Journal of the Nigerian Association of Mathematical Physics
***Volume* 18 (May, 2011), pp 91 - 98**
© J. of NAMP

Exact Vibration Solution for initially stressed Beams resting on Elastic Foundation and subjected to partially distributed masses

[1] B. Omolofe, [2]A. Adedowole, [3]S.O. Ajibola and [4]Jimoh Ahmed

[1]Department of Mathematical Sciences,
The Federal University of Technology, Akure, Ondo State, Nigeria

[2]Department of Mathematics & Industrial Mathematics.
Adekunle Ajasin University, Akungba Akoko. Nigeria.

[3]School of Science and Technology
National Open University of Nigeria, Victoria Island, Lagos State.

[4]Department of Mathematical Sciences, Faculty of Natural Sciences.
Kogi State University, Anyigba, Nigeria.

Abstract

In this paper, the response to moving distributed masses of a simply supported elastic thick beam resting on an elastic foundation with exponential rigidity is presented. The technique is based on the generalized Galerkin's method and integral transformation. Exact solutions are obtained and the convergence of these solutions established. Solutions obtained are calculated for various values of foundation moduli K, axial force N, and damping coefficient ε_0. It is observed that, as the values of these structural parameters increase, the transverse deflections of the finite elastic beam under the actions of moving masses decreases. Furthermore, the conditions under which the vibrating systems will experience resonance phenomenon are highlighted. Results presented in this paper are useful in structural engineering design and could also form basis for further investigations in this area of study.

Introduction:

This paper is concerned the problem of dynamic behaviour of elastic beam with exponentially varying foundation and under a moving partially distributed load. Studies in structural dynamics dealing with moving loads on bridges are enormous and have been enriched in the last few decades by the development of high-speed railway networks in the developed countries. Similarly, there exist remarkable advances in various branches of transport. These advances are characterized by increasingly higher speeds and weights of vehicles. Hence, structures and media over which the vehicles move have been subjected to vibrations and dynamic stresses far larger than ever before. Thus, there is the need for continuous study of the behaviour of bodies subjected to moving loads. Such studies will, for instance, provide safer and more economic design of structures on which the loads move.

The problem of concentrated forces moving with constant velocity along a slender member when damping effects is neglected has been investigated extensively. Steel [12] has worked on the finite beam with moving load, [13] studied the vibration of a beam under random stream of moving forces. Frybal [16] scrutinized non-stationary response of a beam to a moving random force, [17] have considered vibration of an elastic beam subjected to discrete loads, [18] have also studied non-linear vibration of Timoshenko beam due to moving force and the weight of the beam. Florence [20] has handled the problem of traveling force on a [4] have considered dynamics behaviour of beams and rectangular plate under moving loads while [14] treated the oscillations of infinite periodic beams subjected to a moving concentrated force.

In a more recent development, [7] considered the effects of damping and exponentially decaying foundation on the motions of finite beam subjected to concentrated loads. The latter authors [22] also considered dynamic response of structurally damped beams with rotatory inertia correction factor to moving concentrated forces. In these studies, effects of damping forces on the dynamic systems are well investigated. In all these aforementioned studies however, the scope of studies have been limited to the cases in which the traveling load is assumed to be point-like. The more practical cases in which the travelling loads is assumed to be distributed has not been extensively studied. It is well known that in reality, moving loads are usually distributed, but not point-like. Therefore, this paper investigates the dynamic behaviour of elastic beams with exponentially decaying foundation under moving distributed masses.

Corresponding author: ***B. Omolofe: e-mail:-----,*** Tel. +2348062140408 - Adedowole

2. THE BASIC EQUATION

The transverse displacement $W(x,t)$, of a uniform finite beam resting on an elastic foundation of the exponential rigidity and carrying n masses M^i (i=1,2…….n) is governed by the fourth order differential equation [8].

$$D_1\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{\partial^2 W(x,t)}{\partial t^2}+D_2\frac{\partial W(x,t)}{\partial t}+D_3 W(x,y)=\frac{1}{\mu}P(x,t)+N^0\frac{\partial^2 W(x,t)}{\partial x^2}+\frac{R_o\partial^4 W(x,t)}{\partial x^2\partial t^2} \qquad (2.1)$$

where

$D_1=EI/\mu$, $D_2=\varepsilon_0/\mu$, $D_3=F_0 e^{-\lambda x}$, $F_0=k_0/\mu$, $N^0=N/\mu$ and x is the spatial coordinate, E is the young modulus ,I is the moment of inertia, W(x,t) is the transverse displacement response of the vibrating beam, N is the axial force, μ is the mass per unit length, ε_0 is the damping coefficient, K (x) is the non-uniform elastic foundation, R_o is the measure of rotating inertia effect and P (x,t) is the time dependent distributed moving load.

