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Modeling Economic Planning for Exhaustible Natural Resources

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Abstract

Petroleum economics is a key component of any field development plan (FDP) for crude oil and natural gas fields having finite life. This paper presents an analytical equation to model the relationship between initial speculative fund(s) and investment cost(s) in a project with finite life. We define a utility function for three categories of risk. The convolution of the utility function and decline income is our objective function to be maximized subject to the rate of increase of capital. The results show that the return on investments will have the characteristic exponential decline.

1. Introduction:

Crude oil and natural gas are often classified as exhaustible natural resources. Our oil and gas fields are known to have finite lives, determined by the so-called economic (volumetric) rate(s). In the development of these fields, asset managers are faced with basic risks and uncertainties. Classical examples are economic risks and uncertainties connected with taking decisions on field development, with the aim of arriving at optimum economic outcomes for equity holders. Total capital expenditure, CAPEX, with an initial speculative capital, is one item in field development that is loaded with risks and uncertainties.

- What cost of capital is the optimum that a company can bear for field development, with respect to the reserves of the exhaustible natural resources – crude oil and natural gas?
- How will this change with respect to activities in the business environment - financial markets, etc.?
- How will the change affect the economics of the field development?
- Will the expected net present value (worth) of the assets – the crude oil and natural gas, the exhaustible natural resources, be achieved to justify value to equity holders?
- What level of losses will the company (Speculator) be prepared to bear?
- Should the company conduct the field development on basis of expected monetary reward alone without regards to cost of failure?
- What parameter(s) determine(s) the optimum price for an exhaustible resource, such as crude oil and/or natural gas?

These are some of the questions that must be answered in the economic planning for the development of exhaustible natural resources, such as crude oil and natural gas.

Ultimately, a decision on the field development must be made, by weighing the advantage of possible gains (total accruable revenue) against the consequences of possible losses (total expenditure, CAPEX and OPEX). Both the objective(s) and resources of the company must be taken into consideration in any planning and evaluation.

Total revenue (Income) is dependent on the cumulative production volumes of the crude oil and natural gas assets. Cumulative production is highly dependent on reservoir pressures, decline characteristics and reservoir/fluid properties.

Various authors [1, 2, 3] have studied reserve calculation and production decline characteristics of exhaustible natural resources, such as crude oil. A key element to field performance forecasting is the quantification of hydrocarbons that are both economically and technically recoverable. The decline characteristics have a direct relationship to revenues. Arps[1] concluded that revenues from non renewable resources belong to the family of hyperbolic functions. Alderman[4] stressed that production from reservoirs is governed by its decline rate and postulated that decline is exponential. The mathematical simplicity of this assumption is that it gives easy analysis for exploratory programs where decisions are made based on the performances of similar ventures[5].

Using statistical decision methods, Mcray [6] presented an equation to analyze the fractional participation in a given venture, a familiar mode of operation by IOCs in developing exhaustible resources.

In this work, we formulate a model with risk characterization in the exploitation of exhaustible natural resources, such as crude oil and natural gas being a function of utility function..

2. Mathematical Formulation

Nomenclature

$y =$ Monetary value of reserve at time t

$C =$ Investment cost

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D = decline constant

$i =$ Rate of return on investment

$t_f =$ Economic life of the field

$t =$ Time.

$y(0) = M$, initial Asset value

$U(C) =$, Utility function

Utility theory recognizes the subject of risk preference, which is an integral part of decision making in petroleum companies. This guarantees results that give more realistic measures of value to the company. Preference is the attitude towards different dimensions of thoughts, based on objectives and values of a decision maker. This attitude of a decision maker about money is influenced by such factors as his business environment and the general policy of the company at a given time. Thus, risk preference is the attempt to incorporate this attitude and feeling about money into a quantitative parameter called expected utility. This parameter will have all the characteristics of money and furthermore the benefit of recognizing the specific attitude of the decision maker about money. This concept will address situations such as when the expected monetary return is enough to guarantee profit while there is loss to bear from failure.

