Journal of the Nigerian Association of Mathematical Physics
Volume 18 (May, 2011), pp 181 – 186
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A Note on Unsteady Temperature Equation For Gravity Flow of A Power-Law Fluid Through A Porous Medium

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Abstract

We present an analytical study of unsteady temperature energy equation for gravity of a fluid with non – Newtonian behaviour through a porous medium. For the case of radial axisymmetric flow, the governing partial differential equation is transformed into an ordinary differential equation through similarity variables. It is further assumed that the viscosity is temperature dependent. We investigate the effects of velocity on the temperature field. Theorems on existences and uniqueness of solution are formulated based on some criteria and the proofs of the theorem are established.

Keywords: Unsteady temperature equation, Non – Newtonian power-law fluid porous media and gravity flows.

1.0 Introduction:

The steady and unsteady flows of a non – Newtonian fluids in a porous media in which the main driving force is gravity has attracted a large class of applications in engineering practice, particularly in applied geophysics, geology, ground water and oil reservoir engineering. Due to the increasing in the production of heavy crude oils, and else where materials whose flow behaviour in shear cannot be characterized by Newtonian relationships, it has become necessary to have an adequate understanding of the Rheological effects of non – Newtonian fluid flows and, as a result, a new stage in the evolution of fluid dynamics theory is in progress [7].

Some scientists have studied gravity flows of a power-law fluid through a porous medium. These include, [2] investigated the effect of permeability on the temperature rise in a reacting porous medium. [6] presented a paper on unsteady gravity flows of a power-law fluid through a porous medium. [3] presented a note on the flow of a power-law fluid with memory past an infinite plate. [1] investigated the hydromagnetic flow and heat transfer over a stretching sheet. [5] presented a paper on the influence of power-law exponent on an unsteady endothermic reaction. Lamidi and Ayeni [4] investigated the influence of power-law index on an unsteady exothermic reaction. Many useful information on the unsteady gravity flow of a power-law fluid through a porous media was presented but the temperature equation was not considered.

2.0 Mathematical Formulation

The governing equations for the Mathematical formulation are momentum and energy equation. Considering a two dimensional flow in the $x-z$ plane where the free surface is a streamline at a point on the surface, the flow can be expressed by a modified Darcy‘s law.

The unsteady equations are

$$v = -\frac{k}{\mu}\frac{\partial h}{\partial s} \qquad \text{(Darcy‘s law- Momentum)} \qquad (2.1)$$

It is a single phase flow where $\frac{\partial h}{\partial s}$ is the gradient in the flow direction and k is independent of the nature of the fluid but depends on the geometry of the medium.

$$v = -\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial s}\left|\frac{\partial h}{\partial s}\right|^{\frac{1-n}{n}} \qquad \text{(Modified Darcy‘s law)} \qquad (2.2)$$

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Where z = h on the free surface. The rheological parameter n is the power-law exponent which represents shear-thinning (i.e. n<1) and shear-thinning (i.e. n>1) fluids. k is the permeability, ρ is the density and μ_{ef} is the effective viscosity. The Dupuit's approximation yields $\frac{\partial h}{\partial s} \cong \frac{\partial h}{\partial x}$ for small gradients and assume a horizontal flow with h=h(x, t) (t being the time).

$$V_X = -\left(\frac{K\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial x}\left|\frac{\partial h}{\partial x}\right|^{\frac{1-n}{n}} \tag{2.3}$$

$$v_r = -\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial r}\left|\frac{\partial h}{\partial r}\right|^{\frac{1-n}{n}} \tag{2.4}$$

Where v_r is the component of the velocity in the radial direction, whereas for radial axisymmetric flow

$$v_R = -\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial R}\left|\frac{\partial h}{\partial R}\right|^{\frac{1-n}{n}} \tag{2.5}$$

Substituting (2.3) and (2.5) into the continuity equations we obtain

$$\frac{1}{R}\frac{\partial(Rhv_R)}{\partial R} = -\varphi\frac{\partial h}{\partial t} \tag{2.6}$$

For radial axisymmetric flow where h=h(R, t) and by cylindrical coordinate equation (2.6) becomes

$$\frac{1}{R}\frac{\partial}{\partial R}\frac{\partial h}{\partial R}\left(Rh\left|\frac{\partial h}{\partial R}\right|^{\frac{1-N}{N}}\right) = -\Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}\frac{\partial h}{\partial t} \tag{2.7}$$

Defining new variables

$$h(R,t) = t^{\alpha} f(\eta); \eta = Rt^{\beta}$$

By similarity variables we obtain

$$\frac{d}{d\eta}\left[\eta f f'(f')^{\frac{1-n}{n}}\right] = a^2\eta\left[\alpha f - \frac{\alpha+n}{n+1}\eta f'\right] \tag{2.8}$$

Where φ being the porosity, $a^2 = \varphi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}$, $\beta = \frac{-n+\alpha}{n+1}$ and prime denotes differentiation with respect to η.