The time t is assumed to be limited to that interval of time when the mass μ is on the beam that is $0\le C_i t\le L$

In this study, a simply supported beam is considered and thus the following boundary conditions pertain.

$$W(x,t)=0=W(L,t),\ \frac{\partial W(0,t)}{\partial x}=0=\frac{\partial^2 W(L,t)}{\partial x^2} \qquad (2.2)$$

and the initial conditions are

$$W(x,0)=0=\frac{\partial W(x,0)}{\partial t} \qquad (2.4)$$

We assumed that the beam is continuously supported by elastic foundation whose rigidity is of exponential form given by

$$K(x)=K_0 e^{-\lambda x} \qquad (2.5)$$

where λ is a constant and K_0 is the elastic foundation constant.

To treat (2.1), two special cases are considered.

i. When the distributed load P(x,t) is assumed to be a constant type and take the form

$$P(x,t)=PH(x-c_m t) \qquad (2.6)$$

ii. When the distributed load P(x,t) is assumed to be of varying magnitude and take the form

$$P(x,t)=Pe^{-\omega t}H\left(x-c_j t\right) \qquad (2.7)$$

3 Case 1

Dynamic response of structural elastic beams with Rotatory inertia correction factor under constant magnitude moving distributed masses.

The distributed moving loads P(x, t) in equation (2.1) is assumed to be of constant magnitude in this section thus we have

$$P(x,t)=PH(x-c_m t) \qquad (3.1)$$

$c_m$is the velocity of the mth particle of the system. Substituting equation (3.1) into equation (2.1) gives

$$D_1\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{\partial^2 W(x,t)}{\partial t^2}+D_2\frac{\partial W(x,t)}{\partial t}+D_3 W(x,y)-N^0\frac{\partial^2 W(x,t)}{\partial x^2}-\frac{R_o\partial^4 W(x,t)}{\partial x^2\partial t^2}=\frac{1}{\mu}PH\left(x-c_m t\right) \qquad (3.2)$$

Equation (3.2) above represents the motion of a transverse displacement of a uniform finite beam under moving distributed loads. In what follows, we seek the solution of (3.2), though the equation may yield readily to numerical technique, but an analytical solution is desirable as the solution so obtained often shed light on vital information about the vibrating system. Thus, to this end we make use of a versatile analytical technique called assumed mode method.

3.1 SOLUTION PROCEDURE

By applying the assumed mode technique, the dynamic deflection W(x,t) of the vibrating beam, can be written as

$$W_a(x,t)=\sum_{m=1}^{n}R_m(t)P_m(x) \qquad (3.3)$$

where $R_m(t)$ are coordinates in modal space

$P_m(x)$ is the normal mode of vibrations of the beam and is given as

$$P_m(x)=Sin\alpha_m x+A_m Cos\alpha_m x+B_m Sinh\alpha_m x+c_m Cosh\alpha_m x \qquad (3.4)$$

Thus, for a simply supported beam, it can be shown that $A_m=B_m=C_m=0\ \ and\,\alpha_m=\frac{m\pi}{L}$

where A_m, B_m and C_m are constants that can be determined using the boundary conditions.

Thus for a beam with simple supports at both ends equation (3.4) takes the form

$$P_m(x) = Sin\frac{m\pi x}{L} \tag{3.5}$$

In view of the equation (3.5), the transverse displacement response of a simply supported elastic beam, using an assumed mode method can be written as

$$W_a(x,t) = \sum_{m=1}^{n} R_m(t) Sin\frac{m\pi x}{L} \tag{3.6}$$

Substituting equation (3.6) into the governing equation (2.6) and after some simplifications and arrangements we obtain

$$\sum_{m=1}^{\infty}\left\{D_1(\frac{m\pi}{L})^4 R_m(t)Sin\frac{m\pi x}{L} + \ddot{R}_m(t)Sin\frac{m\pi x}{L} + N^0(\frac{m\pi}{L})^2 R_m(t)Sin\frac{m\pi x}{L} + D_2\dot{R}_m Sin\frac{m\pi x}{L} + D_3 e^{-\lambda x} R_m(t) Sin\frac{m\pi x}{L}\right\} - \frac{1}{\mu}PH(x-c_m t) = 0 \tag{3.7}$$

