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Mathematical Modelling of Effect of Ambient Temperature and Relative Humidity on Soil Surface Temperature during Dry Season in Abeokuta, South – Western Nigeria

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Abstract

Temperature distributions on the soil surface strongly depend on the state of the processes of mass and energy exchanges (radiation and convection, evaporation and water condensation, supply of water through precipitation and gaseous exchange). It was assumed that soil medium is homogeneous and parameters describing this medium are changeless in the whole of its volume except that they depend on soil temperature and humidity. This work examines the effect of Ambient Temperature and Relative Humidity on Soil Surface Temperature during Dry Season. The data obtained from the experiment were used to generate a model which can be used to predict the soil surface temperature during the dry season in Abeokuta, South – Western Nigeria once the ambient temperature and the relative humidity are known. The chi-square test showed that there was no significant difference ($p>0.05$) between the expected and observed data. The coefficient of determination (r^2) showed that 92.89% of the experimental data were predicted by the model. The model developed in this work enabled us to use simulation prediction as the basis for temperature determination, which otherwise would be difficult or impossible to perform.

Keywords: Ambient Temperature, Relative Humidity, Soil Surface, Environmental Conditions.

1.0 Introduction:

The environment in which mines are placed is extremely variable in terms of climate, vegetation, soil type, depth of ground water table, and topography. For example, the three countries that have the largest average number of mines deployed per square mile are Bosnia-Herzagovina in a temperate zone, Cambodia in the humid tropics, and Egypt in an arid desert. Variations in environmental conditions influence sensor performance because in general, landmine sensors exploit soil and environmental conditions to discern between mines and other objects. Little effort has been made on evaluating the environmental conditions that affect sensor performance. The performance of sensors based on radar and infrared imaging is expected to vary with soil and environmental conditions [1]. Surface evaporation depends on relative humidity, temperature and wind speed. Evaporation may change surface temperature gradient [6].

Temperature distributions on the soil surface strongly depend on the state of the processes of mass and energy exchanges (radiation and convection, evaporation and water condensation, supply of water through precipitation and gaseous exchange). It was assumed that soil medium is homogeneous and parameters describing this medium are changeless in the whole of its volume except that they depend on soil temperature and humidity [4].

2.0 Empirical Models

An empirical model is one which is derived from and based entirely on data. In such a model, relationships between variables are derived by looking at the available data on the variables and selecting a mathematical form which is a compromise between accuracy of fit and simplicity of mathematics. It will always be possible to arrange a perfect fit, if necessary, by using a sufficiently complicated mathematical formular, but this is hardly a sensible approach. What is usually required is the simplest formular which will give an adequate fit. The important distinction is that empirical models are not derived from assumptions concerning the relationships between variables, and they are not based on physical laws or principles. Quite often, empirical models are used as 'submodels' or parts of a more complicated model. When we have no principles to guide us and no obvious assumptions suggest themselves, we may (with justification) turn to the data to find how some of our variables are related. Therefore the models derived in this work are empirical models.

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2.1 Least Square Approximation

The basic idea of Least Square Approximation is to fit a polynomial function P(x) to a set of data (x,y) having a theoretical solution

$$y = f(x). \quad (1)$$

Many problems arise in Engineering and Science where the dependent variable is a function of two or more independent variables, for example,

$$z = f(x,y) \quad (2)$$

is a two-variable, or bivariate function. Least squares multivariate approximation is used to solve this type of problem.

Given N data points, (x_i, y_i, z_i), fit the best linear bivariate polynomial through the set of data. Consider the linear polynomial:

$$z = a + bx + cy \quad (3)$$

The sum of the squares of the deviations is given by

$$S(a, b, c) = \Sigma(e_i)^2 = \Sigma(Z_i - a - bx_i - cy_i)^2 \quad (4)$$

The function S(a, b, c) is a minimum when

$$\frac{\delta S}{\delta a} = \Sigma 2\left(Z_i - a - bx_i - cy_i\right)\left(-1\right) = 0 \quad (5a)$$

$$\frac{\delta S}{\delta b} = \Sigma 2\left(Z_i - a - bx_i - cy_i\right)\left(-x_i\right) = 0 \quad (5b)$$

$$\frac{\delta S}{\delta c} = \Sigma 2\left(Z_i - a - bx_i - cy_i\right)\left(-y_i\right) = 0 \quad (5c)$$

