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The Effects of Viscosity and Radiation Absorption on Double-Diffusive Convection in a Porous Medium

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Abstract

This study is devoted to the problem of the effect of radiation absorption on the onset of thermosolutal convection in a horizontal fluid layer taking into consideration viscous effects when the fluid is heated from below in a horizontal porous layer. The radiative heat transfer is heated using the differential approximation for optically thin case in the energy equation. The linear stability analysis is used to show how radiation absorption affects the condition leading to onset of instability even in the present of viscosity. In the momentum equation for free-free boundaries it was seen that a positive increment in the radiation absorption parameter delayed the onset of instability.

1.0 Introduction:

Double-diffusive convective can occur when the fluid contains two (or more) components having different molecular diffusivities and the component make opposite contributions to the density gradient. The process which occurs in oceans is called thermosolutal. This situation can be observed of in wide range of oceanography, astrophysics, chemical engineering, spreading of pollutants, non- nuclear waste, solute intrusion in sediments in coastal environment, an in enhanced oil recovery system. [11] ,[4] and [10].There have been investigations on chemical stability of a horizontal fluid layer heated from below was carried out by [15] and [1]. There has been very recent research on double- diffusive by D'Hernoncouit *et al* and Hills [5]. This paper is aimed at the study of the onset of thermosolutal instability in the presence of viscosity in the momentum equation using linear stability theory and normal mode analysis. For free boundaries, it is seen that positive increment in the radiation absorption parameter delayed the onset of instability.

MATHEMATICAL FORMULATION

We consider a binary fluid layer of height $d > 0$, which are bounded between two horizontal parallel plates located at $\hat{z} = \pm\frac{d}{2}$ in a porous medium taking into consideration viscous effects. Temperature T_1 *and* T_2 ,Solutal mass concentration C_1 *and* C_2, are imposed both at bottom and top respectively.

We assume the density to be linearly dependent on the temperature T and concentration C as in [8], [9], [5] and [6].

$$\rho = \rho_0[1 - \beta_T(T' - T_0) + \beta_C(C' - C_0)], \tag{1}$$

where ρ_0 is the density of the fluid mixture at $T = T_0$ and $C = C_0$. T_0 and C_0 are initial temperature and concentration inside the fluid layer respectively . β_T and β_C are the thermal expansions for temperature and solutal concentration respectively. The horizontal coordinates is $\hat{x}$ and $\hat{z}$ for the vertical component which increases vertically upward. The usual Boussinesq approximation equation of flow, energy and solute transport is used taking into consideration the effect of radiation absorption with viscosity. ([12], [7] and [6])

$$\nabla' \cdot \vec{V}' = 0 \tag{2}$$

$$\frac{\mu}{k}\vec{V}' = -\nabla' P' - \rho g\vec{k} + J' \times B' \tag{3}$$

$$(\rho_0 C_p)_m \frac{\partial T'}{\partial t'} + (\rho_0 C_p)_f (\vec{V}' \cdot \nabla' T') = \kappa \nabla'^2 T' - \nabla' \cdot \vec{q}' \tag{4}$$

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$$\phi \frac{\partial C'}{\partial t'} + \vec{V}' \cdot \nabla' C' = D_m \nabla'^2 C' \quad (5)$$

Respectively and
Radiative heat flux

$$\nabla'^2 q_r - 3\alpha^2 q_r - 16\sigma\alpha T'^3 \nabla' T' = 0 \quad (6)$$

$$\nabla' \cdot \vec{V}' = 0 \quad (7)$$

$$\frac{\mu}{k}\vec{V}' = -\nabla' P' - g\rho_0[1 - \beta_T(T' - T_O) + \beta_C(C' - C_0)]\hat{k} - \sigma_c B_0^2(u\,\hat{i} + v\,\hat{j}) \quad (8)$$

$$\frac{\mu}{k}\vec{V}' = -\nabla'(P' + g\rho_0 z') + g\rho_0\beta_T(T' - T_0)\vec{k} - g\rho_0\beta_C(C' - C_0)\vec{k} - \sigma_c B_0^2(u\,\hat{i} + v\,\hat{j})$$

$$M\frac{\partial T'}{\partial t'} + \vec{V}' \cdot \nabla' T' = \frac{\kappa}{(\rho_0 C_P)_f}\nabla'^2\left(1 + \frac{16T_0^3\sigma}{3\alpha\kappa}\right)T' \quad (9)$$

$$\phi \frac{\partial C'}{\partial t'} + \vec{V}' \cdot \nabla' C' = D_m \nabla'^2 C' \quad (10)$$

