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Free Convective Flow of a Reacting Fluid between Vertical Porous Plates

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Abstract

This study investigates free convective flow between vertical porous plates. The energy and momentum equations which arise from the definitions of temperature and velocity are written in dimensionless forms. The resulting second order equations are solved to obtain expressions for the velocity, temperature, mass transfer skin friction, and rate of heat transfer.

Keywords: Convective flow, reacting fluid, vertical porous plates.

1.0 Introduction:

Transient convection flow in vertical channels has been extensively studied because of its wide scope of applications which include cooling of electrical and electronic equipment like electric transformer, vertical circuit board, small domestic mobile, winter oil heaters [1]. Jha and Ajibade [6] studied free convective flow between vertical porous plates with periodic heat input under suction/injection. The temperature and velocity fields are separated into steady and periodic parts and the resulting second order ordinary differential equations are solved to obtain the solution to the problem.

In 2007, Toki and Tokis examined unsteady free convection flows of a viscous and incompressible fluid near a porous infinite vertical plate under an arbitrary time-dependent heating of the plate.

Buckmaster and Ludford [2] gave some useful results on laminar flames and in a recent work [7] extended the model investigated by Toki and Tokis to include reaction and magnetism.

Some other helpful results on convective flow were obtained by [3, 4, 5]. In the present work we extend the model studied by [1] by including chemical reaction. We remark that the resulting model is considered for highly exothermic reaction.

2. Mathematical Formulation

The governing equations for the flow in dimensionless form are given by Ajibade (2009, $\delta_1 = \delta_2 = 0$)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Dc \tag{2.1}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2} + \delta_1 e^{\theta} \tag{2.2}$$

$$\frac{\partial c}{\partial t} = \frac{1}{Sc}\frac{\partial^2 c}{\partial y^2} - \delta_2 e^{\theta} \tag{2.3}$$

together with the boundary and initial conditions

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$$u(0,t) = u(1,t) = u(y,0) = 0 \quad (2.4)$$

$$\theta(0,t) = \theta(1,t) = \theta(y,0) = 0 \quad (2.5)$$

$$c(0,t) = 0 \quad (2.6)$$

$$c(1,t) = 1 \quad (2.7)$$

$$c(y,0) = \frac{1}{2}(1 - \cos \pi y), \quad (2.8)$$

Where u is the velocity field, θ is temperature and c is the chemical concentration. Pr is the Prandtl number and Sc is the Schmidt number, δ_1 and δ_2 are Arrhenius constants defined by

$$\delta_1 = \frac{QAh^2}{EvT_v pc\rho} e^{-\frac{E}{RT_0}}, \ \delta_2 = \frac{Ah^2}{c_0 v} e^{-\frac{E}{RT_0}} \ and$$

$$D = \frac{\beta^* c_0}{\beta T_0}$$

3. Method of Solution

Assume $P_r = S_c = 1, \ \delta_1 = \delta_2 = \delta, \ D = 1.$

Let $\phi = \theta + c.$

Then

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2}, \ y \in (0,1)$$

$$\phi(0,t) = 0, \phi(1,t) = 1, \ \phi(y,0) = \frac{1}{2}(1 - \cos \pi y)$$

Buckmaster and Ludford have shown that the temperature θ blows up as $t \to 1/\delta$. Moreover,

as $t \to 1/\delta$

$$\theta \to -In(1 - \delta t)$$

Hence we can obtain

u and c as $t \to 1/\delta$.

The solutions are:

$$\theta = -In(1 - \delta t)$$

$$c = \phi + In(1 - \delta t),$$

and

$$\phi = y + \sum_{n=2}^{\infty} a_n e^{-n^2\pi^2 t} \sin n\pi y \ ,$$

where

$$a_n = \frac{1}{n\pi}\left[1 + (-1)^n\right] + \frac{n}{(n^2 - 1)\pi}\left[1 - (-1)^n\right]$$

Hence

$$u(y,t) = \sum_{n=1}^{\infty}\left[\frac{2}{\delta n \pi}\left((-\delta t) - (1 - \delta t) In (1 - \delta t) + \frac{2(-1)^{n+1}}{n\pi} t\right)\right] \sin n\pi y$$

When $\delta = 0$ (no chemical reaction), $u = \sum_{n=1}^{\infty} \frac{1}{n^3 \pi^3}\left(1 + (-1)^{n+1}\right) e^{-n^2\pi^2 t} \sin n\pi y$