Dividing equations (5) by 2 and rearranging yields the normal equations:

$$aN + b\sum x_i + c\sum y_i = \sum Z_i \quad (6a)$$

$$a\sum x_i + b\sum x_i^2 + c\sum x_i y_i = \sum x_i Z_i \quad (6b)$$

$$a\sum y_i + b\sum x_i y_i + c\sum y_i^2 = \sum y_i Z_i \quad (6c)$$

Equations (6) can be solved for a, b, and c by Gauss elimination.

A linear fit to a set of bivariate data may be inadequate. Consider the quadratic bivariate polynomial:

$$z = a + bx + cy + dx^2 + ey^2 + fxy \quad (7)$$

The sum of the squares of the deviations is given by

$$S(a,b,c,d,e,f) = \sum (Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)^2 \quad (8)$$

The function S(a, b, …, f) is a minimum when

$$\frac{\delta S}{\delta a} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)\left(-1\right) = 0 \quad (9a)$$

$$\frac{\delta S}{\delta b} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)\left(-x_i\right) = 0 \quad (9b)$$

$$\frac{\delta S}{\delta c} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)\left(-y_i\right) = 0 \quad (9c)$$

$$\frac{\delta S}{\delta d} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)\left(-x_i^2\right) = 0 \quad (9d)$$

$$\frac{\delta S}{\delta e} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)\left(-y_i^2\right) = 0 \quad (9e)$$

$$\frac{\delta S}{\delta f} = \sum 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-x_i y_i) = 0 \quad (9f)$$

Dividing equations (9) by 2 and rearranging yields the normal equations:

$$aN + b\sum x_i + c\sum y_i + d\sum x_i^2 + e\sum y_i^2 + f\sum x_i y_i = \sum Z_i \quad (10a)$$

$$a\sum x_i + b\sum x_i^2 + c\sum x_i y_i + d\sum x_i^3 + e\sum x_i y_i^2 + f\sum x_i^2 y_i = \sum x_i Z_i \quad (10b)$$

$$a\sum y_i + b\sum x_i y_i + c\sum y_i^2 + d\sum x_i^2 y_i + e\sum y_i^3 + f\sum x_i y_i^2 = \sum y_i Z_i \quad (10c)$$

$$a\sum x_i^2 + b\sum x_i^3 + c\sum x_i^2 y_i + d\sum x_i^4 + e\sum x_i^2 y_i^2 + f\sum x_i^3 y_i = \sum x_i^2 Z_i \quad (10d)$$

$$a\sum y_i^2 + b\sum x_i y_i^2 + c\sum y_i^2 + d\sum x_i^2 y_i^2 + e\sum y_i^4 + f\sum x_i y_i^3 = \sum y_i^2 Z_i \quad (10e)$$

$$a\sum x_i y_i + b\sum x_i^2 y_i + c\sum x_i y_i^2 + d\sum x_i^3 y_i + e\sum x_i y_i^3 + f\sum x_i^2 y_i^2 = \sum x_i y_i Z_i \quad (10f)$$

Equations (10) can be written as the matrix equation

$$Ac = b \quad (11)$$

Where A is the 6 x 6 matrix, c is the 6 x 1 column vector of polynomial coefficients (i.e., a to f), and b is the 6 x 1 column vector of nonhomogeneous terms. The solution to equation (11) is

$$c = A^{-1}b \quad (12)$$

Where A^{-1} is the inverse of A. [2].

3.0 Materials and Methods

The equipment used in this work included a Hoboware data logger and sensors (temperature and relative humidity) manufactured in the United States of America by Onset Corporation. The experiment was carried out on a field in Lafenwa area of Abeokuta (latitude 7^o 3´ N and longitude 3^o 3´ E), Ogun State Nigeria.

