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An equation for the dimensionless friction factor of consolidated sandstone

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Abstract

An equation that relates the dimensionless friction factor of a porous medium its Reynolds number, during incompressible and compressible fluid flow through the medium is proposed. The equation is a curve fit of a graph drawn from the results of Ohirhian for water (liquid) and that of Akpokene for Nitrogen (gas) flow through synthetic consolidated sandstone cores.

The equation is valid for Reynolds number in the range 25 to 62.5. This range of the Reynolds number is dictated by the low ratio of upstream to downstream pressure (1.45:1.00) of the Ruska Gas Permeameter that is used to perform the gas flow experiment.

1.0 Introduction:

As far back as 1930, [2] developed an equation that relates the dimensionless friction factor to the Reynolds number for fluid flow in smooth pipes. [3] developed an equation that relates the dimensionless friction factor tot eh relative roughness of a pipe in fully developed turbulent flow in rough pipes. [4] proposed that equations for fluid flow in pipes can be modified to equations for fluid flow in porous media by the replacement of the diameter of a pipe by the diameter of a porous medium. His diameter of a porous medium is the diameter of an equivalent pipe multiplied by the square root of the porosity of the medium.

This study aims at finding an equation that relates the dimensionless friction factor to the Reynolds number for porous medium. Two experiments were performed by flowing liquid and gas through synthetic consolidated porous and permeable cores. The liquid used is water ,as in the experiment of Darcy (1859) and Nitrogen as the gas in the second experiment. Nitrogen gas was used because the Ruska Gas Permeameter uses nitrogen. The dimensionless friction factors obtained from both experiments were plotted against the Reynolds number from the two experiments.

The plot was parabolic and it was curve fitted to produce an equation for the dimensionless friction factor during non laminar (non Darcian) flow in porous media.

1. Development of Equations

Ohirhian [4] modified the Darcy-Weisbach equation for the lost head during fluid flow in a pipe, to define a new equation for the lost head during fluid flow in a porous medium. The modified Darcy-Weishbach lost head equation is:

$$H_L = \frac{f_p Lv^2}{2gd\sqrt{\phi}} \qquad (1)$$

where,

H_L = Lost head

f_p = Dimensionless friction factor for porous medium

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L = Length of porous medium
v = Average velocity of flowing fluid
d = Diameter of cylindrical porous medium
g = Acceleration due to gravity
φ = Porosity

Ohirhian [4] then combined the Bernoulli (energy balance) equation for liquid flow in a pipe with the new equation for lost head in a porous medium (H_L) to obtain:

$$\frac{\Delta p}{\gamma} + (h + L) \quad \sin\theta = \frac{f_p L v^2}{2 g d \sqrt{\phi}} \qquad (2)$$

where,

Δp = Pressure drop across porous medium

γ = Specific weight of flowing liquid

h = Height of the column of the flowing fluid above the porous medium

θ = Angle of inclination of the porous medium with the horizontal axis, in degrees

During vertical downhill liquid flow, in which the liquid discharges into the atmosphere, equation (2) reduces to:

$$f_p = 1.2337 \frac{d^5 \phi^{2.5}}{Q^2} \left[\frac{p}{\rho L} + g \left(\frac{h}{L} + 1 \right) \right] \qquad (3)$$

where,

p = Upstream pressure of the liquid that is displaced through the porous medium
Q = Volumetric flow rate
$\rho =$ Mass density of liquid that flows through the porous medium

In equation (3), the velocity in equation (2) is replaced by V = Q/A.

Ohirhian [4] also modified the Reynolds number for a pipe, to obtain a Reynolds number for a cylindrical porous medium. The modified Reynolds number (R_{NP}) for a liquid is:

$$R_{NP} = \frac{\rho v d \sqrt{\phi}}{\mu} = \frac{4 \rho Q}{\pi \mu d \sqrt{\phi}} \qquad (4)$$

where, $\mu =$ Absolute viscosity of flowing liquid

Ohirhian [4] plot of f_p versus the reciprocal of the modified Reynolds number for various sandstone cores, defined straight lines during laminar (Darcian) flow. The straight lines converged into a single parabola in the non-laminar (non Darcian) flow. The non Darcian flow started at $R_{NP}^{-1} = 0.055$ ($R_{NP} = 18.8$). It became well developed for all $R_{NP}^{-1} \leq 0.040$ ($R_{NP} \geq 25$). Table 1 is part of the results of Ohirhian [4] experiment.