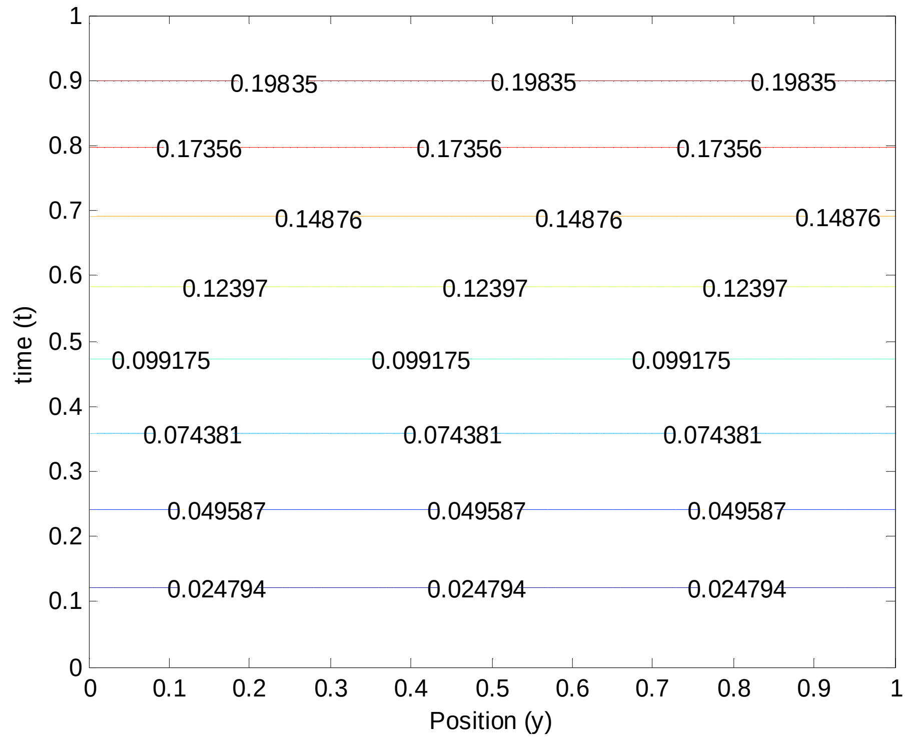

Fig1: Contour graph of Temperature θ at different values of time (t) and Position (y) With δ=0.2

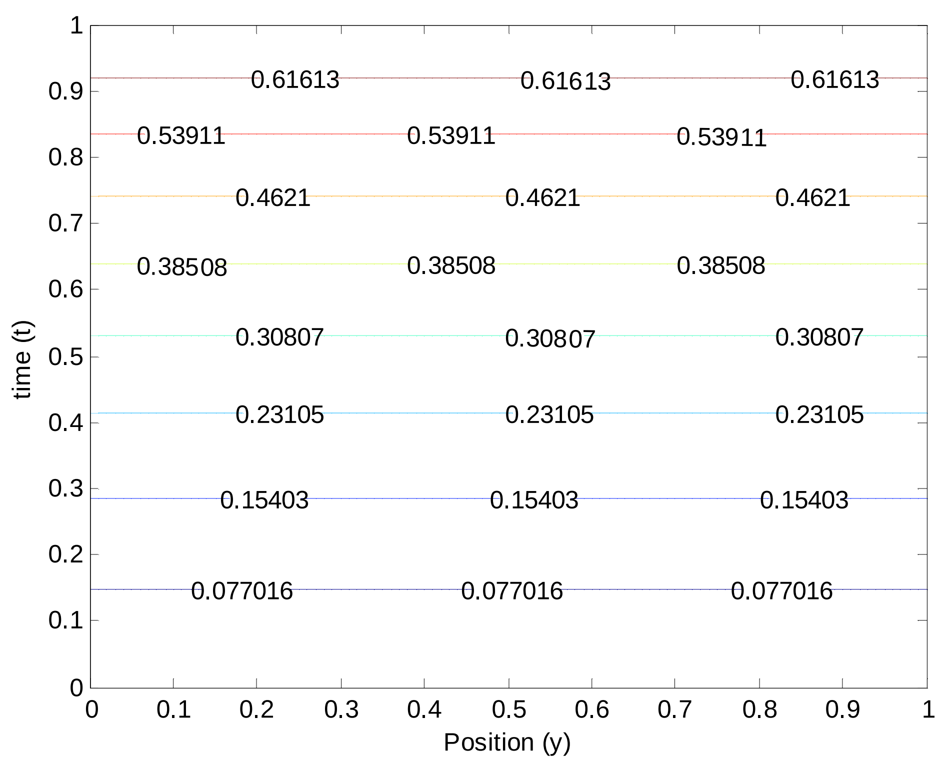

Fig2: Contour graph of Temperature θ at different values of time (t) and Position (y) With δ=0.5