Solving equation (2.8) we obtain

$$f(\eta) = \eta^{\alpha\left(\frac{n+1}{\alpha+n}\right)} \tag{2.9}$$

The flow can also be expressed by a modified energy equation

$$\rho c_p\left(\frac{\partial T}{\partial t}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.10}$$

Where

$\mu = m\left(-\frac{\partial u}{\partial y}\right)^n, T = temperature, c_p = specific$ heat at constant pressure, $\mu =$ viscosity, k= thermal conductivity, ρ == density,

rheological parameter n = power-law index and η =apparent viscosity.

Let T (R, t) = t^{α} g (η), T = T(R, t), η = Rt^{β}

When k is not a constant

By similarity variables we obtain

$$\eta(\gamma g + \beta\eta g') = b\frac{d(\eta g')}{d\eta} + c\eta\left[\frac{d}{d\eta}\left(-\frac{df}{d\eta}\right)^{\frac{1}{n}}\right]^2 \tag{2.11}$$

Where b = $\frac{k}{\rho c_p}$, c= $\frac{\mu}{\rho c_p}\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{2}{n}}$

Let g (η_1) = 1, $g'(\eta_1) = 0$ (2.12)

$$f(\eta) = \eta^{\alpha\left(\frac{n+1}{\alpha+n}\right)} \tag{2.13}$$

3.0. Methods of Solution

We prove existence and uniqueness theorem, the problem has a solution and the solution is unique.

From equation (2.11) - (2.13)

Let n =1

$$\eta(\gamma g + \beta\eta g') = b\frac{d(\eta g')}{d\eta} + c\eta\left[\frac{d}{d\eta}\left(\frac{df}{d\eta}\right)^2\right] \tag{3.1}$$

$$f(\eta) = \eta^{\frac{2\alpha}{\alpha+1}} \tag{3.2}$$

We obtain

$$\eta(\gamma g + \beta\eta g') = b\frac{d(\eta g')}{d\eta} + c\left[\left(\frac{16}{81}\right)\eta^{-1/3}\right] \tag{3.3}$$

Let $\gamma = 0$

We obtain

$$\eta(\beta\eta g') = b\frac{d(\eta g')}{d\eta} + c\left[\left(\frac{16}{81}\right)\eta^{-\frac{1}{3}}\right] \tag{3.4}$$

$$g(1) = 1 \tag{3.5}$$

$$g'(1) = \alpha \tag{3.6}$$

where α is a guessed value of the temperature gradient of the system.

Theorem 3.1: Problem (3.4) – (3.6) has a unique solution.

Proof:

Let $x_1 = \eta, x_2 = g, x_3 = g'$

Then $x_1' = 1, x_2' = x_3$

$$x_3' = \frac{\left(b - \beta x_1^2\right) x_3}{b x_1} - \frac{16c}{81} x_1^{-4/3} \tag{3.7}$$

Satisfying

$$1 \le x_1 \le 1$$

$$1 \le x_2 \le 1$$

$$-\alpha \le x_3 \le \alpha \tag{3.8}$$

The partial derivatives $\left|\frac{\partial f_i}{\partial x_j}\right|, i, j = 1,2,3$ are Lipschitz continuous and are bounded on D.Therefore; problem (3.4)-(3.6) has a unique solution. This completes the proof.

Theorem 3.2

From equation (2.6)-(2.8)

We obtain

$$b\eta g'' + g'\left(b - \beta\eta^2\right) - \eta\gamma g + c\left[\alpha \frac{n+1}{\alpha+n}\left(\frac{\alpha-1}{\alpha+n}\right)\right]^2 \eta^{\frac{\alpha-2}{\alpha+n}} = 0 \tag{3.9}$$

$$g(1) = 1 \tag{3.10}$$

$$g'(1) = \delta \tag{3.11}$$

Problem (3.9)-(3.11) has a unique solution.