The problem definition is to describe mathematically, the plan of investment that will maximize the total utility of a petroleum company, having a fixed initial speculative reserve, M. We make the following assumptions:

- The company has no other income than that obtained through the investment.
- The maximum utility at a given time is a certain function of the rate of expenditure.
- The income from this investment can be modeled by rate of decline formulation(s).
- The rate of return on investment must justify the company investment rate

The growth of a decision maker's capital depends on the present state of his capital and the expenditure. The initial invested capital can be recovered through amortization. That is:

$$\frac{dy}{dt} = iy(t) - C(t). \qquad (1)$$

If we define net income as $v = iy - C$, then the rate of growth of capital is

$$\frac{dv}{dt} = iv - \frac{dc}{dt}. \qquad (2)$$

The canonical solution to eq.(2) can be readily obtained by describing integration factor as

$$v(t) = -e^{iT}\int_0^T e^{-i\tau}\frac{dc}{d\tau}d\tau + Ae^{it}. \qquad (3)$$

Where A is a constant of integration and can be evaluated at time zero. Hence $A = -c(0)$ and the solution to eq.(3) becomes

$$v(t) = c(t) + e^{it}\int e^{-i\tau}c(\tau)d\tau - c(0). \qquad (4)\cdot$$

Obviously, our capital base at any time after the investment is

$$iy(t) = e^{it}\int e^{-i\tau}c(\tau)d\tau - c(0). \qquad (5)$$

Equation (5) gives analysis of our capital base at time t as difference in the cumulative expenditure at that time minus initial investment discounted at interest rate i.

Now if D is the decline rate of pattern of earning, then our objective function is to maximize

$$\int_0^{t_f} e^{-D\tau}U(c(\tau))dt. \qquad (6)$$

And by defining $c(t) = -\frac{dy}{dt} + iy$, our objective function becomes a functional U of growth rate and capital base. We then require maximizing

$$\int_0^{t_f} e^{-D\tau}U\left(\frac{dy}{dt} - iy\right)d\tau. \qquad (7)$$

With the initial and final state of income as $y(0) = M$, $y(t_f) = 0$

The calculus of variations provides a method of finding the function $y(t)$ that satisfies a problem of this type. Specifically, we aim to find the function that makes the integral stationary, which is the value of the integral that makes the function a local maximum.

We describe a functional with first order variation in y. The Euler-Lagrange's formula for analyzing such integral is

$$\frac{d}{dt}\left(e^{-Dt}\frac{\partial U\left(iy(t)-\frac{dy(t)}{dt}\right)}{\partial y'}\right)=e^{-Dt}\frac{\partial U\left(iy-dy/dt'\right)}{\partial y}. \tag{8}$$

This equation is a differential equation for $y(t)$ if the form of U is known. It follows that

$$ie^{-Dt}U'(iy-y')=-\frac{d}{dt}e^{-Dt}U'(iy-y'). \tag{9}$$

$$ie^{-Dt}U'(iy-y')=DU'-\frac{d}{dt}e^{-Dt}U'(iy-y') \tag{10}$$

Equation 10 simplify to

$$(i-D)U'(C)=-\frac{d}{dt}U'(C). \tag{11}$$

By variable separable

$$U'(C)=U'(0)e^{-(D-i)t}. \tag{12}$$

From Bernoulli's assertion gives a simple utility function that the intuitive notion of diminishing marginal enjoyment in form of decreasing $U'(C)$ and infinite marginal enjoyment at zero expenditure is

$$U(C(t))=2\sqrt{C(t)}. \tag{13}$$

And

$$U'(C(t))=1/\sqrt{C(t)}. \tag{14}$$

Substituting in eqn. 12,

$$C(t)=C(0)e^{-2(D-i)t)}. \tag{15}$$

Thus,

$$\frac{dy}{dt}-iy=-C(0)e^{-2(D-i))t}. \tag{16}$$

The solution is

$$y(t)=e^{it}y(0)+\frac{C(0)}{i-2D}\left[e^{it}-e^{-2(D-i)t}\right]. \tag{17}$$

Now at $t=t_f$, $y(t_f)=0$, therefore, equation (17) give $C(0)$ as $C(0)=\frac{(2D-i)y(0)}{1-e^{-(2D-i)t_f}}$. (18)

Define the following dimensionless variables with reference to initial capital of the investor as.

$$C_d(t)=\frac{c(t)}{y(0)};\quad \text{and, } y_d=\frac{y(t)}{y(0)} \text{ from these we obtain}$$

$$y_d=\left(1-\frac{C_D(0)}{2D-i}\right)e^{it}+\frac{C_d(0)}{2D-i}e^{-2(D-i)t}. \tag{19}$$

And dimensionless initial investment is

$$C_D(0)=\frac{(2D-i)}{1-e^{-(2D-i)t_f}}. \tag{20}$$

Thus utility function of initial investment is an inverse of equation (20).