This work aimed at examining the effect of Ambient Temperature and Relative Humidity on the Soil Surface Temperature in Abeokuta, South – West, Nigeria. The experiment was carried during the dry season in the area. One temperature sensor was laid on the soil surface to read the soil surface temperature while the two other sensors (ambient temperature and relative humidity) were hanged up to read the ambient temperature and the relative humidity respectively. The sampling interval is one second with logging interval of one hour.

4.0 Results and Discussion

The data from the experiment is presented in Table 1. A 6 x 6 matrix was generated from the data using equations (10) and the matrix solved using Microsoft Student Encarta.

Let the variables x, y, and z in equations (10) correspond to T_a (ambient temperature) P (relative humidity), and $T_{surface}$ (soil surface temperature) respectively. The matrix is presented in equation (13).

$$\begin{pmatrix} 24 & 728 & 1735 & 22548 & 133097 & 50805 \\ 728 & 22548 & 50805 & 713772 & 3787274 & 1518832 \\ 1735 & 50805 & 133097 & 1518832 & 10682848 & 3787274 \\ 22548 & 713772 & 1518832 & 23061809 & 109695879 & 46392887 \\ 133097 & 3787274 & 10682848 & 109695879 & 885414545 & 297251112 \\ 50805 & 1518832 & 3787274 & 46392887 & 297251112 & 10965879 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 825 \\ 25993 \\ 55787 \\ 836010 \\ 4053051 \\ 1691589 \end{pmatrix} \quad (13)$$

Solving the matrix equation (13) gives the values of *a* to *f* as,

$a = 21.2952355549017$, $b = -2.1152828101165$, $c = 1.0734156978063$,

$d = 0.0931357360374$, $e = -0.0010628278883$, $f = -0.0389548543642$

Substituting the values of *a* to *f* above into equation (7) yields

$$T_{surface}(P, T_a) = 21.2952355549017 - 2.11528281011165T_a + 1.0734156978063P + 0.0931357360374T_a^2 - 0.0010628278883P^2 - 0.0389548543642T_a P \quad (14)$$

Where $T_{surface}$ is the Soil Surface Temperature (in degree centigrade), T_a is Ambient Temperature (in degree centigrade) and P is the Relative Humidity (in percentage).

Evaluating equation (14) using the experimental data gives Table 2.

Table 1: Table of observed values of soil surface temperature as a function of ambient temperature and relative humidity

Time	Ambient Temp, °C	Relative Humidity, %	Surface Temp., °C
0	25.016	93.3	25.404
1	26.061	93.0	26.109
2	30.217	84.6	29.54
3	33.209	74.3	33.131
4	37.48	63.0	38.365
5	36.905	50.9	48.504
6	36.389	51.8	49.309
7	36.905	48.3	53.553
8	36.227	44.5	56.898
9	35.797	43.5	50.059
10	34.783	45.8	41.123
11	33.678	48.2	37.178
12	30.773	62.9	33.287
13	29.34	70.8	30.722
14	28.543	75.1	29.414
15	28.023	78.1	28.891
16	27.284	81.0	28.122
17	26.426	84.8	27.21
18	25.963	87.8	26.573
19	25.744	89.2	26.475
20	25.939	90.8	26.402
21	25.574	90.3	26.598
22	25.671	91.6	26.134
23	25.744	91.0	26.475

Table 2: Table of expected values of soil surface temperature as a function of ambient temperature and relative humidity

$T_{surface}(P, T_a)$ (°C)	Expected Value (°C)	Observed Value (°C)
$T_{surface}(93.3,\ 25.016)$	27.160	25.404
$T_{surface}(93.0, 26.061)$	26.130	26.109
$T_{surface}(84.6, 30.217)$	26.532	29.540
$T_{surface}(74.3, 33.209)$	32.047	33.131
$T_{surface}(63.0, 37.48)$	44.795	38.365
$T_{surface}(50.9, 36.905)$	49.29	48.504
$T_{surface}(51.8, 36.389)$	47.457	49.309
$T_{surface}(48.3, 36.905)$	50.511	53.553
$T_{surface}(44.5, 36.227)$	50.222	56.898
$T_{surface}(43.5, 35.797)$	49.405	50.059
$T_{surface}(45.8, 34.783)$	45.789	41.123
$T_{surface}(48.233.678)$	42.208	37.178