Subject to the boundary condition

$$\left.\begin{aligned} w' = 0, T' = T_1, C' = C_1 \quad at \quad z' = -d/2, \\ w' = 0, T' = T_2, C' = C_2 \quad at \quad z' = d/2. \end{aligned}\right\} \quad (11)$$

where $\vec{V}'$ is the Darcy velocity; T' is temperature; g is acceleration due to gravity; ρ_0 is density; C_P is specific heat capacity; q_r is radiative heat flux; k is permeability of the porous medium; κ is thermal conductivity; P' is pressure; D_m is mass diffusitivity of species through the fluid saturated medium; α is absorbing coefficient or penetration depth, and σ is Stefan-Boltzmann constant.

The governing equation for radiative heat transfer (6) is generally nonlinear. From equation different limits may be considered depending on the absorption coefficient, α.

The governing non-dimensional equations for (7) to (10) together with the boundary condition of (11) are given by

$$\nabla \cdot \vec{V} = 0 \quad (12)$$

$$\frac{1}{P_r}\frac{\partial \vec{V}}{\partial t} = -\nabla P + \nabla^2 \bar{V} - \frac{1}{D_a}\bar{V} + R_a T\,\hat{k} - R_s C\,\hat{k} \quad (13)$$

$$\frac{1}{M}\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla)T = (1 + F^2)\nabla^2 T \quad (14)$$

$$\varepsilon\frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla)C = \frac{1}{Le}\nabla^2 C \quad (15)$$

With the corresponding boundary conditions:

$$w = 0, T = \pm 1/2, C = \pm 1/2 \text{ at } z = \mp 1/2. \quad (16)$$

Where $R_a = g\beta_T d^3(T_1 - T_2)$, thermal Rayleigh number; $R_S = g\beta_C d^3(C_1 - C_2)$, solutal Rayleigh number; $D_a = \frac{k}{d^2}$, Hartmann number; $F^2 = \frac{16\sigma T_0^3}{3\alpha K}$, radiation absorption parameter; $\varepsilon = \phi M$, normalized porosity, and $Le = \frac{\alpha}{D_m}$, Lewis number, $P_r = \frac{\gamma}{Ca\alpha_m}$, Prandtl number. This is a measure of the ratio of momentum boundary layer to thermal boundary layer of the fluid $P_r = \frac{\gamma}{Ca\alpha_m}$ P_r for most gases is of order unity and for liquids P_r may vary between 0.1 and 0.001 (Haghis and Brighton (1991)).

Linearization
We study the stability of the motionless solutions to the system (12)-(15) together with boundary conditions of (11) as given by equation (16).

$$\frac{d^2T_S}{dz^2}=0=\frac{d^2C_S}{dz^2}$$

$$T_s=-z,\ C_s=-z,\ P_s=1/2z^2(R_s-R_a),\vec{V}=0 \tag{17}$$

Accesing the stability of the steady state solutions (17), we define perturbation of the form by [2] and [3]:

$$\vec{V}=0+u, T=T_s+\theta, C=C_s+S, P=P_s+p \tag{18}$$

Where $\theta << T_S, \varphi << C_S$.
Substituting equation (18) into (12) to (15) and boundary condition of (17) and neglecting non-linear terms we have

$$\nabla\cdot u=0 \tag{19}$$

$$\frac{1}{p_r}\frac{\partial\vec{u}}{\partial t}=-\nabla P+\nabla^2\vec{V}-\frac{1}{D_a}\vec{u}+R_a\theta\vec{k}-R_s\varphi\vec{k}$$

$$\left(\frac{1}{P_r}\frac{\partial}{\partial t}-\nabla^2+\frac{1}{D_a}\right)u=-\nabla P+(R_a\theta-R_sS)\vec{k} \tag{20}$$