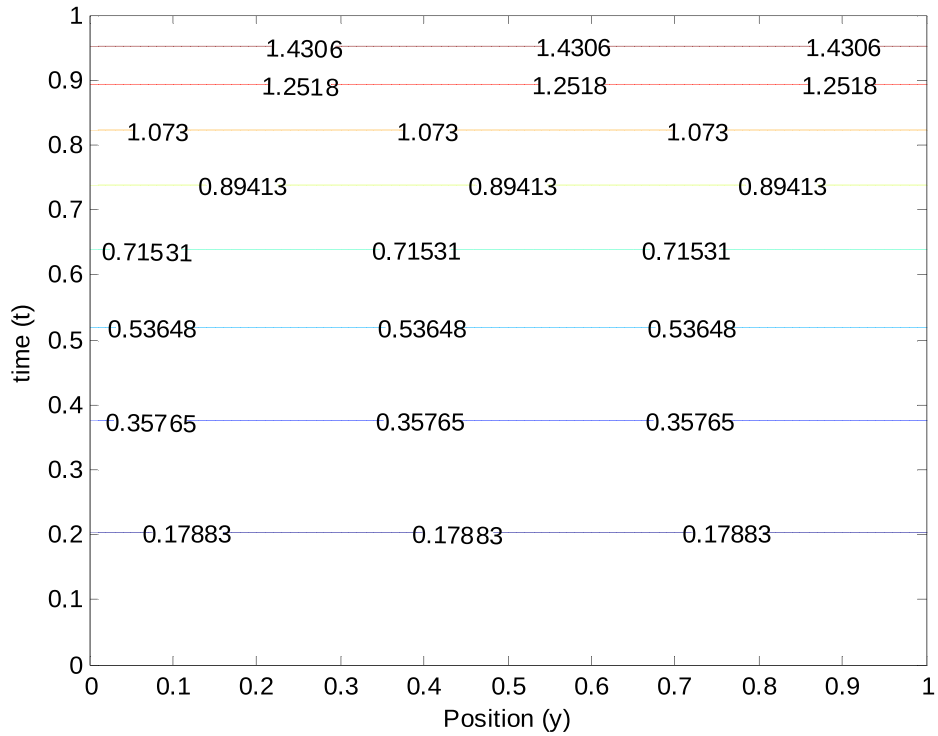

Fig3: Contour graph of Temperature θ at different values of time (t) and Position (y) with δ=0.8

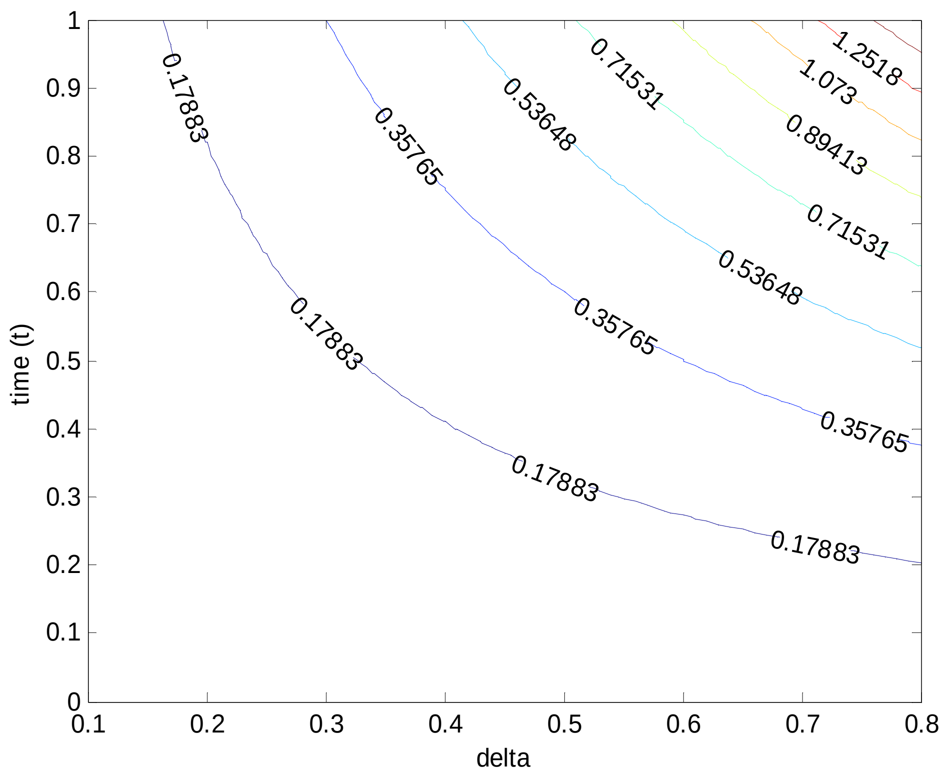

Fig4: Contour graph of Temperature θ at different values of time (t) and δ with Position y=0.5