$T_{surface}(62.9, 30.773)$	32.812	33.287
$T_{surface}(70.8, 29.34)$	29.673	30.722
$T_{surface}(75.1, 28.543)$	28.441	29.414
$T_{surface}(78.1, 28.023)$	27.725	28.891
$T_{surface}(81.0, 27.284)$	27.258	28.122
$T_{surface}(84.8, 26.426)$	26.993	27.210
$T_{surface}(87.8, 25.963)$	26.903	26.573
$T_{surface}(89.2, 25.744)$	26.926	26.475
$T_{surface}(90.8, 25.939)$	26.562	26.402
$T_{surface}(90.3, 25.574)$	26.913	26.598
$T_{surface}(91.6, 25.671)$	26.677	26.134
$T_{surface}(91.0, 25.744)$	26.717	26.475

5.0 Chi – Square Test

Chi – Square distribution is one of the most widely used theoretical probability distributions in inferential statistics, e.g., in statistical significant tests. The best – known situations in which the chi – square distribution is used are the common chi – square tests for goodness of fit of an observed distribution to a theoretical one, and of the independence of two criteria of classification of qualitative data [7, 8].

According to [5], chi – square test is used to test if a sample of data came from a population with a specific distribution. Chi – Square is a family of distributions commonly used for significance testing. Pearson's chi – square is by far the most common type of chi – square significance test [3].

In this work, the Observed Values and Expected Values were compared and subjected to statistical analysis using Chi Square to test if their were significant difference between the observed data and the data from the theoretical models.

The chi – square was computed for the model in equation (14) using the formula below:

$$\chi^2 = \sum \frac{(Observed\ data - Expected\ data)^2}{Expected\ data} \sim \chi^2_{0.05, n-1} \qquad (15)$$

Where n = 24 (number of data points), and $\chi^2_{0.05, n-1}$ is the χ^2 tabulated which gives 35.507.

The chi – square calculated for the model was computed using equation (15) which gives 3.838151.

Comparing the χ^2 **Calculated** and the χ^2 **tabulated,** there was no significant difference between the expected and observed values.

6.0 Coefficient of Determination

Coefficient of Determination is the proportion of variability in a data set that is accounted for by the statistical model. It provides a measure of how well future outcomes are likely to be predicted by the model. It is used in the contest of statistical models whose main purpose is the prediction of future outcomes on the basis of other related information. [7, 8]. The values vary from 0 to 1.

The coefficient of determination was computed for the model. This was aimed at determining the percentage of the experimental data that was explained by the model. The formula used to calculate the coefficient of determination is

$$Coeeficient\ of\ Deter\min ation\ (R^2) = \left[\frac{\left[n\Sigma x^2 - (\Sigma x)^2\right]\left[n\Sigma y^2 - (\Sigma y)^2\right]}{\sqrt{\left[n\Sigma x^2 - (\Sigma x)^2\right]\left[n\Sigma y^2 - (\Sigma y)^2\right]}} \right]^2 \qquad (16)$$

Where x *is the observed value while y is the* exp*ected value*

The Coefficient of Determination for the model was computed and a value of 0.928913 was obtained, meaning that 92.89% of the observed data were explained by the model.

7.0 Conclusion

Surface evaporation depends on relative humidity, temperature and wind speed. Evaporation may change surface temperature gradient. This work examines the effect of Ambient Temperature and Relative Humidity on Soil Surface Temperature during Dry Season. The experimental data obtained from the experiment were used to generate a model which can be used to predict the soil surface temperature during the dry season in Abeokuta, South – Western Nigeria once the ambient temperature and the relative humidity are known.

The chi-square test showed that there was no significant difference ($p>0.05$) between the expected and observed data. The coefficient of determination (r^2) showed that 92.89% of the experimental data were predicted by the model. The model developed in this work enabled us to use simulation prediction as the basis for temperature determination, which otherwise would be difficult or impossible to perform.

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