The solution technique requires that the RHS of equation (3.4) be orthogonal to the function $\sin\frac{m\pi x}{L}$

Thus, multiplying equation (3.7) by $\sin\frac{m\pi x}{L}$ and integrating from 0 to L with respect to x after some simplifications and rearrangements yield

$$\sum_{m=1}^{\infty}\left\{H_1R_m(t) + H_2\ddot{R}_m(t) + H_3R_m(t) + H_4\dot{R}_m(t) + H_5R_m(t) + H_6R_m(t)\right\} = \frac{1}{\mu}\int_0^L PH(x-c_m t)\sin\frac{k\pi x dx}{L} \tag{3.8}$$

where

$$H_1 = D_1\left(\frac{m\pi}{L}\right)^4 \int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \quad H_2 = N^0\int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx$$

$$H_3 = \int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \quad H_4 = r_0\frac{(m\pi)^2}{L}\int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx,$$

$$H_5 = D_2\int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \quad H_6 = F_0\int_0^L e^{-\lambda x} Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \tag{3.9}$$

and over-dot represents the partial derivative with respect to time t.

We note the following properties of the Heaviside function

(i) $$\frac{d}{dx}H(x-ct) = \delta(x-ct) \tag{3.10}$$

(ii) $$f(x)H(x-ct) = \begin{cases} 0, & x \le ct \\ f(x), & x \ge ct \end{cases} \tag{3.11}$$

Thus using equations (3.10) & (3.11), equation (3.8) can be written after some rearrangements as

$$\sum_{m=1}^{\infty}\left\{\ddot{R}_m(t) + Q_1\dot{R}_m(t) + Q_2R_m(t)\right\} = Q_3 Cos\theta t \tag{3.12}$$

Where

$$Q_1 = \frac{H_5}{H_3+H_4}, \quad Q_2 = \frac{H_1+H_2+H_6}{H_3+H_4},$$

$$Q_3 = \frac{1}{\mu}\frac{P(-\theta)}{(H_3+H_4)} \quad \text{and} \quad \theta = \frac{k\pi C_m}{L} \tag{3.13}$$

Now considering only the m^th particle of the dynamical system we have

$$\ddot{R}_m(t) + Q_1\dot{R}_m(t) + Q_2R_m(t) = Q_3 Cos\theta t \tag{3.14}$$

Subjecting the second order ordinary differential equation (3.14) to a transformation

$$(\tilde{\bullet}) = \int_0^{\infty}(\bullet)e^{-st}dt \tag{3.15}$$

in conjunction with the initial conditions defined in equation (2.3), yields the following algebraic equation

$$(S^2 + Q_1S + Q_2)R_m(s) = Q_3\frac{S}{S^2+\theta^2} \tag{3.16}$$

It is straight forward to show that equation (3.5) after some simplifications and rearrangements yields

$$R_m(S) = \frac{Q_3}{(\gamma_1-\gamma_2)}\left(\frac{S}{S^2+\theta^2}\times\frac{1}{S-\gamma_1} - \frac{S}{S^2+\theta^2}\times\frac{1}{S-\gamma_2}\right) \tag{3.17}$$

where

$$\gamma_1 = \frac{-Q_1+\sqrt{Q_1^2-4Q_2}}{2} \text{ and } \gamma_2 = \frac{-Q_1-\sqrt{Q_1^2-4Q_2}}{2} \tag{3.18}$$

In order to obtain the Laplace inversion of equation (3.17), we shall adopt the following representations

$$g(s) = \frac{S}{S^2+\theta^2}, \qquad f_1(s) = \frac{1}{S-\gamma_1} \qquad \text{and} \quad f_1(s) = \frac{1}{S-\gamma_2} \tag{3.19}$$

So that the Laplace inversion of equation (3.17) is the convolution of f_i and g defined as

$$f_i * g = \int_0^t f_i(t-\mu)g(u), i = 1,2 \tag{3.20}$$

Thus the Laplace inversion of (3.17) is given by $R_m(t) = Q_3\left[e^{\gamma_1 t}Z_1 - e^{\gamma_2 t}Z_2\right]$ (3.21)

Where $Z_1 = \int_0^t e^{-\gamma_1 u}\cos\theta u du$ and $Z_2 = \int_0^t e^{-\gamma_2 u}\cos\theta u du$ (3.22)