3. Results and Discussion

The approved cost for an oil and gas property would be for management of available resources such as geology, engineering, laboratory and pilot studies among other, and make decision about what should be the value of resource. For undiscounted values of revenue, the cost of the field is independent of the rate of return, i. In this case, equation 20 shows that the cost of the field should depend upon the decline constant, and from equation 19, the revenue net revenue depends as well on decline constant. Using this equation and estimates of decline constant will provide required relation between value and initial speculating cost.

$$y_d=\left(1-\frac{C_D(0)}{2D}\right)+\frac{C_d(0)}{2D}e^{-2(D)t}. \tag{21}$$

Table 1 and 2 have data generated from equation 19 for cases when the parameter $2D-i<0$ and $2D-i>0$, respectively. The profiles are such that one is convex and the other is concave. Figures three and four represent the cost per unit reserve for same condition. For the favorable case, the cost increases initially and drops down to zero, while it never reach zero for other.

4. Conclusion

We clearly demonstrated that rate of return and decline constant are two fundamental factors that govern the value of oil and gas venture with outcomes that are deterministic [7]. We employed optimization principle called Euler-Lagrange extrema technique where the class of admissible variations is in the subspace of life of the venture, $[0, t_F]$. Here whole series of control action predetermined through a wide range of available data for field development.

Planning is the problem of determining the optimal procedure for attaining a set of objectives. The object in the case is the pattern of earning for a given field that has finite life and the capital must be available for reinvestment. Thus we conclude that value of recoverable reserve follows Alderman's rule. Thus, this model can be used in bid evaluation and production planning whose specializations are based on minimum cost problems. It has little concerns with the operating expenses. These expenses always vary with the unit of production and analysis of these costs is dependent on local conditions and legislation. Necessary costs such as transportation cost, production and severance taxes, and royalty payment such as property and ad-valorem taxes are time dependent, but can become fixed after a short interval of time.

Table1: Representative data for $D < i/2$

TIME	res @ -1	res @ -.75	res @ -.5	res @ -.25
0	0.00	0	0	0
1	1.72	1.4893	1.2974	1.1361
2	6.39	4.6423	3.4366	2.5949
3	19.09	11.3170	6.9634	4.4680
4	53.60	25.4474	12.7781	6.8731
5	147.41	55.3614	22.3650	9.9614
6	402.43	118.6895	38.1711	13.9268
7	1095.63	252.7550	64.2309	19.0184
8	2979.96	536.5717	107.1963	25.5562
9	8102.08	1137.4117	178.0343	33.9509
10	22025.47	2409.3899	294.8263	44.7300
11	59873.14	5102.1678	487.3839	58.5705
12	162753.79	10802.7786	804.8576	76.3421
13	442412.39	22870.9717	1328.2833	99.1614
14	1202603.28	48419.3369	2191.2663	128.4618
15	3269016.37	102505.2264	3614.0848	166.0843

Table2: Representative data for $D > i/2$

TIME	res @ .125	res @ .25	res @ .5	res @ .75	res @ 1	res @ 1.25
0	0	0	0	0	0	0
1	0.9400	0.8848	0.7869	0.7035	0.6321	0.5708
2	1.7696	1.5739	1.2642	1.0358	0.8647	0.7343
3	2.5017	2.1105	1.5537	1.1928	0.9502	0.7812

4	3.1478	2.5285	1.7293	1.2670	0.9817	0.7946
5	3.7179	2.8540	1.8358	1.3020	0.9933	0.7985
6	4.2211	3.1075	1.9004	1.3185	0.9975	0.7996
7	4.6651	3.3049	1.9396	1.3263	0.9991	0.7999
8	5.0570	3.4587	1.9634	1.3300	0.9997	0.8000
9	5.4028	3.5784	1.9778	1.3318	0.9999	0.8000
10	5.7080	3.6717	1.9865	1.3326	1.0000	0.8000
11	5.9773	3.7443	1.9918	1.3330	1.0000	0.8000
12	6.2150	3.8009	1.9950	1.3332	1.0000	0.8000
13	6.4247	3.8449	1.9970	1.3333	1.0000	0.8000
14	6.6098	3.8792	1.9982	1.3333	1.0000	0.8000
15	6.7732	3.9059	1.9989	1.3333	1.0000	0.8000

