta_{ij.}$ The allocation will result in crossing out a row or a column, according as to whether the supply in the row or the demand in the column is satisfied.
The process is repeated until all demands and supplies are satisfied.

(vii) Row Minimum Method

Row minimum method, denoted as M7 in this study, starts with the first row and chooses the lowest cost cell in the row so that either the capacity of the first supply is exhausted or the demand of the jth distribution centre is satisfied or both. Three cases arise as follows.

i. If the capacity of the first supply is completely satisfied, cross out the first row and proceed to the second row.
ii. If the demand at j^{th} distribution centre is satisfied, cross out the j^{th} column and reconsider the first row with the remaining capacity.
iii. If the capacity of the first supply as well as demand at j^{th} distribution centre are completely satisfied, cross out the row as well as the j^{th} column and move down to the second row.

Continue this process for the resulting reduced transportation table units until the last row [3].
The above seven initial methods were applied to the 100 random problems generated and the results obtained are in Table1, Appendix 1.
After obtaining the number of iterations, we carried out an analysis of variance (ANOVA) on the mean number of iterations given by each method. ANOVA is a powerful parametric technique for analysis differences between sample means. It does not only indicate that three or more means differ, but also can be used to examine which two means significantly differ. Advantages of ANOVA include the fact that it saves time and labour by comparing all sample means simultaneously, and more importantly it reduces the probability of committing a type I error.

The statistical analysis tested whether there was actually any significant difference between averages of the number of iterations given by the seven initial methods. It also showed the position of each method if they were ranked. One-way ANOVA is considered to be the appropriate method of analysis because each method has one-way variable and also ANOVA test assumes that the results of these initial methods are normally distributed. This is considered so due to the large number of the results. For each method, there are one hundred observations for each method which makes a total of seven hundred observations in all.

The result obtained using SAS is in table 2, Appendix2 while that using SPSS is in table3, Appendix3.

5.0 ANALYSIS OF THE RESULTS OBTAINED

From the results obtained in table 3, the following analysis is made when we consider the confidence interval to be 95%.

The Northwest corner method (M1) is significantly different ($p < 0.05$) from the rest of the methods, giving the worst result in almost all cases.

There is no significant difference ($p > 0.05$) between Row-Column minimum method (M4), Column minimum method (M5),Russell's method (M6) and Row-minimum method (M7) when compared with the Least cost method (M2), but there is significant difference ($p < 0.05$) between the Least cost method (M2), North west corner method (M1) and Vogel's method (M3) .

Comparison between Vogel's method (M3) and the rest of the methods shows that there is significant difference ($p < 0.05$) with North west corner method (M1), Least cost method (M2), Row-Column minimum method (M4),Column minimum (M5), and Row minimum method (M7) but there is no significant difference ($p > 0.05$) with Russell's method (M6).

From the table, it also shows that there is no significant difference ($p > 0.05$) between Row-Column minimum method (M4) when compared with the Least cost method (M2), Column minimum method (M5), and Row- minimum method (M7) but there is significant difference ($p < 0.05$) between Row-Column minimum method (M4) when compared with North west corner method (M1), Russell's method (M6) and Vogel's method (M3).

Considering the Column minimum method (M5) from the table, it shows that there is no significant difference ($p > 0.05$) with the Least cost method (M2), Row-Column minimum method (M4) and Row-minimum method (M7) but there is significant difference ($p < 0.05$) with North west corner method (M1), Russell's method (M6) and Vogel's method (M3).

It also shows from the table that when Russell's method (M6) is compared with North west corner method (M1), Column minimum method (M5) and Row-Column minimum method (M4) there is significant difference ($p < 0.05$) but there is no significant difference ($p > 0.05$) with Vogel's method (M3), Least cost method (M2) and Row minimum method (M7).

There is significant difference ($p < 0.05$) between the Row-minimum method (M7) when compared with North west corner method (M1) and Vogel's method (M3) but there is no significant difference ($p > 0.05$) when Row- minimum method (M7) is compared with Least cost method (M2), Row-Column minimum method (M4), Russell's method (M6) and Column-minimum method.