Proof:

Let $x_1 = \eta, x_2 = g, x_3 = g'$

As

$$x_1' = 1, x_2' = x_3$$

$$x_3' = \frac{\gamma x_2}{b} \frac{x_{3\left(b - \beta x_1^2\right)}}{b x_1} - \frac{c\alpha^2}{b}\left[\frac{n+1}{\alpha+n}\left(\frac{\alpha-1}{\alpha+n}\right)\right]^2 x_1^{-\frac{2+n}{\alpha+n}}$$

Satisfying

$$1 \le x_1 \le 1$$

$$1 \le x_2 \le 1$$

$$-\delta \le x_3 \le \delta \tag{3.12}$$

$$\left|\frac{\partial f_3}{\partial x_1}\right| \le \left|\frac{\delta(\beta + b)}{b} + \frac{c\alpha^2}{b}\left(\frac{2+n}{\alpha+n}\right)\left[\frac{n+1}{\alpha+n}\right)\left(\frac{\alpha+1}{\alpha+n}\right]^2\right| \tag{3.13}$$

Hence, $\left|\frac{\partial f_i}{\partial x_j}\right|, i.j = 1,2,3$ are Lipschitz continuous since the derivative of $\frac{\partial f_i}{\partial x_j}$ are bounded on D. Therefore, the problem has a unique solution. This completes the proof.

From equation (2.6) together with the boundary conditions

$$g(1) = 1 \quad (3.14)$$

$$g(\infty) = 0 \quad (3.15)$$

Replace

$$g(\infty) = 0$$

by

$$g'(1) = \delta = -0.1 \quad (3.16)$$

Suppose c = 0

$$\eta(\gamma g + \beta\eta g') = b(g' + \eta g'') \quad (3.17)$$

Take γ = β = b = 1

Equation (3.17) becomes

$$\eta(g + \eta g') = (g' + \eta g'') \quad (3.18)$$

Let us use series

Suppose

$$g = 1 + be^{-\eta} + ce^{-2\eta} \quad (3.19)$$

We obtain

$$g = 1 + (-0.272)e^{-\eta} + (0.739)e^{-2\eta} \quad (3.20)$$

Finding the minimum point of g at

$$g'(\eta) = 0 \quad (3.21)$$

$$g''(\eta) \succ 0 \quad (3.22)$$

We obtain

$$\eta = 1.69$$

$$g'' = 0.051$$

Remark: So g has a minimum point at η = 1.69

4.0 Results

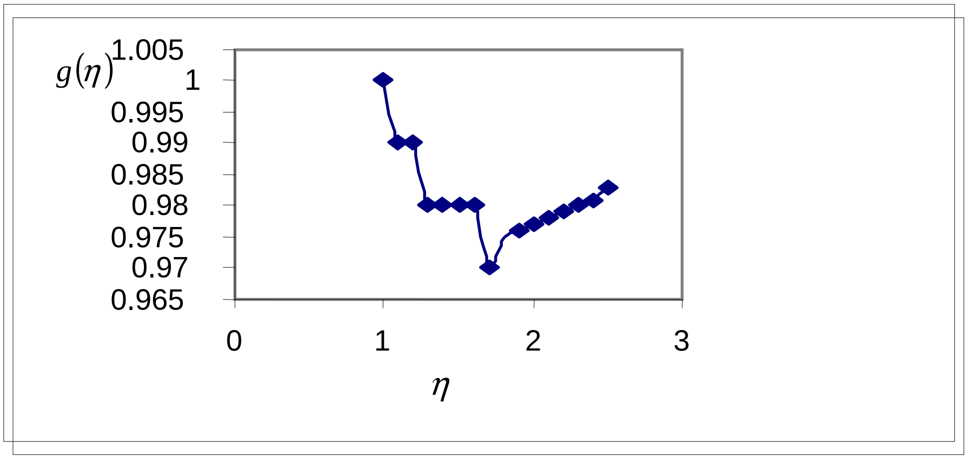

Fig. 1: Graph of the temperature function of g against the similarity variable η when $\gamma = 1$, b = -0.272 and c =0.739.

Discussion/Conclusion

We proposed suitable expression for unsteady temperature equation for gravity flows of a power-law fluid through a porous medium. The unsteady profile and the temperature profile were studied for various values of power-law exponent α and n. The fluid is Newtonian for n>1.We show that the problem has a solution and the solution is unique. It is seen from the graph that g has a minimum point.

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