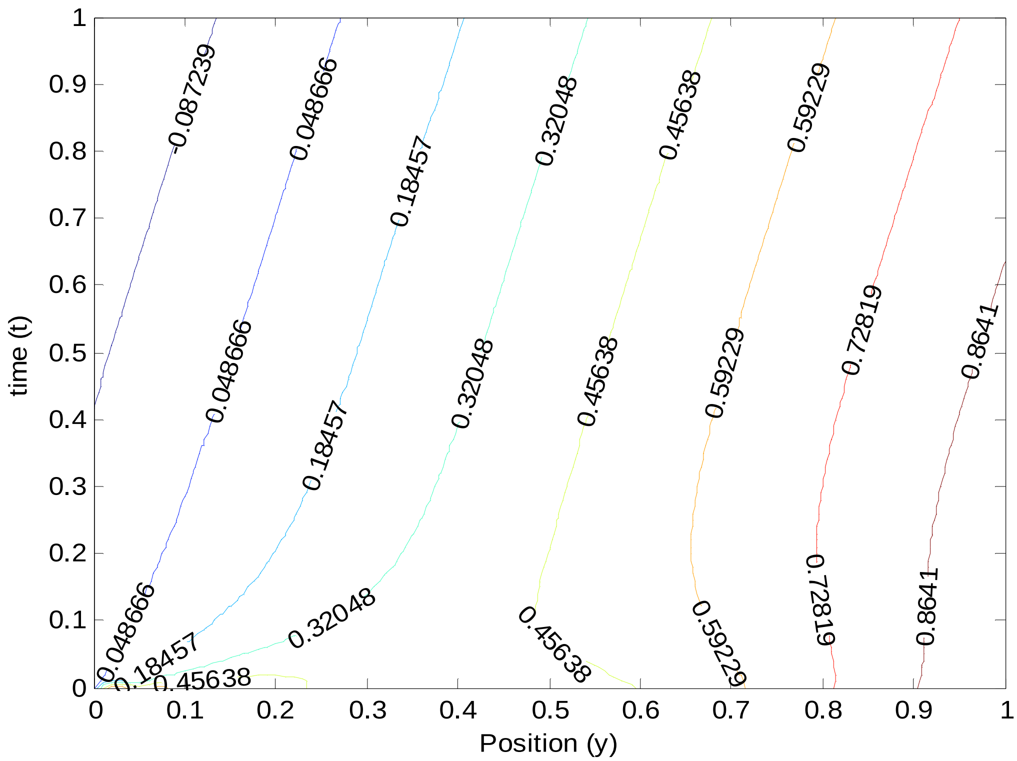

Fig5: **Contour graph of Concentration (c) at different values of time (t) and Position (y) with δ=0.2**

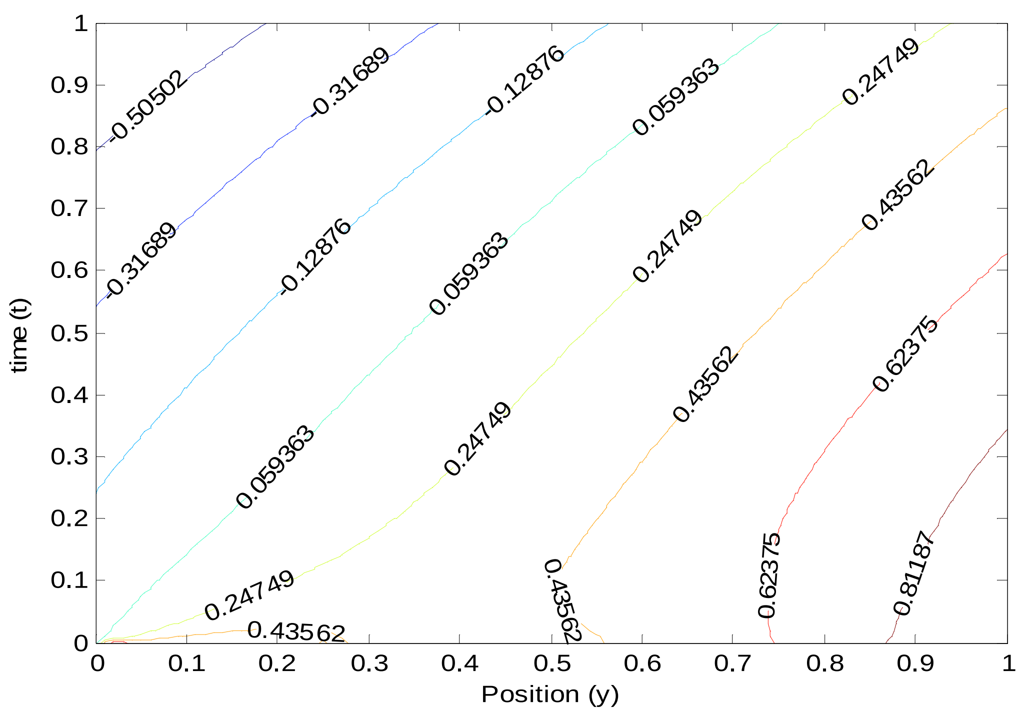

Fig6: **Contour graph of Concentration (c) at different values of time (t) and Position (y) with δ=0.5**

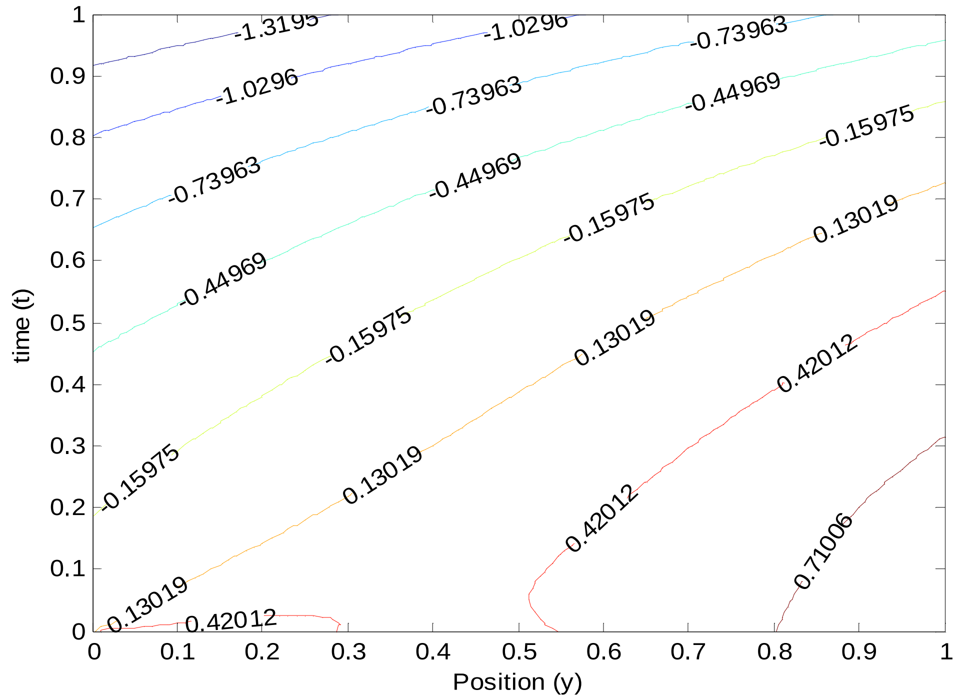

Fig7: Contour graph of Concentration (c) at different values of time (t) and Position (y) with δ=0.8