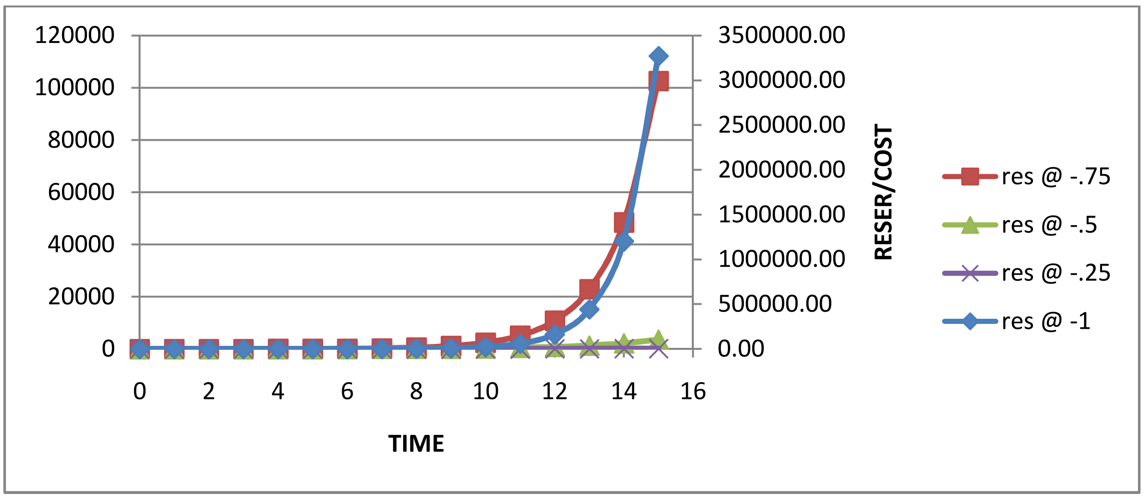

Fig1: plot of reserve value-cost ratio vs Time, for $D < i/2$

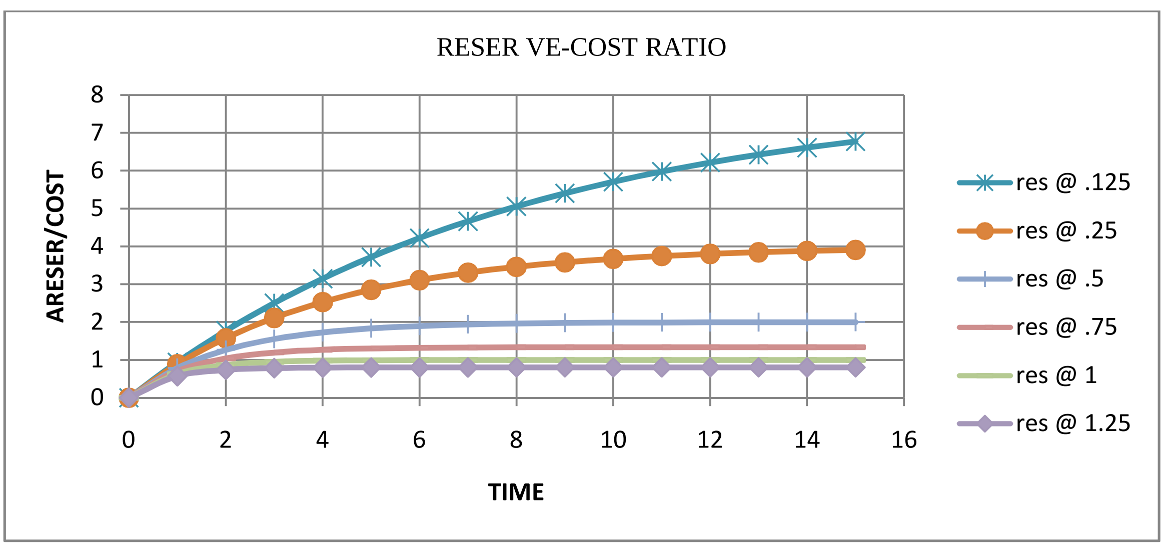

Fig2: plot of reserve value-cost ratio vs Time, for $D < i/2$

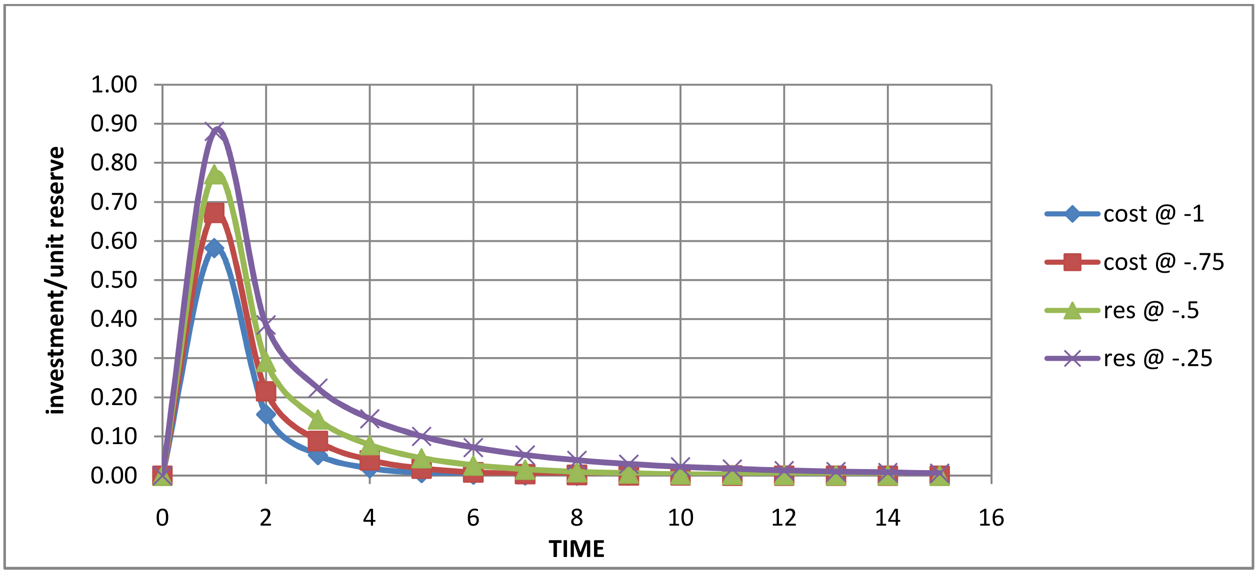