Thus in view of equation (3.21) and taking into account integrals (3.22) we get

$$R_m(t) = \frac{Q_3\gamma_1}{(\gamma_1-\gamma_2)(\theta^2+\gamma_1^2)}\left((e^{\gamma_1 t}-\cos\theta t)-\frac{\theta}{\gamma_2}\sin\theta t\right) -- \frac{Q_3\gamma_2}{(\gamma_1-\gamma_2)(\theta^2+\gamma_2^2)}\left((e^{\gamma_2 t}-\cos\theta t)-\frac{\theta}{\gamma_2}\sin\theta t\right) \tag{3.23}$$

which on inversion yields

$$W(x,t) = \sum_{m=1}^{\infty}\left[\frac{Q_3\gamma_1}{(\gamma_1-\gamma_2)(\theta^2+\gamma_1^2)}\left((e^{\gamma_1 t}-\cos\theta t)-\frac{\theta}{\gamma_1}\sin\theta t\right) - \frac{Q_3\gamma_2}{(\gamma_1-\gamma_2)(\theta^2+\gamma_2^2)}\left((e^{\gamma_2 t}-\cos\theta t)-\frac{\theta}{\gamma_2}\sin\theta t\right)\right]\sin\frac{m\pi x}{L} \tag{3.24}$$

which represents the dynamic response of a structurally damped elastic thick beam resting on exponentially decaying foundation and subjected to partially distributed constant moving load.

In what follows we now establish the convergence of the series solution (3.24)

4.0. THE CONVERGENCE OF THE SERIES SOLUTION

To show that the series solution (3.24) converges, we only need to demonstrate that the coefficient $R_m(t)$ of the spatial sine term in equation (3.24) is convergent. While we note we can write

$$R_m(t) \le \frac{Q_3}{(\gamma_1-\gamma_2)}\left[\frac{\gamma_1\left(1+\left\{\frac{\theta}{\gamma_2}-e^{\gamma_1 t}\right\}\right)}{(\theta^2+\gamma_1^2)} + \frac{\gamma_2\left(1+\left\{\frac{\theta}{\gamma_2}-e^{\gamma_2 t}\right\}\right)}{(\theta^2+\gamma_2^2)}\right] \tag{4.1}$$

Now, using equation (3.13), we have

$$R_m(t) < -\frac{\frac{Pk\pi C_m}{L}}{(\gamma_1-\gamma_2)\left(\frac{r_o\pi^2}{2}+\frac{L}{2}\right)}\left[\frac{\gamma_1\left(1+\left\{\frac{\theta}{\gamma_2}-e^{\gamma_1 t}\right\}\right)}{(\theta^2+\gamma_1^2)} + \frac{\gamma_2\left(1+\left\{\frac{\theta}{\gamma_2}-e^{\gamma_2 t}\right\}\right)}{(\theta^2+\gamma_2^2)}\right]\times\frac{1}{m^4} \tag{4.2}$$

Therefore, the coefficients $R_m(t)$ and the solution $W(x,t)$ converges as m^{-4}.

5.0 Case II

Dynamic response of structurally elastic beams with Rotatory inertia correction factor Foundation under Harmonic variable magnitude moving distributed masses.

The dynamic response of finite beam with exponentially decaying foundation under harmonic variable magnitude moving distributed masses is investigated. Thus, the load P(x, t) is given as

$$P(x,t) = Pe^{-\omega t}H\left(x-c_j t\right) \tag{5.1}$$

where all parameters x, c_i and t are as defined previously, using equation (5.1) in equation (2.1) and taking into account equation (2.5)

$$D_1 \frac{\partial^4 W(x,t)}{\partial x^4} + \frac{\partial^2 W(x,t)}{\partial t^2} + D_2 \frac{\partial W(x,t)}{\partial t} + D_3 W(x,y) - N^0 \frac{\partial^2 W(x,t)}{\partial x^2} - \frac{R_o \partial^4 W(x,t)}{\partial x^2 \partial t^2}$$
$$= \frac{1}{\mu} P e^{-\omega t} H\left(x - c_j t\right) \tag{5.2}$$

Equation (5.2) is the governing equation describing the motion of structurally elastic beam with exponentially decaying foundation subjected to fast moving distributed loads of varying magnitude. Closed-form solution to equation (5.2) is sought as in the previous section.