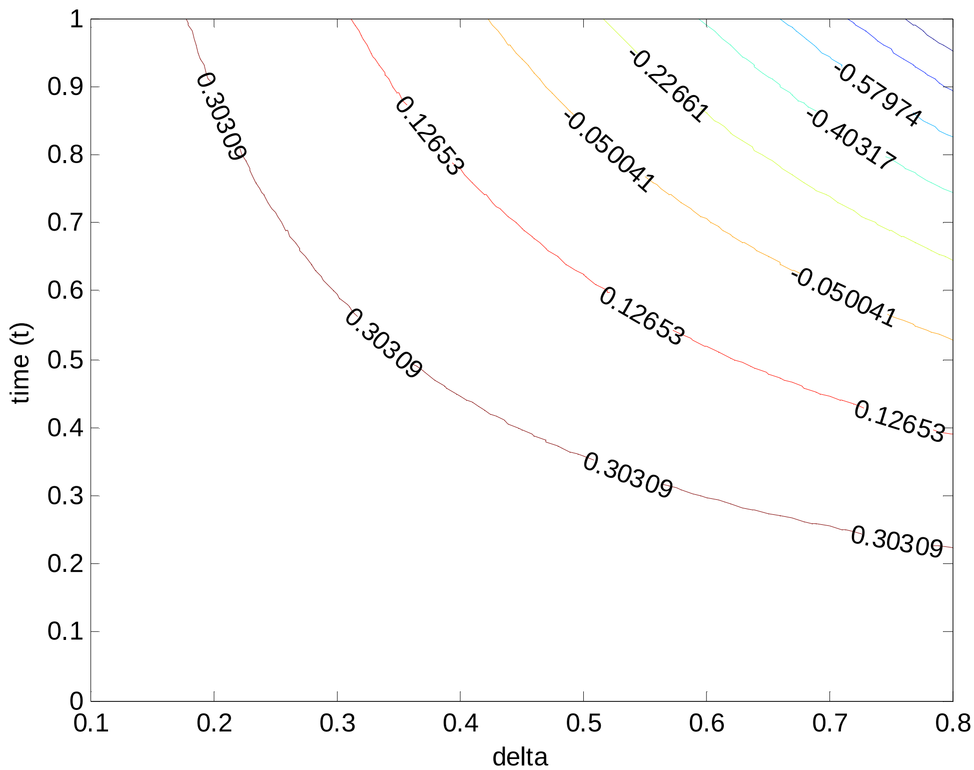

Fig8: Contour graph of Concentration (c) at different values of time (t) and δ with Position y=0.5

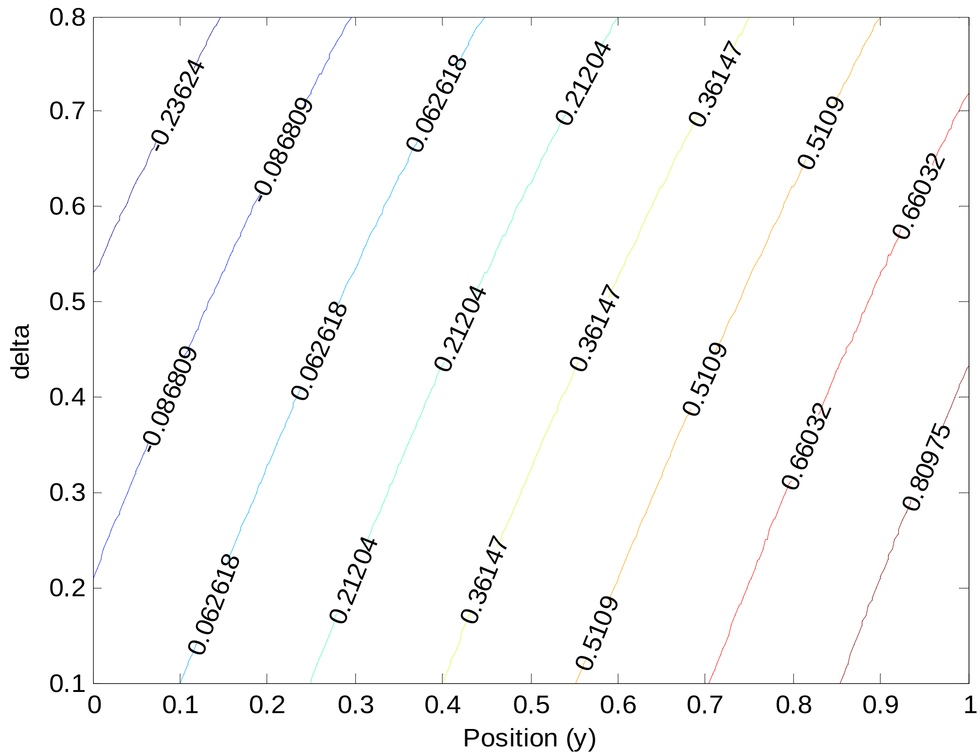

Fig9: Contour graph of Concentration (c) at different values of δ and Position (y) with t=0.4