On the other hand however, if we look at table 2, Appendix2, the methods are simply categorised into four groups. The first group contains the Northwest corner method as the worst method. The second group contains four methods viz: Least cost method, Row minimum method, Column mini method and Row-Column minimum method with no significant difference between them. The third group contains three methods viz: Least cost method, Row minimum method and Russell's method with no significant difference between them. The last group contains the best two methods, Vogel's approximation method and Russell's method.

6.0 CONCLUSION

From the results of the analysis obtained using the C-program, SAS and SPSS, we observe that the seven initial methods are generally grouped into three.

The main virtue of the North-west corner method (M1) is that it is quick and easy. However, because it pays no attention to the unit costs, usually the solution is far from the optimal i.e. it yields the worst result.

Vogel's (M3) and Russell's (M6) yield the best starting basic solution and give an initial solution very near to the optimal. However, computation is slow because they take longer time in implementing their steps.

The remaining four methods, Column minimum method (M5), Row minimum method (M7), Least cost method (M2) and Row-Column minimum method (M4) form the third group and relatively easier to compute, but they are not as good as Vogel's and Russell's methods.

APPENDIX 1

Table 1: Results obtained using Seven Initial Methods

P	M1			M2			M3			M4			M5			M6			M7		
	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL
1	3	520	435	2	475	435	2	457	435	2	475	435	2	475	435	2	475	435	2	505	435
2	3	137	98	1	119	98	0	98	98	1	119	98	1	105	98	1	119	98	1	105	98
3	3	221	169	4	238	169	3	221	169	4	235	169	2	191	169	3	192	169	3	227	169
4	4	1095	646	2	727	646	0	646	646	1	661	646	4	1023	646	0	646	646	2	727	646
5	3	68	53	1	54	53	1	54	53	2	74	53	1	54	53	0	53	53	3	68	53
6	2	128	118	3	156	118	0	118	118	3	162	118	1	121	118	0	118	118	3	152	118
7	2	1170	385	1	425	385	1	385	385	1	425	385	1	425	385	1	425	385	1	425	385
8	2	6600	5920	1	6460	5920	0	5920	5920	2	7120	5920	2	7120	5920	0	5920	5920	1	6462	5920
9	3	62	54	0	54	54	0	54	54	0	54	54	1	63	54	0	54	54	0	54	54
10	5	1950	1580	3	1870	1580	0	1580	1580	1	1730	1580	3	1980	1580	0	1580	1580	1	1710	1580
11	1	78	70	2	75	70	1	71	70	1	75	70	2	75	70	1	71	70	2	81	70
12	2	296	231	1	237	231	0	231	231	1	241	231	2	246	231	0	231	231	1	241	231
13	3	57880	40320	0	40320	40320	0	40320	40320	0	40320	40320	4	41380	40320	0	40320	40320	1	40320	40320
14	3	4925	3600	2	36500	3600	0	3600	3600	0	3600	3600	0	3600	3600	2	3646	3600	0	3600	3600
15	5	363	290	3	321	290	1	305	290	4	320	290	1	295	290	1	295	290	3	320	290
16	2	985	720	1	730	720	0	720	720	0	720	720	0	720	720	1	770	720	0	720	720
17	2	4100000	3320000	0	3320000	3320000	0	3320000	3320000	2	3620000	3320000	0	3320000	3320000	0	3320000	3320000	0	3320000	3320000
18	4	980	875	2	890	875	1	900	875	2	1010	875	3	980	875	1	900	875	2	910	875
19	5	3280000	1790000	2	2060000	1790000	1	1800000	1790000	3	2060000	1790000	2	2300000	1790000	1	1800000	1790000	1	2140000	1790000
20	4	8800	7400	2	7550	7400	1	7500	7400	2	8600	7400	2	8600	7400	1	8000	7400	1	8100	7400
2	5	1713	1655	5	1678	1655	0	1655	1655	3	1660	1655	5	1678	1655	0	1655	1655	3	1670	1655