Table 1: Values of f_p versus reciprocal of R_{NP} during liquid flow

Serial No	$Q\left(\frac{cm}{sec}\right)$	fp × 10^0	$\frac{1}{R_{NP}}$
1	0.11613	0.2088	0.039
2	0.13200	0.1859	0.034
3	0.14600	0.1700	0.031
4	0.10710	0.3974	0.042
5	0.1400	0.2601	0.038
6	0.1500	0.2791	0.036
7	0.1846	0.1958	0.029

Ohirhian (2010) combined the following equations:

(a) Euler Equation (energy balance for any fluid)
(b) Ohirhian [4] modification of the Darcy – Weisbach Equation
(c) Continuity Equation for a gas
(d) Equation of State for a gas

He arrived at a general differential equation for the flow of a gas in a cylindrical porous medium. The equation is:

$$\frac{d\ p^2}{d\ell_p} = \frac{(AAp \pm B_p\ p^2)}{(1 - \frac{C_p}{p^2})} \tag{5}$$

where,

$$AA_p = \frac{1.621139\, f_p W^2 zRT}{g d_p^5 M},\ B_p = \frac{2M \sin\theta}{zRT},\ C_p = \frac{KW^2 zRT}{gMd_p^4}$$

and

W = Weight flow rate of a gas
z = Gas deviation factor (z – factor)
R = Universal gas constant
T = Absolute temperature
M = Molecular weight of a gas
K = A constant = 1.0328 for a sweet natural gas.

The denominator of equation (5) is the contribution of kinetic effect to the pressure drop across a given length of a cylindrical porous medium. The kinetic effect is small and can be neglected. Ignoring the kinetic contribution, [4] used the Runge–Kutta algorithm to provide a solution to equation (5). His solution to equation (5) during downhill flow of a gas in which the gas discharges into the atmosphere is:

$$f_p W^2 = \frac{\left[J_p + \frac{S_1 L}{6}\left(1 - x_f + 0.5^2 x_f - 0.3 x_f^3\right)\right]}{BB_p^a\left[z_1 T_1\left(1 - x_f + 0.5 x_f^2 - 0.3 x_f^3\right) + z_{av}^f T_{av}\left(5.2 - 2.2 x_f + 0.6 x_f^2\right)\right]} \tag{6}$$

where

$$J_p = P_i^2 - \frac{p_1^2}{6}\left(-5.2 x_f + 2.2 x_f^2 - 0.6 x_f^3\right) - p_2^2 \quad \text{if } BB_p^a \geq S_1$$

$$J_p = P_2^2 - p_1^2 - \frac{p_1^2}{6}\left(-5.2 x_f + 2.2 x_f^2 - 0.6 x_f^3\right) \quad \text{if } BB_p^a < S_1$$

$$BB_p^a = \frac{1.621139\ RL}{6 g d_p^5 M} = \frac{0.270110\ RL}{g d_p^5 M}$$

$$S_1 = \frac{2M \sin\theta p_1^2}{z_1 T_1 R},\ x_f = \frac{2M \sin\theta L}{z_{av}^f T_{av} R}$$

z_{av}^f = Gas deviation factor at the midsection of the porous medium calculated with Tav and p_{av}^f, where $T_{av} = 0.5(T_1 + T_2)$ and $p_{av}^f = \frac{2p_1 p_2}{p_1 + p_2}$

P_1 = Gas inlet pressure
P_2= Gas exit pressure

During isothermal flow in which there is no significant variation of the gas deviation factor (z), between the inlet and the exit ends of a porous medium, equation (6) simplifies to:

$$f_p W^2 = \frac{\left[J_p + \frac{S_1 L}{6}\left(1 - x_f + 0.5x_f^2 - 0.35x_f^3\right)\right]}{BB_p^a \left[z_1 T_1 \left(1 - x_f + 0.5x_f^2 - 0.35x_f^3\right) + z_1 T_1 \left(5.0 - 2.0x_f + 0.7x_f^2\right)\right]} \qquad (7)$$

where

z_1 = Gas deviation factor which is same at inlet and exit ends of the porous medium

T_1 = Gas temperature which is same at inlet and exit ends of the porous medium

$$J_p = P_1^2 - \frac{p_1^2}{6}\left(-5.0x_f + 2.0x_f^2 - 0.7x_f^3\right) - p_2^2$$

In a horizontal flow equation (7) reduces to:

$$f_p = \frac{p_1^2 - p_2^2}{W^2 BB_p^a \left(z_2 T_2 + 4.96 z_{av} T_{av}\right)} \qquad (8)$$

During isothermal horizontal flow in which there is no variation in z, equation (8) reduces to:

$$f_p = \frac{p_1^2 - p_2^2}{6W^2 BB_p^a z_2 T_2} \qquad (9)$$

The definition by [5] of the Reynolds number for gas flow through a porous medium is:

$$R_{Np} = \frac{4W}{\pi g \mu_g d_p} = \frac{36.88575 G_g P_b Q_b}{Rgd_p \mu_g z_b T_b} \qquad (10)$$

Where,

f_p = Dimensionless friction factor of porous medium that is dependent on the Reynolds number of porous medium.