In this view, use is made of an assumed mode method already alluded to and by this method, the transverse deflection $W_b(x,t)$ of a thick beam under the action of variable magnitude moving load can be written as

$$W_b(x,t) = \sum_{j=1}^{n} R_j(t) P_j\left(x\right) \tag{5.3}$$

where R_j (t) are coordinates in modal space and $P_j(x)$ are the normal modes of free vibration. Thus for a simply supported beam equation (5.3) becomes

$$W_b(x,t) = \sum_{m=1}^{n} R_j(t) Sin \frac{j\pi x}{L} \tag{5.4}$$

Using equation (5.4) in equation (5.2) and following the same arguments similar to the previous section and after some simplifications and rearrangements one obtains

$$\sum_{j=1}^{\infty} \left\{ \ddot{R}_j(t) + Q_1 \dot{R}_j(t) + Q_2 R_j(t) \right\} = Q_3 e^{-\omega t} \cos \frac{i\pi x c_j}{L} \tag{5.5}$$

where c_j is the velocity of the j^{th} particle of the system and other parameters are as defined previously.

By considering only the j^{th} particle of the dynamical system we get

$$\ddot{R}_j(t) + Q_1 \dot{R}_j(t) + Q_2 R_j(t) = Q_3 e^{-\omega t} \cos \eta t \tag{5.6}$$

Equation (5.6) is analogous to equation (3.14). Subjecting equation (5.6) to Laplace transform in conjunction with the boundary condition (2.2) and using convolution theory we obtain

$$R_j(t) = \frac{\eta Q_3}{(\theta_1 - \theta_2)} \left[\left\{ \beta_2 \theta_4 \left(\cos \eta t - e^{\theta_4 t} \right) \right\} + \beta_1 \theta_3 (e^{\theta_3 t} - \cos \eta t) + (\beta_1 - \beta_2) \sin \eta t \right] \tag{5.7}$$

which on inversion yields

$$W_b(x,t) = \sum_{j=1}^{n} \frac{\eta Q_3}{(\theta_1 - \theta_2)} \left[\left\{ \beta_2 \theta_4 \left(\cos \eta t - e^{\theta_4 t} \right) \right\} + \beta_1 \theta_3 (e^{\theta_3 t} - \cos \eta t) + (\beta_1 - \beta_2) \sin \eta t \right] Sin \frac{j\pi x}{L} \tag{5.8}$$

which represents the dynamic response of a structurally damped elastic thick beam resting on exponentially decaying foundation and subjected to partially distributed harmonic variable magnitude moving load. It can be shown, following the same procedures as in the previous sections that series solution (5.8) converges rapidly.

6.0 DISCUSSION OF CLOSED FORM SOLUTIONS

The response amplitude of a dynamical system such as this may grow without bound. Condition under which this happens is termed resonance conditions. Evidently, from equation (3.22) a structurally elastic damped thick beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads will experience resonance effects when

$$\gamma_1 = \gamma_2, -\gamma_1^2 = \theta^2 or - \gamma_2^2 = \theta^2 \tag{6.1}$$

and the velocity at which this occurs is termed the critical velocity and it is given by

$$C_a^2 = \left(4Q_2 + 2Q_1(Q_1^2 - 4Q_2)^{1/2} - 2Q_1^2\right) \times \left(\frac{L}{k\pi}\right)^2 \tag{6.2}$$

While the same system traversed by a structurally elastic damped thick beam in equation (5.8) and subjected to harmonic variable magnitude moving loads will experience resonance effects whenever

$$\theta_1 = \theta_2, -\theta_3^2 = \eta^2 or - \theta_4^2 = \eta^2 \tag{6.3}$$

And the velocity at which this occurs, is known as critical velocity and it is given by the relation

$$C_b^2 = \left(4Q_2 + 2Q_1(Q_1^2 - 4Q_2)^{1/2} - 2Q_1^2\right) \times \left(\frac{L}{i\pi}\right)^2 - \omega \tag{6.4}$$

Equations (6.1) and (6.3) show that for the same natural frequency, the critical speed for the system consisting a structurally damped beam on exponentially decaying foundation and under the actions of constant magnitude travelling load is smaller than that of the system involving elastic beam under the actions of harmonic variable magnitude moving load. Thus, resonance is reached earlier in latter than in the former.

7 NUMERICAL RESULT S AND DISCUSIONS

In order to illustrate the analytical results, the structurally damped thick beam is taken to be of length /12.192m.