$$\frac{1}{M}\frac{\partial\theta}{\partial t}-w=(1+F^2)\nabla^2\theta \tag{21}$$

$$\varepsilon\frac{\partial\varphi}{\partial t}-w=\frac{1}{Le}\nabla^2\varphi \tag{22}$$

$$w=0,\theta=0=\varphi \text{ at } z=\pm\frac{1}{2} \tag{23}$$

The momentum equation (20) is reduced to a scalar equation by taking the double curl and using continuity equation (19) while keeping only the vertical components of the velocity, we obtain

$$\left(\frac{1}{P_r}\frac{\partial}{\partial t}-\nabla^2+\frac{1}{D_a}\right)\nabla^2 w=R_a\nabla_h{}^2\theta\vec{k}+R_S\varphi\vec{k} \tag{24}$$

Where $\nabla_h{}^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the Laplace operator.

EFFECT OF PERTURBATION AND NORMAL MODE ANALYSIS
The classical method of Chandrsekhar [2] for the linear stability analysis was followed. We study the stability of the normal mode disturbances by the choice of two dimensional waves in the horizontal plane xy-plane. Following [3] we employ the normal mode representation of the form:

$$w=W(z)f(x,y)e^{\Omega t},\quad \theta=\Theta(z)f(x,y)e^{\Omega t},\quad S=\Psi(z)f(x,y)e^{\Omega t}. \tag{25}$$

Putting equation (25) into equations (21), (22) and (24), and letting $D=\frac{\partial}{\partial z}$ we obtain

$$(D^2-a^2-\frac{\Omega}{M}\alpha)\Theta=-\alpha W_1, \tag{26}$$

$$(D^2-a^2-Le\Omega\varepsilon)\Psi=-LeW_1, \tag{27}$$

$$(D^2-a^2)\left(D^2-a^2-\frac{1}{D_a}-\frac{\Omega}{P_r}\right)W_1=a^2R_a\Theta-a^2R_s\Psi \tag{28}$$

subject to the condition

$$W=0,\Theta=\Psi \text{ at } z=\pm\frac{1}{2} \tag{29}$$

and $D^2W=0$ on a free-surface (30)

Here a^2 is the wave number arising from the separation of variables (Christopherson), and $\alpha = \dfrac{1}{1+F^2}$.

We study the stability of all possible disturbances for all wave numbers from the system (26)-(30) by eliminating Θ and Ψ from the system. To achieve this we operate on both sides of (28) [6].

With $(D^2 - a^2 - \dfrac{\Omega}{M}\alpha)\ (D^2 - a^2 - Le\Omega\varepsilon)$ and obtain

$$(D^2-a^2)(D^2-a^2-\frac{1}{D_a}-\frac{\Omega}{P_r})(D^2-a^2-\frac{\Omega\alpha}{M})(D^2-a^2-Le\varepsilon\Omega)W_1$$

$$=a^2R_a\left(D^2-a^2-\frac{\Omega\alpha}{M}\right).\left(D^2-a^2-Le\varepsilon\Omega\right)\Theta-a^2R_S\left(D^2-\varepsilon^2-\frac{\Omega\alpha}{M}\right)\left(D^2-a^2-Le\varepsilon\Omega\right)\psi$$

Equation (31) is subject to the conditions:

$$W_1 = D^2W_1 = D^4W_1 = ... = 0 \text{ at } z = \pm 1/2. \tag{32}$$

Following Chandrasekhar [2], Drazin and Reid [3] for a fluid layer with idealized free-free boundaries in which we seek solution of equation (31) in the form

$$W_1 = W_0 \sin \pi z \quad (w_0 \text{ is a constant}) \tag{33}$$

Substituting (33) into (31) yields

$$\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{D_a}+\frac{\Omega}{P_r}\right)\left(\pi^2+a^2+\frac{\Omega\alpha}{M}\right)\left(\pi^2+a^2+Le\varepsilon\Omega\right)=$$

$$a^2R_a\alpha\left(\pi^2+a^2+Le\varepsilon\Omega\right)-LeR_Sa^2\left(\pi^2+a^2+\frac{\Omega\alpha}{M}\right)$$