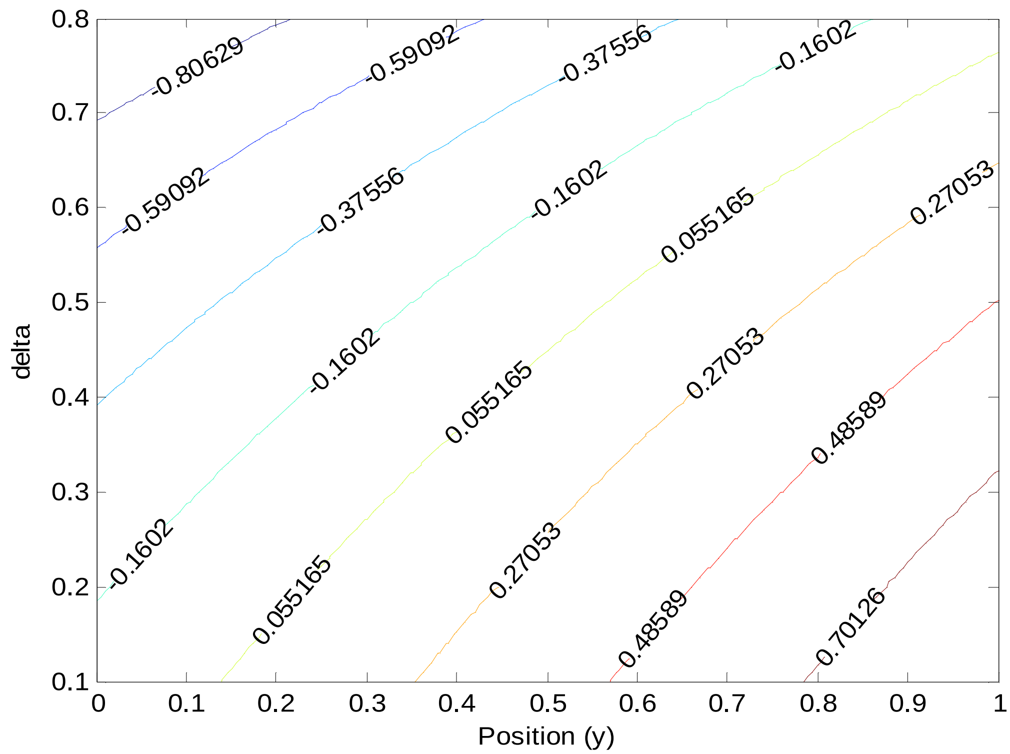

Fig10: Contour graph of Concentration (c) at different values of δ and Position (y) with t=0.8

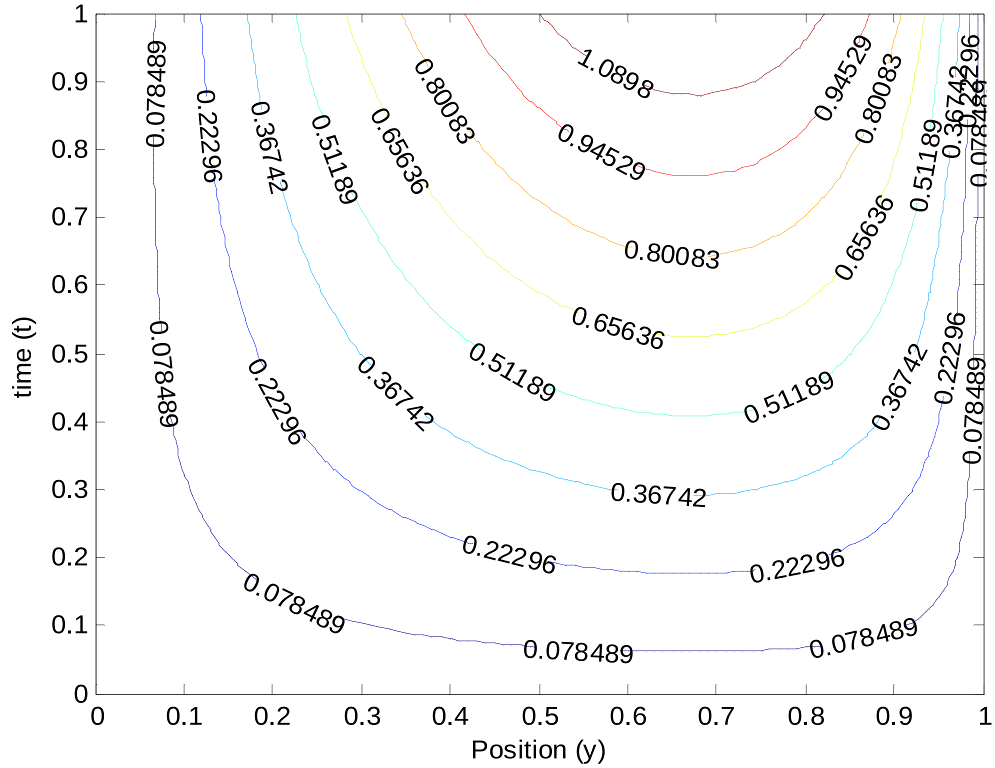

Fig11: Contour graph of velocity u at different values of time t and Position (y) with δ=0.2