R_{Np} = Reynolds number of the porous medium.

W = Weight flow rate of the gas

γ_b = Specific weight of fluid at P_b and T_b

Q_b = Volumetric rate of fluid, measured at P_b and T_b

P_b = Base pressure, absolute unit

T_b = Base Temperature, absolute unit

z_b = Gas deviation factor at p_b and T_b usually taken as 1

G_g = Specific gravity of gas (air = 1) at standard condition

M = Molecular weight of gas

R = Universal gas constant

μ_g = Absolute viscosity of the gas

g = Acceleration due to gravity

In this study, the values of the dimensionless friction factor (fp) as defined in equation (6) are plotted against the Reynolds number as defined in equation (8). The porosity of the permeable cores was measured by the helium porosimeter. The dimensionless friction factor (fp) was measured with the Ruska gas permeameter. This Ruska instrument uses nitrogen as fluid. Results obtained by [1] during his measurement of the porosity of the porous and permeable cores are shown in table (2). Akpokene's Ruska gas permeameter measurements are shown in Table (3).

Table 2: Measurement of Porosity.

Serial No.	Sand Grain	Sand/cement Ratio	Pore Volume + Line Volume (cm^3)	Bulk Volume + Line Volume (cm^3)	Porosity
1 2	710m to 2.8mm	1:1 1:2	10.03 9.85	19.00 19.00	0.3073359 0.2934362
3 4	355m to 500m	1:1 2:1	9.95 9.15	19.00 19.00	0.311583 0.2393822
5 6	150m to 355m	1:1 2:1	9.95 10.15	19.00 19.00	0.3011583 0.3166023
7 8 9	710m 500m 710/500m (1:1)	Aradite Aradite Aradite	10.93 9.90 10.35	19.00 19.00 19.00	0.3368339 0.2972972 0.3320463
10 11	1.4m	3:1 4:1	10.40 10.60	19.00 19.00	0.3359073 0.3513513
12 13 14	710m	3:1 4:1 7:1	9.40 9.85 11.10	19.00 19.00 19.00	0.2586872 0.2934362 0.3899613
15		4:1	9.75	19.00	0.2883142

Line Volume = $6.05cm^3$

Table 3: Helium Permeameter results

Sample Serial No.	Q (in cm of flow tube)	Temperature (^oC)	Diameter (cm)	Length (cm)	Q(cm^3/sec). Read from Instrument Curve
1	8.6	33.0	2.54	2.57	7.5
2	8.1	33.0	2.45	2.50	6.9
3	7.9	33.4	2.48	2.55	6.7
4	7.5	33.3	2.61	2.55	6.2
5	10.4	34.0	2.57	2.54	9.5
6	9.4	34.0	2.47	2.50	8.3
7	11.9	31.0	2.56	2.54	11.2
8	11.8	31.0	2.45	2.55	11.0
9	11.8	31.0	2.57	2.54	11.0
10	11.6	31.0	2.60	2.53	10.6
11	11.7	31.0	2.59	2.53	10.8
12	11.3	31.0	2.59	2.54	10.4
13	11.4	31.0	2.58	2.54	10.5
14	11.4	31.0	2.60	2.40	10.5
15	10.1	31.0	2.58	2.51	9.2

2. Sample Calculation of Dimensionless Friction Factor

The equations developed in this study for the calculation of the dimensionless friction factor are used to calculate the dimensionless friction factor of the first core in Tables 2 and 3, as a practical example.

Example 1

During an experiment to calculate the dimensionless friction factor of porous and permeable core, the following data were obtained.