Other values used are velocity, c of the distributed loads which is taken to be 8.128m/s. The flexural rigidity EI is $6068242m^3/s^2$. The values of foundation moduli are varied between $0N/m^3$ and 4×10^8 N/M^3, the values of axial force N are varied between 0 N/M^3 and $2 \times 10^8 N/M^3$ and the values of rotatory inertia r_o are varied between 0 and 2 . The transverse displacement response of a structurally damped thick beam resting on exponentially decaying foundation and under the actions of traveling distributed forces are calculated and plotted against time for both constant and variable magnitude cases and for various values of foundation moduli K_0, damping coefficient ε_0, axial force N and rotatory inertia r_o.

Figures 7.1 and 7.5 display the effect of axial force N on the transverse deflection of a structurally damped thick beam resting on variable elastic foundation under the actions of traveling distributed forces for both constant and variable magnitude loads respectively for fixed values of K_0, ε_0, N and r_o. The graphs show that the response amplitude decreases as axial force N increases. In figures 7.2 and 7.6, the response amplitude of a structurally damped thick beam resting on variable elastic foundation when the traversing distributed forces are of constant and variable magnitude respectively are depicted. It is clearly seen that as the values of foundation moduli K_0 increase, for fixed values of the damping coefficient ε_0, axial force N and rotatory inertia r_o, the response amplitudes decrease.

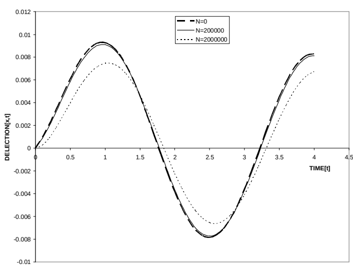

Figure 7.1: Transverse displacement response of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of axial force N and for fixed values of foundation moduli K_0 = 400,000, rotatory inertia r_o=0.5 and damping coefficient ε_0 = 78

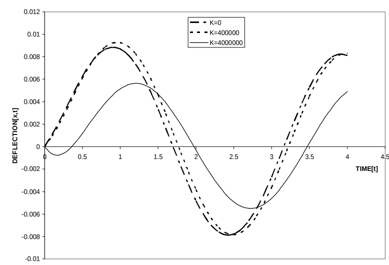

Figure 7.2: Displacement response of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of foundation moduli K_0 and for fixed values of axial force N = 200,000, rotatory inertia r_o=0.5 and damping coefficient ε_0 = 78

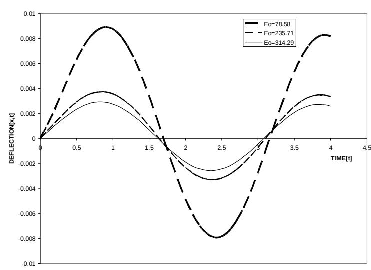

Figure 7.3: Response amplitude of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of damping coefficient ε_0 and for fixed values of axial force N = 200,000, rotatory inertia r_o=0.5 and foundation moduli K_0 = 400,000

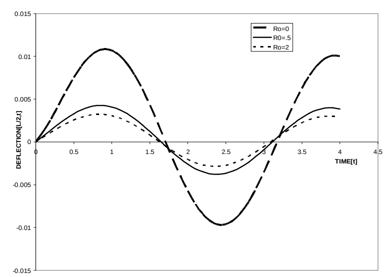

Figure 7.4: Displacement response of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of r_o and for fixed values of foundation moduli K_0, axial force N = 200,000 and damping coefficient ε_0 = 78

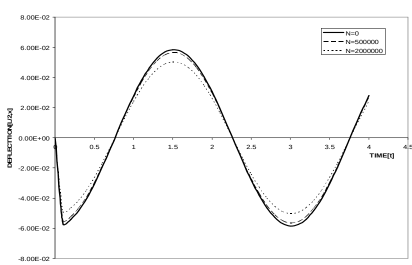

Figure 7.5: Transverse displacement response of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of axial force N and for fixed values of foundation moduli K_0 = 400,000, rotatory inertia r_o=0.5 and damping coefficient ε_0 = 43

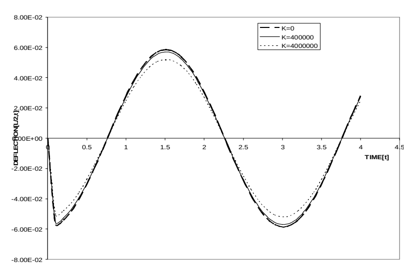

Figure 7.6: Deflection profile of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of foundation moduli K_0 and for fixed values of axial force N = 200,000, rotatory inertia r_o=0.5 and damping coefficient ε_0 = 43