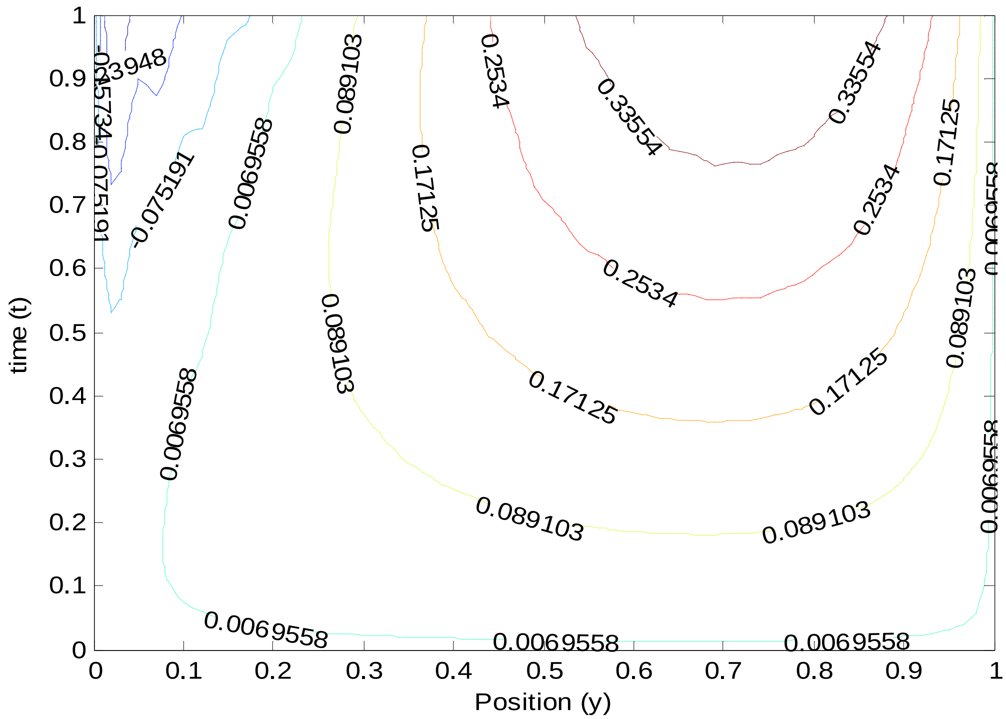

Fig12: Contour graph of velocity u at different values of time t and Position (y) with δ=0.5

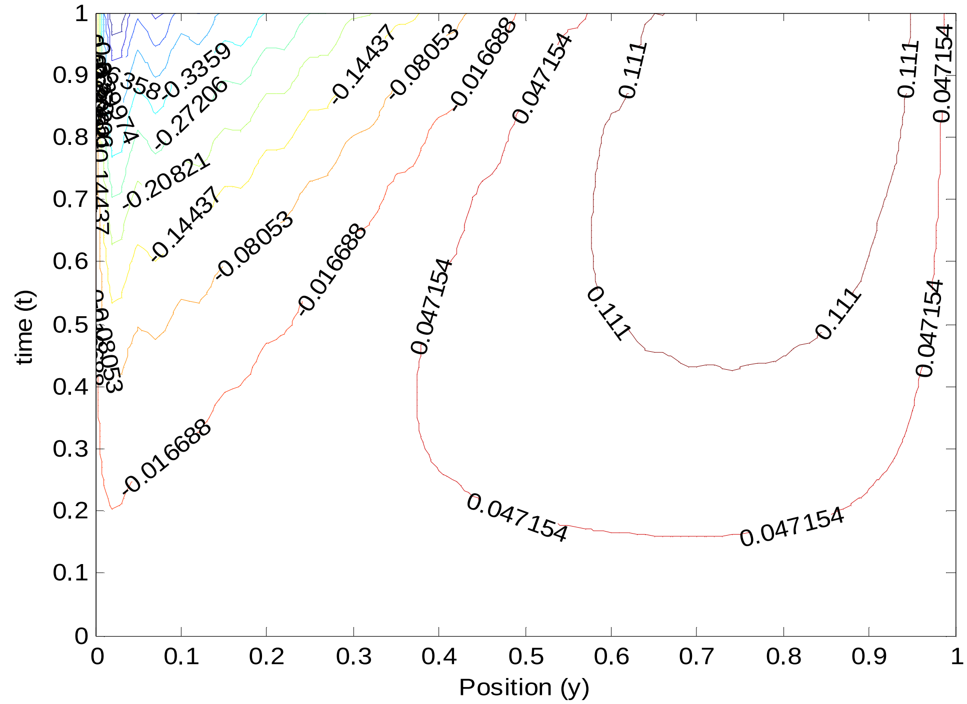

Fig13: Contour graph of velocity u at different values of time t and Position (y) with δ=0.8