P	M1 No IT	M1 INBFS	M1 OPTSL	M2 No IT	M2 INBFS	M2 OPTSL	M3 No IT	M3 INBFS	M3 OPTSL	M4 No IT	M4 INBFS	M4 OPTSL	M5 No IT	M5 INBFS	M5 OPTSL	M6 No IT	M6 INBFS	M6 OPTSL	M7 No IT	M7 INBFS	M7 OPTSL
1																					
22	20	17900	8830	6	9530	8830	3	9180	8830	12	11040	8830	13	11330	8830	3	8895	8830	10	1058	8830
23	1	87	79	2	85	79	1	81	79	2	94	79	1	87	79	2	94	79	2	127	79
24	2	2740	2100	1	2340	2100	0	2100	2100	1	2340	2100	0	2100	2100	0	2100	2100	1	2340	2100
25	19	18080	9100	5	9800	9100	3	9630	9100	3	9650	9100	5	9980	9100	3	9345	9100	7	11020	9100
26	1	4450	4350	1	4400	4250	1	4400	4250	0	4250	4250	1	4400	4250	1	4300	4250	0	4250	4250
27	5	1150	625	1	640	625	0	625	625	1	640	625	1	645	625	0	625	625	2	635	625
28	3	61	48	1	49	48	1	48	48	2	64	48	0	48	48	0	48	48	2	63	48
29	3	1440	1190	1	1220	1190	2	1200	1190	1	1200	1190	2	1600	1190	1	1430	1190	1	1310	1190
30	1	11580	9600	2	10720	9600	2	9820	9600	3	10620	9600	1	9700	9600	0	9600	9600	2	12920	9600
31	11	7000	1230	5	3270	2230	4	2680	2230	7	5220	2230	6	4610	2230	5	3410	2230	6	4790	2230
32	5	1595	1035	0	1035	1035	0	1035	1035	1	1110	1035	3	1370	1035	2	1350	1035	1	1110	1035
33	6	500	255	0	255	255	0	255	255	1	259	255	3	375	255	0	255	255	1	315	255
34	2	2580	2310	1	2310	2310	1	2310	2310	2	2590	2310	4	2635	2310	0	2310	2310	0	2310	2310

Key:
P: problem number,
NO IT: Number of iterations
INBFS: Initial basic feasible solution
OPTLS: Optimal solution

M1: North-west corner method
M2: Least cost method
M3: Vogel's method
M4: Row-Column minimum method
M5: Column minimum me
M6: Russell's method
M7: Row-minimum method

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P	M1			M2			M3			M4			M5			M6			M7		
	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL
35	4	3100	2630	1	2660	2630	1	2765	2630	2	2820	2630	3	2835	2630	0	2630	2630	0	2630	2630