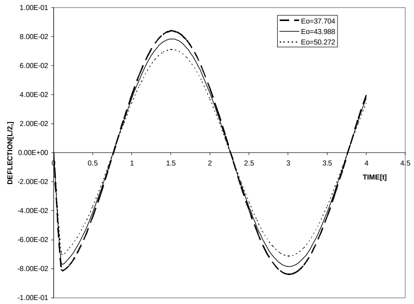

Figure 7.7: Transverse displacement response of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of damping coefficient ε_0 and for fixed values of foundation moduli K_0 = 400,000, rotatory inertia r_o=0.5 and axial force N=200,000

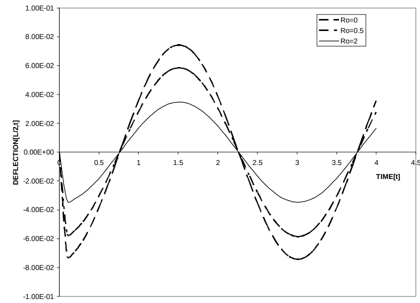

Figure 7.8: Response amplitude of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of r_o and for fixed values of damping coefficient ε_0=37 ,axial force N = 200,000 and foundation moduli K_0 = 400,000

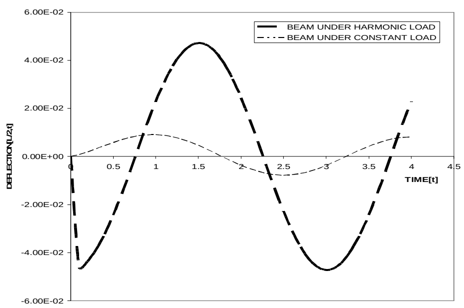

Fig 7.9:Comparison of the response amplitude of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant and variable magnitude moving loads for fixed values of damping coefficient ε_0=37 , axial force N = 200,000, rotatory inertia r_o=0.5 and and foundation moduli K_0 = 400,000

The transverse displacement response of a structurally prestressed damped thick beam resting on variable elastic foundation under the actions of traveling distributed forces for both constant and variable magnitude loads are depicted in figures 7.3 and 7.7 respectively. The graphs show that higher values of damping coefficient ε_0 and for fixed values of foundation moduli K_0 = 400,000, rotatory inertia r_o=0.5 and axial force N=200,000 reduce the deflection profiles in both cases.

The effect of rotatory inertia r_o on the transverse deflection of a structurally prestressed damped thick beam resting on variable elastic foundation under the actions of traveling distributed forces for both constant and variable magnitude loads for fixed values of K_0, ε_0, and N are displayed in figures 7.4 and 7.8 respectively. The results show that as the rotatory inertia r_o increases the maximum amplitude of the beam decreases. Finally, Figure 7.9 depicts the comparison of the response amplitude of a simply supported structurally damped thick beam resting on exponentially decaying foundation and subjected to constant and variable magnitude moving loads for fixed values of damping coefficient ε_0=63 ,axial force N = 200,000, rotatory inertia r_o=0.5 and and foundation moduli K_0 = 400,000. Clearly, the response amplitude of variable magnitude moving load is higher than that of the constant magnitude moving load.

7. CONCLUSIONS

The problem of the dynamic response of a structurally damped thick beam with exponentially decaying foundation resting on variable elastic foundation under the action of traveling distributed masses has been investigated. Also, numerical analysis has been carried out by way of illustration, for structurally damped beam under the action of constant magnitude moving distributed masses and harmonic variable magnitude moving distributed masses. The analysis shows that:

(i) the transverse displacement response of the elastic beam decreases as the value of axial force N increases for fixed values of foundation modulus K_o, rotatory inertia r_0 and damping ratio ε_0 .

(ii) the higher the value of foundation stiffness K_0 the lower the deflection profile of the beam for fixed value of the beam parameters axial force N, rotatory inertia r_0 and damping coefficient ε_0 .

(iii) for fixed values of axial force N, rotatory inertia r_0 and foundation modulus $K_{0,}$ the response amplitude of the vibrating beam reduces as the values of damping coefficient ε_0 increase.

(iv) as the values of rotatory inertia r_0 increase, the displacement response of the beam reduces for fixed values of axial force N, foundation moduli K_0 and damping coefficient ε_0.