From which we obtain

$$R_a = \frac{\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{1}{D_a}+\frac{\Omega}{P_r}\right)\left(\pi^2+a^2+\frac{\Omega\alpha}{M}\right)\left(\pi^2+a^2+Le\varepsilon\Omega\right)+LeR_Sa^2\left(\pi^2+a^2+\frac{\Omega\alpha}{M}\right)}{\alpha a^2\left(\pi^2+a^2+Le\varepsilon\Omega\right)} \quad (34).$$

RESULTS AND DISCUSSION

The effects of radiation absorption with viscosity on double- diffusive convection in a porous medium, we set $R_a = R_{a_s}$ and $\Omega = 0$. $\dfrac{1}{Da} = \chi$ in equation (34) and obtain

$$R_{a_s} = \frac{\left(\pi^2+a^2\right)^3\left(\pi^2+a^2+\chi\right)+LeR_Sa^2\left(\pi^2+a^2\right)}{\alpha a^2\left(\pi^2+a^2\right)} \tag{35}$$

Following Chardrasekher [2], the critical value of the wave number $a = a_C$ is obtain by finding the minimum of $R_{a_s}(a)$.

$$\frac{\partial R_{a_s}(a_C)}{\partial a_C{}^2} = 0 \tag{36}$$

This yields a sixth order polynomial in a_C given by

$$2a_C{}^6 + (3\pi^2+\chi)a_C{}^4 - \pi^4(\pi^2+\chi) = 0 \tag{37}$$

$$2p^3 + (3\pi^2+\chi)p^2 - \pi^4(\pi^2+\chi) = 0$$

Where $\chi = 0,\ p = \dfrac{\pi^2}{2} \Rightarrow a_C = \dfrac{\pi}{\sqrt{2}}$.

let $\chi = \dfrac{1}{D_a} = 0.01$ in equation (37)

Since the wave number a_C is real and positive we take the critical wave number $a_C = 2.222 \approx \frac{\pi}{\sqrt{2}}$.

In the absence of F, χ

$$R_{a_S cri} = \frac{(\pi^2 + a_C^2)^3 + LeR_S a_C^2}{a_C^2}$$

$$= \frac{(\frac{3\pi^2}{2})^3 + LeR_S \frac{\pi^2}{2}}{(\frac{\pi^2}{2})}$$

$$= \frac{\frac{27\pi^6}{8} + LeR_S \frac{\pi^2}{2}}{(\frac{\pi^2}{2})} \quad = \frac{\frac{27\pi^{6\,2}}{8}}{(\frac{\pi^2}{2})} + LeR_S$$

This result is in agreement with the results of Israel-Cookey et al [6].

In the present of F, χ

$$R_{a_S cri} = \frac{(1+F^2)\left[\left(\pi^2 + \frac{\pi^2}{2}\right)^2\left(\pi^2 + \frac{\pi^2}{2} + \chi\right) + LeR_S \frac{\pi^2}{2}\right]}{\frac{\pi^2}{2}}$$

$$= \frac{2(1+F^2)}{\pi^2}\left[\frac{9\pi^2(3\pi^2 + 2\chi)}{8} + \frac{LeR_S \pi^2}{2}\right]$$

$$= \frac{(1+F^2)}{4}\left[9\pi^2(3\pi^2 + 2\chi) + 4LeR_S\right] \qquad (38)$$

The result of the numerical behaviour of viscosity and radiation absorption on the thermosolutal convection the software ‘‘mathematica’’ is used [14]. The following numerical results are presented in figure 1- 3 and it shows a linear relationship that exists between the thermal Rayleigh number Ra and solutal Rayleigh number Rs and also the relationship between thermal Rayleigh number and wave number k.