Fig3: plot of reserve value-cost ratio vs Time, for $D < i/2$

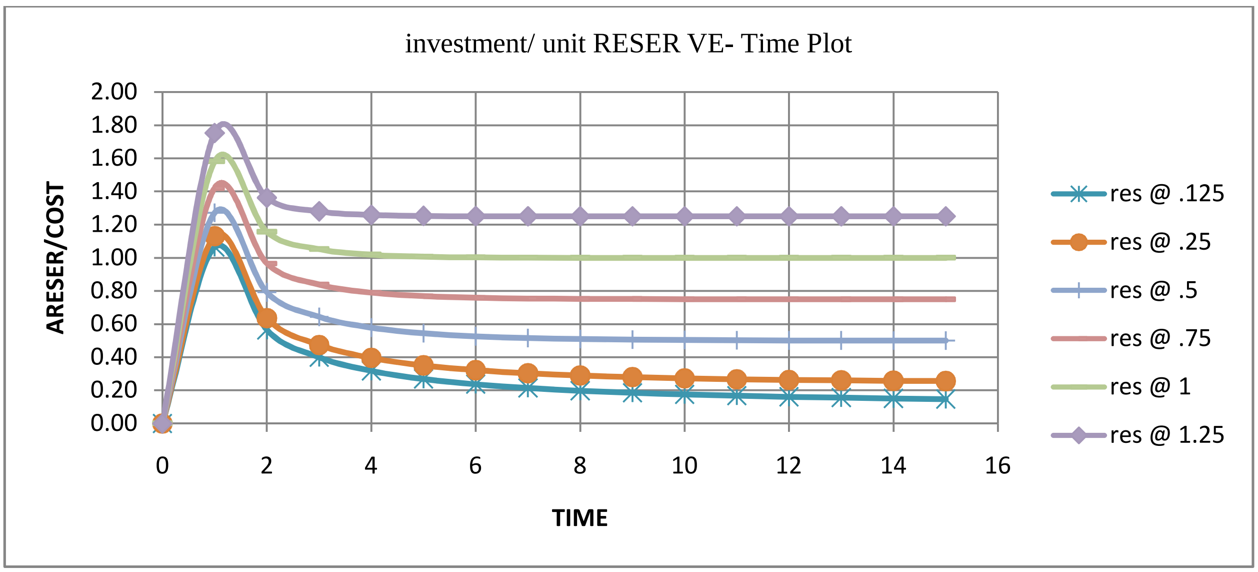

Fig4: plot of reserve value-cost ratio vs Time, for $D < i/2$

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