(v) the critical velocity of the vibrating system involving structurally damped beam under the action of distributed moving masses increases as the values of the beam parameters namely foundation stiffness K_0, axial force N, rotatory inertia r_0 and damping coefficient ε_0 increase thereby reducing the risk of resonance.

REFERENCES

[1] E. Esmailzadeh and M. Ghorashi (1995); Vibration Analysis of Beams Traversed by Uniform Partially Distributed Moving Masses. *Journal of Sound and Vibration.* 184(1), 9-17.

[2] Y.H. Lin and J.A. Gbadeyan (1997); Comments on Dyanmic Behaviour of Beams and Rectangular Plates under moving Loads. *Journal of Sound and Vibration.* 200(5), 721-728.

[3] Gbadeyan J.A. and Oni S.T. (1992); Dynamic Response to moving concentrated masses of elastic plate on a non-winkler elastic foundation. *Journal of Sound and Vibration,* 154(2), 343-358.

[4] Gbadeyan J.A. and Oni S.T. (1995); Dynamic Behaviour of Beams and Rectangular Plates under moving Loads. *Journal of Sound and Vibration*, 182(5), 677-695.

[5] Sadiku S. and Leipholz H.H.E. (1989); On the Dynamics of Elastic System with moving concentrated masses. *Ingenieur Achives*, 57, 223-242.

[6] Wu J.S., Lee M.L. and Lai T.S. (1987); The Dynamic Analysis of a Flat Plate under a moving load by the finite element method. *International Journal of Numerical Methods in Engineering,* 24, 743-762.

[7] B. Omolofe and B.M. Oseni (2008); Effects of Damping and Exponentially Decaying Foundation on the Motions of Finite thin beam subjected to traveling loads. *Journal of the Nigerian Association of Mathematical Physics*. Vol. 13, 119-126.

[8] S.T. Oni and S.N. Ogunyebi, (2008): Dynamical analysis of a prestressed elastic beam with general boundary conditions under the action of uniform distributed masses. *Journal of Nigerian Association of Mathematical Physics*. Vol. 12, 87-102.

[9] A.N. Krylov, (1905): Mathematical Collection of Papers of Academy of Sciences. Vol. 61, *Piterburg*.

[10] A.N. Lowan, (1935): On the transverse oscillations of beams under the action of moving variable loads; *Phil. Mag. Ser. 7, Vol. 19, No. 127, pp 708-715*.

[11] Fryba, L. (1972): Vibration of Solids and Structures under Moving Loads. *Groningen*: Noordhoff.

[12] C.R. Steel, (1967): The finite beam with moving loads. Applied Mechanics, Vol. 34, pp 111-118.

[13] R. Iwankiewicz, and P. Sniady, (1984), Vibration of a beam under random stream of moving forces, *Journal of Structural Mechanics 12; 13-26.*

[14] P.M. Belotserkovskiy, (1996): On the Oscillations of infinite periodic beams subjected to a moving concentrated force. *Journal of Sound and Vibration* 193(3), 706-712.

[15] S. Timoshenko, D.H. Young and W. Weaver (1974): Vibration problems in Engineering. New York: John Willey; fourth edition.

[16] L. Frybal (1976): Non-Stationary response of a beam to a moving random force. *Journal of Sound and Vibration* 46, 323-338.

[17] M. Kurihara and T. Shimogo, (1978): Vibration of an elastic beam subjected to discrete loads. *ASME Journal of Mechanical Design, 100,* 514-519.

[18] R.T. Wang and T.H. Chou, (1998): Non-Linear Vibration of Timoshenko beam due to a moving force and the weight of beam. *Journal of Sound and Vibration 218*, 117-131.

[19] J.S. Licari and E.N. Wilson (1962): Dynamic response of a beam subjected to a moving force system. *Proceedings of the fourth U.S. National Congress of Applied Mechanics 1,* 419-425.

[20] A.L. Florence, (1965): Traveling force on a Timoshenko Beam. *Journal of Applied Mechanics, 32,* 351-358.

[21] M.A. Hilal and H.S. Zibdeh (2000): Vibration analysis of beams with general boundary conditions traversed by a moving force. *Journal of Sound and Vibration 229(2),* 377-388.

[22] B. Omolofe and M. Oseni (2000): Dynamic response of structurally damped beams with rotatory inertia correction factor to moving concentrated forces. *Nigerian Journal of Mathematics and applications* Vol. 19, pp100-113,(2009).