36	4	710	360	0	360	360	0	360	360	2	460	360	0	360	360	0	360	360	0	360	360
37	2	560	530	2	515	530	1	545	530	2	560	530	0	530	530	2	535	530	2	560	530
38	21	16270	5780	6	6580	5780	4	6020	5780	9	7200	5780	10	7600	5780	6	6465	5780	10	7650	5780
39	2	117	85	2	93	85	0	85	85	2	94	85	1	88	85	1	90	85	2	127	85
40	4	10000	8750	2	9100	8750	2	9600	8750	2	9100	8750	2	9800	8750	2	9600	8750	3	10250	8750
41	1	430	424	1	430	424	0	424	424	1	430	424	0	424	424	0	424	424	1	430	424
42	3	660	390	2	560	390	0	390	390	0	590	390	2	620	390	3	574	390	2	560	390
43	2	2740	2100	1	2340	2100	0	2100	2100	2	2340	2100	0	2100	2100	1	2580	2100	2	2340	2100
44	19	16390	1540	5	2140	1540	0	1540	1540	6	2180	1540	8	2940	1540	6	3280	1540	8	2960	1540
45	20	16450	5930	6	6780	5930	5	6280	5930	8	7360	5930	7	7210	5930	6	7400	5930	9	8180	5930
46	1	4450	4250	1	14400	4250	1	4400	4250	1	4450	4250	1	4400	4250	1	4400	4250	0	4250	4250
47	4	1976	776	1	781	776	1	781	776	2	791	776	2	902	776	1	836	776	1	801	776
48	17	11970	1170	6	1890	1170	5	1880	1170	7	2190	1170	10	5900	1170	7	2720	1170	8	2720	1170
49	10	11260	5220	6	7580	5220	2	5610	5220	3	5900	5220	5	6890	5220	4	6830	5220	1	5580	5220
50	11	9000	5090	2	5340	5090	2	5290	5090	5	6420	5090	4	6220	5090	3	5840	5090	3	5760	5090
51	1	5300	5240	0	5240	5240	0	5240	5240	1	5300	5240	1	5300	5240	0	5240	5240	1	5300	5240
52	3	111	74	0	74	74	0	74	74	0	74	74	0	74	74	0	74	74	1	80	74
53	1	7130	6650	3	8977	6650	0	6650	6650	2	9870	6650	1	9530	6650	5	9620	6650	1	8010	6650
54	2	36250	13200	1	17800	13200	0	13200	13200	0	13200	13200	0	13200	13200	0	13200	13200	0	13200	13200
55	3	1405	850	3	1405	ss850	0	850	850	0	850	850	0	850	850	0	850	850	0	850	850
56	2	959	855	0	855	855	0	855	855	3	990	855	3	990	855	0	855	855	1	900	855
57	10	11260	5650	7	9340	5650	2	5790	5650	4	6150	5650	3	6110	5650	4	7690	5650	5	7080	5650
58	1	560	530	0	530	530	0	530	530	1	590	530	1	570	530	0	530	530	1	560	530
59	4	65500	37500	0	37500	37500	0	37500	37500	1	40000	37500	1	40000	37500	0	37500	37500	1	38500	37500
60	2	1660	1280	0	1280	1280	0	1280	1280	0	1280	1280	0	1280	1280	1	1310	1280	1	1210	1280
61	1	690	600	1	605	600	0	600	600	3	735	600	2	705	600	2	710	600	1	690	600
62	2	1015	743	2	814	743	1	779	743	2	834	743	1	779	743	1	807	743	2	1110	743
63	3	4065	2635	1	2760	2635	0	2635	2635	1	2670	2635	1	2670	2635	0	2635	2635	0	2365	2365
64	3	116	100	3	112	100	0	101	100	3	116	100	3	116	100	0	100	100	4	105	100
65	5	292	184	0	184	184	0	184	184	1	216	84	1	216	84	0	184	184	0	184	184
66	3	206	134	1	161	134	0	134	134	1	161	134	0	134	134	1	152	134	1	161	134
67	3	2897	2221	1	2238	2221	0	2221	2221	2	2688	2221	1	2459	2221	1	2238	2221	2	2688	2221

Key:

P: problem number,
NO IT: Number of iterations

M1: Northwest corner method
M2: Least cost method

M5: Column minimum method
M6: Russell’s method

INBFS: Initial basic feasible solution
OPTSL: Optimal solution

M3: Vogel's method
M4: Row-column minimum method

M7: Row minimum method

P	M1			M2			M3			M4			M5			M6			M7		
	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL	No IT	INBFS	OPTSL
68	2	262	94	0	94	94	0	94	94	0	94	94	0	94	94	0	94	94	0	94	94
69	3	390	355	1	375	355	0	355	355	0	355	355	1	375	355	1	485	355	2	390	355
70	1	765	760	0	760	760	0	760	760	1	795	760	0	760	760	0	760	760	1	795	760
71	4	865	715	0	715	715	0	715	715	1	815	715	0	715	715	0	715	715	0	715	715
72	4	3700	1800	0	1800	1800	0	1800	1800	2	3500	1800	2	3500	1800	0	1800	1800	1	2200	1800
73	2	102	76	2	83	76	1	80	76	2	85	76	2	111	76	1	92	76	2	79	76
74	2	128	114	3	156	114	0	114	114	1	162	114	2	118	114	0	114	114	3	152	114
75	5	1076	770	1	824	770	0	770	770	2	801	770	1	824	770	0	770	770	1	824	770
76	21	18200	10050	3	10400	10050	3	10400	10050	6	11090	10050	7	11170	10050	3	10600	10050	6	11070	10050
77	5	259	164	1	170	164	0	164	164	3	167	164	1	170	164	0	164	164	3	199	164
78	19	13850	850	3	1030	850	1	850	850	1	990	850	4	2240	850	3	1010	850	6	3030	850
79	4	228	143	1	163	143	0	143	143	1	163	143	2	193	143	0	143	143	0	143	143
80	2	310	180	0	180	180	0	180	180	0	180	180	0	180	180	1	200	180	0	180	180
81	2	199	193	1	196	193	0	193	193	1	196	193	0	193	193	0	196	193	1	196	193
82	2	5700	5020	1	5560	5020	1	5560	5020	2	6220	5020	2	6620	5020	0	5020	5020	1	5560	5020
83	3	4740	2980	0	2980	2980	0	2980	2980	2	3100	2980	1	3430	2980	2	3200	2980	2	3100	2980
84	2	5020	4780	1	4860	4780	0	4780	4780	2	7560	4780	1	5040	4780	0	4780	4780	1	4860	4780
85	4	1260	1030	0	1030	1030	0	1030	1030	1	1170	1030	0	1030	1030	0	1030	1030	1	1190	1030
86	2	97	59	1	62	59	0	59	59	0	59	59	0	59	59	1	62	59	1	62	59
87	2	176	149	1	150	149	0	149	149	1	150	149	0	149	149	0	149	149	1	159	149
88	5	1950	1580	3	1870	1580	0	1580	1580	2	1690	1580	2	1750	1580	0	1580	1580	2	1710	1580
89	4	3528	2424	4	2968	2424	0	2424	2424	4	2968	2424	1	2712	2424	0	2424	2424	4	2968	2424
90	3	2680	1900	3	2050	1900	0	1900	1900	1	1950	1900	0	1900	1900	0	1900	1900	2	2050	1900
91	3	8475	7630	1	7665	7630	1	7665	7630	0	7630	7630	1	7665	7630	0	7630	7630	2	8010	7630
92	2	12200	12075	1	12825	12075	0	12075	12075	2	13175	12075	0	12075	12075	0	12075	12075	2	13175	12075
93	2	4800	4120	1	4660	4120	0	4120	4120	2	4320	4120	1	5320	4120	0	4120	4120	1	4660	4120

94	3	472	301	0	301	301	0	301	301	1	316	301	2	341	301	0	301	301	2	310	301
95	3	2730	2090	1	2360	2090	0	2090	2090	1	2360	2090	1	2360	2090	0	2090	2090	1	2360	2090
96	3	237	209	1	236	209	0	209	209	1	236	209	1	236	209	2	268	209	1	2236	209
97	3	4925	3300	2	3650	3330	1	3600	3330	0	3330	3330	0	3330	3330	3	3850	3330	1	3400	3330
98	2	4160000	3320000	2	4160000	3320000	0	3320000	3320000	2	3620000	3320000	0	3320000	3320000	1	3520000	3320000	0	3320000	3320000
99	3	55500	39500	1	42000	39500	1	42000	39500	0	39500	39500	0	39500	39500	2	48500	39500	1	42000	39500
100	3	330	2235	0	235	235	0	235	235	3	385	235	3	285	235	1	254	235	0	235	235

Key:

P: problem number,
NO IT: Number of iterations
INBFS: Initial basic feasible solution
OPTSL: Optimal solution

M1: Northwest corner method
M2: Least cost method
M3: Vogel's method
M4: Row-column minimum method

M5: Column minimum method
M6: Russell's method
M7: Row minimum method

APPENDIX 2

STATISTICAL ANALYSIS ONE-WAY ANOVA

Table 2: Analysis of the results using SAS version 9.0

Dependent Variable: Number of iterations.