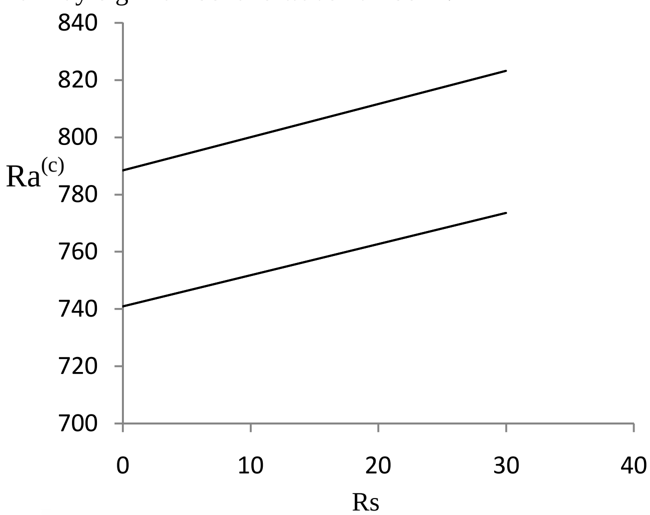

Figure 1: Dependence of Thermal Rayleigh Number $Ra^{(c)}$ on solutal Rs for various values of Radiation Absorption F.

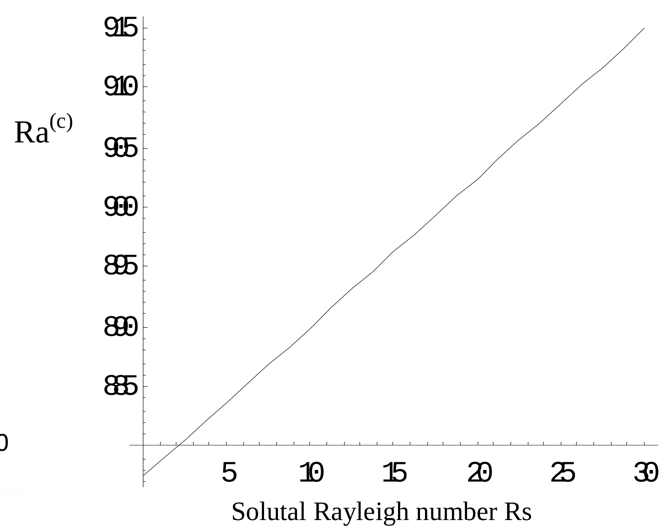

Figure 2: Dependence of Thermal Rayleigh Number $Ra^{(c)}$ on solutal Rayleigh Rs for Radiation Absorption F.

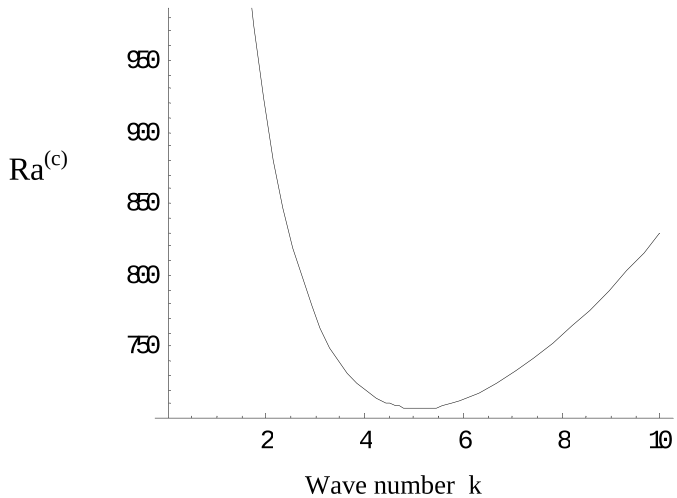

Figure 3: Dependence of Thermal Rayleigh number on wave number k

CONCLUSIONS

The effect of radiation absorption on the onset of thermosolutal convection with the effect of viscosity a linear relationship was establish with critical thermal Rayleigh number Ra and solutal Rayleigh number Rs for fixed value of Le and χ and of various values of F. The increase in radiation absorption parameter delayed the onset of instability in the system.

For given values of Rs, Ra increases quadratically with wave number (k) for increase values of F.

The onset of instability is presented in Figures 1 and 2 for various values of radiation absorption in the presence of viscosity. The linear relationship that exist between Ra and Rs is also shown in the figures.

1) a positive increase in radiation absorption parameter delays the onset of instability
2) the effect of viscosity unsettles the system thereby delays the onset of instability

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