Length (L) = 2.57 cm
Diameter (d) = 2.54 cm
Flow rate (Q) = 7.5 cm^3/ sec
Temperature = 33 ^OC
Pressure upstream of core (p_1) = 1.45 atm absolute
Pressure downstream of core (p_2) = 1.00 atm absolute
Viscosity of Nitrogen at 33 ^OC = 0.018 cp

Problem

Calculate the dimensionless friction factor (f_p)

Solution

Average flowing pressure = $\dfrac{2\,p_1\,p_2}{p_1 + p_2} = \dfrac{2 \times 1.45 \times 1}{1.45 + 1.0} = 1.183673$ atm

Temperature is constant at 33 ^OC. conversion from ^OC to ^OF is given by:

$^oF = 32 + 1.8\,^oC$; then, 33 ^OC = 91.40 ^OF = 551.4 ^OR

Molecular weight of Nitrogen = 28.013

Specific weight of Nitrogen (γ_b) = $\dfrac{p_b\,M}{z_b\,T_b\,R}$

By use of 1 atm (14.7 psia) as base pressure (p_b) and flowing temperature as base (T_b),

$$\gamma_b = \frac{14.7 \times 144 \times 28.013}{z_b \times 551.4 \times 1545}$$

Psuedo - critical properties of N_2 are: p_c = 493.0 psia , T_O = 227.8 ^OR. Then,

p_r = 14.7 / 493 = 0.029817. T_r = 551.4 / 227.8 = 2.420544

At these values of the reduced variables, z_b is approximately unity. The z_b is taken as 1.0 in this study. Then, γ_b = 0.0696056 lb / ft^3.

Q_b = 7.5 cm^3/ sec = 7.5 ×3.531467 ft^3/ sec = 2.648600 ft^3/ sec

W = $\gamma_b Q_b$ = 0.0696056×2.648600 lb/ sec = 1.843571 E- 5 lb/ sec

μ_g = 0.018 cp = 0.018×2.088543E-5 lb sec / ft^2 = 3.759174 E-7 lb sec / ft^2

$d_p = d\sqrt{\phi} = 2.54\sqrt{0.3073359}$ cm = 1.408122 cm = 1.408122×3.280840 E-2 ft.

L = 2.57 cm = 2.57 ×3.280840 E-2 ft = 0.0843176 ft.

$$BB_p^a = \frac{0.270110\,R\,L}{g\,d_p^5\,M} = \frac{0.270110 \times 1545 \times 0.0843176}{32.2 \times 0.0461982^5 \times 28.013} = 185373$$

p_1 = 1.45 atm = 1.45 ×14.7 × 144 psf = 3069.36 psfa

p_2 = 1.00 atm = 1.00 ×14.7 × 144 psf = 2116.80 psfa

Taking the core to be horizontal, we get:

$$f_p = \frac{p_1^2 - p_2^2}{6W^2\,BBp^a\,z_2\,T_2} = \frac{3069.36^2 - 2116.80^2}{6 \times (1.843574E-5)^2 \times 185373 \times 1 \times 551.4} = 0.237002\,E\,8$$

Taking the core to be vertical and flow as isothermal, equation 7 is used to get

$$x_f = \frac{2M\sin\theta L}{z_{av}{}^{f} T_{av} R} = \frac{2\times 28.013\times 0.089317\times 1}{1\times 551.4\times 1545} = 5.545141E-6$$

$$1 - x_f + 0.5x_f^2 - 0.35x_f^3 = 1 - 5.545141E-6 + 0.5\times(5.545141E-6)^2 - 0.35\times(5.545141E-6)^3$$
$$= 0.999994$$

$$-5.0x_f + 2.0x_f^2 - 0.7x_f^3 = -5.0\times 5.545141E-6 + 2.0\times(5.545141E-6)^2 - 0.7\times(5.545141E-6)^3$$
$$= -2.051702E-5$$

$$5.0 - 2.0x_f + 0.7x_f^2 = 5.0 - 2.0\times 5.545141E-6 + 0.7\times(5.545141E-6)^2 = 4.99990$$

$$S_1 = \frac{2M\sin\theta p_1^2}{z_1 T_1 R} = \frac{2\times 28.013\times 0.0843318\times 1}{1\times 551.4\times 1545} = 619.424171$$

BB_p^a remains as in the case of horizontal flow. That is $BB_p^a = 185373$

$BB_p^a > S_1$, then,

$$J_p = P_1^2 - \frac{p_1^2}{6}\left(-5.0x_f + 2.0x_f^2 - 0.7x_f^3\right) - p_2^2$$

$$= 3069.36^2 - \frac{3069.36^2}{6}\left(-2.051702E-5\right) - 2116.80^2 = 4940161$$

$$\frac{S_1 L}{6}\left(1 - x_f + 0.5x_f^2 - 0.35x_f^3\right) = \frac{619.424171\times 0.0843176\times 0.999994}{6} = 8.704674$$

$$z_1T_1\left(1 - x_f + 0.5x_f^2 - 0.35x_f^3\right) = 1\times 551.4\times 0.999994 = 551.396691$$

$$z_1T_1\left(5.0 - 2.0x_f + 0.7x_f^2\right) = 1\times 551.4\times 4.999990 = 2756.99448\quad 6$$