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	916.33777143	152.722857	24.00	0.0001
Error	693	441.650000	6.364574		
Total	699	5326.987143			

Multiple Comparison: Means with the same letter are not significantly different.

	Duncan Grouping		Mean	N	MT
		A	4.53000	100	M1
		B	1.8800	100	M4
		B			
		B	1.8400	100	M5
		B			
	C	B	1.8100	100	M7
	C	B			
	C	B	1.6400	100	M2
C					
	C	D	1.1200	100	M6
		D			
		D	0.65000	100	M3

Note that even though the mean for M6 above is 1.12 and the mean for M3 is 0.65 but it is shown that there is no significant difference between them. This is in order and can be clearly seen from Appendix 3 where the p value between the methods is 0.188 implying that there is no significant difference between them.

APPENDIX 3

Table 3: Table of Multiple Comparison between Methods using SPSS version 17.0

					95% Confidence Interval	
(I) var	(J) var	Mean Difference (I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
M1	M2	2.890000*	.35678	.000	2.1895	3.5905
	M3	3.88000*	.35678	.000	3.1795	4.5805
	M4	2.65000*	.35678	.000	1.9495	3.3505
	M5	2.69000*	.35678	.000	2.9895	3.3905
	M6	3.41000*	.35678	.000	2.7095	4.1105
	M7	2.72000*	.35678	.000	2.0195	3.4205
M2	M1	-2.89000*	.35678	.000	-3.5905	-2.1895
	M3	.99000*	.35678	.006	.2895	1.6905
	M4	-.24000	.35678	.501	-.9405	.4605
	M5	-.20000	.35678	.575	-.9005	.5005
	M6	.52000	.35678	.145	.1805	.1.22005
	M7	-.17000	.35678	.634	-.8705	.3.5305
M3	M1	-3.88000*	.35678	.000	-4.5805	-3.1795
	M2	-.99000*	.35678	.006	-1.6905	-.2895
	M4	-1.23000*	.35678	.001	-1.9305	-.5295
	M5	-1.19000*	.35678	.001	-1.8905	-.4895
	M6	-.47000	.35678	.188	-1.1705	.2305
	M7	-1.16000*	.35678	.001	-1.8605	-.4595
M4	M1	-2.65000*	.35678	.000	-3.3505	-1.9495
	M2	.24000	.35678	.501	-.4605	.9405
	M3	1.23000*	.35678	.001	.5295	1.9305
	M5	.04000	.35678	.911	-.6605	.7405
	M6	.76000*	.35678	.034	.0595	1.4605
	M7	.07000	.35678	.845	-.6305	.7705
M5	M1	-2.69000*	.35678	.000	-3.3905	-2.9895
	M2	.20000	.35678	.575	-.5005	.9005
	M3	1.19000*	.35678	.001	.4895	1.8905
	M4	-.04000	.35678	.911	-7405	.6605
	M6	.72000*	.35678	.044	-.0195	1.4205
	M7	-.03000	.35678	.933	-.6705	.7305
M6	M1	-3.41000*	35678	.000	-4.1105	-2.7095
	M2	-.52000	35678	.145	-1.2205	.1805
	M3	.47000	35678	.188	-.2305	1.1705
	M4	-.76000*	35678	.034	-1.4605	-.0595
	M5	-.72000*	35678	.044	-1.4205	.0195
	M7	-.69000	35678	.054	-1.3905	.0105
M7	M1	-2.72000*	.35678	.000	-3.4205	-2.0195
	M2	.170000	.35678	.634	-.5305	.8705
	M3	1.16000*	.35678	.001	.4595	1.8605
	M4	-.07000	.35678	.845	-.7705	.6305
	M5	-.03000	.35678	.933	-.7305	.6705
	M6	.69000	.35678	.054	.0105	1.3905

*. The mean difference is significant at the 0.05 level.

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