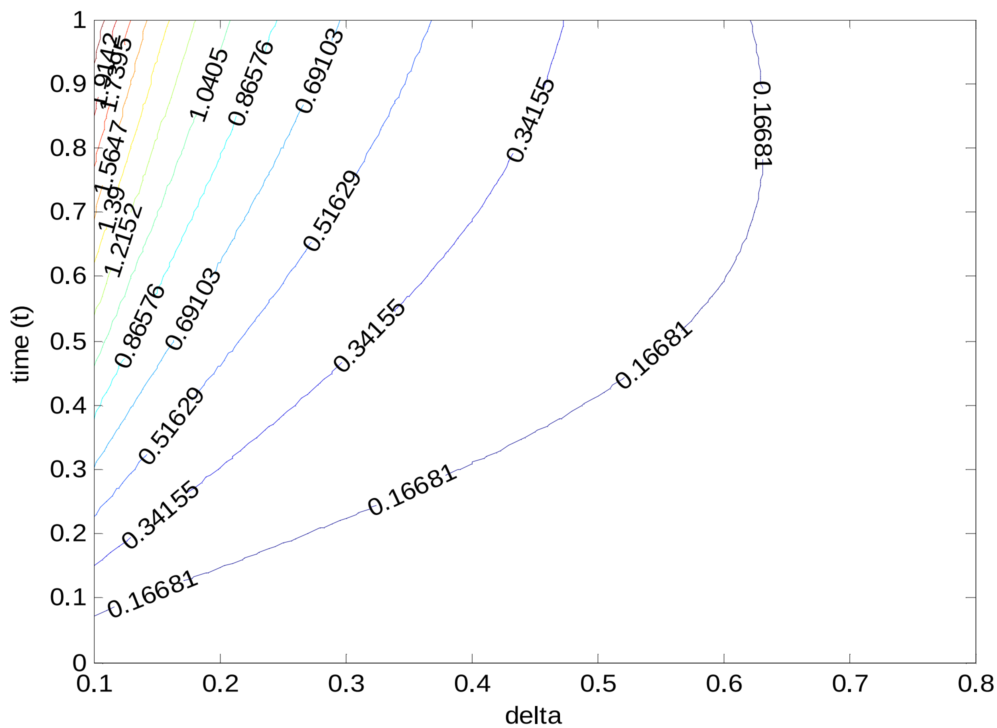

Fig14: Contour graph of velocity u at different values of time t and δ with y=0.5

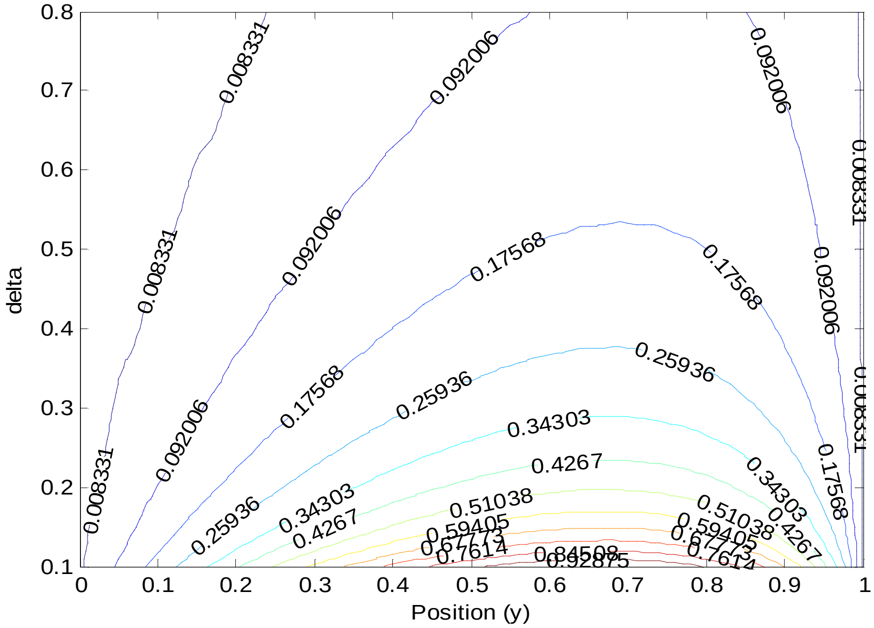

Fig15: Contour graph of velocity u at different values of δ and Position (y) with time t=0.4

Discussion of Results

Figures 1 to 4 show that the temperature distribution increases with time (t). The figures also show that the temperature increases with Arrhenius constant, (δ).

Figure 4 compares the simultaneous effect of both the time (t) and δ on the temperature profiles of the fluid; it is shown that increasing value of both t and δ increases the temperature of the fluid.

The contours of concentration are displayed in figures 5 to 10 at various time (t) and δ. In figures 5 to 7, the concentration of the reacting fluid decreases with time t.

Figure 8 shows the concentration as a decreasing function of time (t) and δ until it is finally used up.

The contours of velocity are presented in figures 11 to 15 at various time (t) and δ across the channel. Velocity is shown in figures 11 to 13 to increase with time t.

Figures 12 to 13 show that the velocity decreases as time increases with δ within a certain range of values space variable. It is noticed that velocity reverses its direction of flow near the lower plate (y = 0).

Figure 14 shows that the velocity increases with time but decreases with increasing value of Arrhenius constant, δ, leading to a reversed flow.

Figure 15 shows that velocity decreases as delta increases. It also shows that velocity is higher near the plate with higher concentration (y = 1).

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