Then,

$$f_p = \frac{(4940128.57 + 8.704674)}{(1.843574E-5)^2\times 185373\,(551.396692 + 2756.99448\,6)} = \frac{4940137.2750}{0.2084417} = 23700330.9\quad 6 = 0.237003\quad E\,8$$

The values of the dimensionless friction factor (f p) ,where the core is horizontal and where the core is vertical are very close. The effect of inclination is more severe on long porous media.

The cores are considered horizontal, in calculating the values of the dimensionless friction factor s shown in Table 4

3. Sample Calculation of Reynolds Number

$$R_{Np} = \frac{4W}{\pi g \mu_g d_p} = \frac{36.88575 G_g P_b Q_b}{R g d_p \mu_g z_b T_b}$$

Example 2

Calculate the Reynolds number of the first core in Table 4.

Solution

In the pound second feet (psf) system of units,

p_b = 1 atm = 1.00 ×14.7 × 144 psf = 2116.80 psf

Q_b = 7.5 cm^3/ sec = 7.5 ×3.531467 ft^3/ sec = 2.648711E - 4 ft^3/ sec

$$G_g = \frac{\text{Molecular weight of Nitrogen}}{\text{Molecular weight of Air}} = \frac{28.013}{28.97} = 0.966966$$

$$d_p = d\sqrt{\phi} = 2.54\sqrt{0.3073359}\ \text{cm} = 1.408122\ \text{cm} = 1.408122\times 3.280840\ E\text{-}2\ \text{ft}$$
$$= 0.046198\ \text{ft.}$$

R = 1545, g =32.2 ft / sec^2, z_b = 1.0, T_b = 33 ^{O}C = 551.4 ^{o}R.

μ_g = 0.018 cp = 0.018×2.088543 lb sec / ft^2 = 3.759174 E-7 lb sec / ft^2 . Then,

$$R_{NP} = \frac{36.885750 \times 0.966966 \times 2116.8 \times 2.648711 \text{ E - 4}}{1545 \times 32.2 \times 0.046198 \times 3.759174 \text{ E - 7} \times 1 \times 551.4} = 41.977400$$

$R_{NP}^{-1} = 0.0238223$

Other values of R_{NP}^{-1} are shown in Table 4.

The values of f_p were plotted against R_{NP}^{-1} for well developed non Darcian flow. The graph is shown in Figure 1.The graph can be represented by a parabola. A least square parabola curve fit of the graph produced the equation:

$$f_p = [a(R_{NP}^{-1})^2 + b(R_{NP}^{-1}) + c] \times 10^8 \qquad (11)$$

Where,

a = - 1856.19687
b = 90.3048304
z = - 0.8662895

Example 3

Calculate the dimensionless friction factor, given that:

(a) $R_{NP}^{-1} = 0.0238$ (b) $R_{NP}^{-1} = 0.0245$ (c) $R_{NP}^{-1} = 0.0263$ (d) $R_{NP}^{-1} = 0.0171$

(e) $R_{NP}^{-1} = 0.016$

Solution

(a) f_p = [-1856.19687(0.0238)^2 + 90.3048504(0.0238) - 0.8662895] × 10^8 = 0.2315 × 10^8
(b) f_p = [-1856.19687(0.0245)^2 + 90.3048504(0.0245) - 0.8662895] × 10^8 = 0.2320 × 10^8
(c) f_p = [-1856.19687(0.0263)^2 + 90.3048504(0.0263) - 0.8662895] × 10^8 = 0.2248 × 10^8
(d) f_p = [-1856.19687(0.0171)^2 + 90.3048504(0.0171) - 0.8662895] × 10^8 = 0.1352 × 10^8
(e) f_p = [-1856.19687(0.0160)^2 + 90.3048504(0.0160) - 0.8662895] × 10^8 = 0.1034 × 10^8

These values may be compared with experimental values of:

(a) f_p = 0.2370 × 10^8
(b) f_p = 0.2139 × 10^8
(c) f_p = 0.2144 × 10^8
(d) f_p = 0.1424 × 10^8
(e) f_p = 0.0887× 10^8

obtained from the graph shown in Figure 1.

Table 4: Calculated Dimensionless Friction Factor and Reciprocal of Reynolds Number

Serial No	Diameter (cm)	Length (cm)	Temperature ^oC	Flow Rate (Cm^3/sec)	Porosity ϕ	fp×10^8	R_{NP}^{-1}
1	2.54	2.57	33.0	7.5	0.30734	0.2370	0.0238
2	2.45	2.50	33.0	6.9	0.29344	0.2139	0.0245
3	2.48	2.55	33.4	6.7	0.30116	0.2526	0.0259
4	2.61	2.55	33.3	6.2	0.23938	0.2144	0.0263

5	2.57	2.54	34.0	9.5	0.301150	0.1510	0.0190
6	2.47	2.50	34.0	8.3	0.31660	0.1868	0.0214
7	2.56	2.54	31.0	11.2	0.33683	0.1400	0.0169
8	2.45	2.55	31.0	11.0	0.29730	0.0847	0.0154
9	2.57	2.54	31.0	11.0	0.33205	0.1424	0.0171
10	2.60	2.53	31.0	10.6	0.3359	0.1679	0.0180
11	2.59	2.53	31.0	10.8	0.3514	0.1776	0.0180
12	2.59	2.54	31.0	10.4	0.2587	0.0887	0.0160
13	2.58	2.54	31.0	10.4	0.2934	0.1192	0.0170
14	2.60	2.40	31.0	10.5	0.3900	0.2620	0.0196
15	2.58	2.51	31.0	9.2	0.2853	0.1437	0.0190

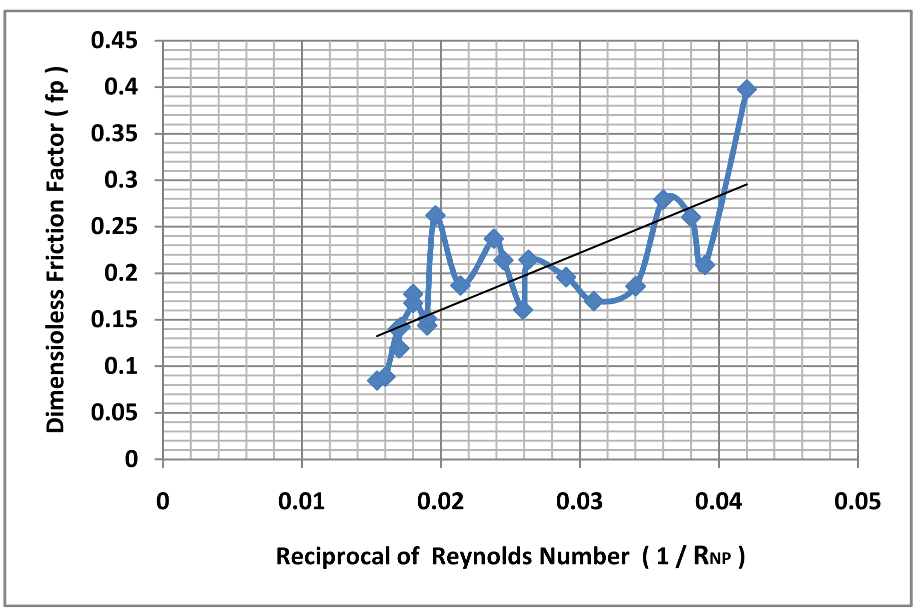

Figure 1: Graph of f $_p$ versus R_{NP}^{-1}

5. Conclusion

A new equation that relates dimensionless friction factor of a porous medium to its Reynolds number has been developed. The equation is valid for Reynolds number for porous media in the range 25 to 62.5

References

[1] Akpokene, A.O. (1983) "Friction Factors of certain Porous Materials", project submitted to the department of Petroleum Engineering, University of Benin, NIGERIA.

[2] Drew, T.B., Koo, E.C., and Mc Adams, W.H. (1930). Transactions. Am. Inst. Chem. Eng. 28, 56

[3] Nikuradse, J. (1933) Forschung shelf, p. 301.

[4] Ohirhian, P.U. (2008). 'A new Dimensionless Friction Factor for porous media', Journal of porous media, vol. 11, number 5, Begell House publishers, CT, U.S.A., pp. 497 – 506.

[5] Ohirhian, P.U. (2010). "Steady State Compressible Flow in Porous Media", Natural Gas, InTech - Open Access Publisher, University Campus STeP Ri, Slavka Krautzeka 83/A, 51000 Rijeka,CROATIA