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Convergence of Closed Form Solutions of The Initial-Boundary Value Moving Mass Problem of Rectangular Plates Resting On Pasternak Foundations.

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Abstract

The problem of assessing the effects of Pasternak foundation on the response to several moving masses of isotropic rectangular plates for all variants of classical end supports was studied in Oni [1]. Closed form solution technique involved the use of the generalized two-dimensional integral transform and modified asymptotic method of Struble. This paper establishes the convergence of the analytical solutions of two illustrative examples of this class of problems and the robustness of the solution technique.

Introduction:

This paper seeks to establish the convergence of the closed form solution of the problem of flexural vibrations under moving loads of isotropic rectangular plates on a Pasternak elastic foundation obtained in [1]. The problem involves a non-homogeneous fourth order Partial differential equations [2, 3, 4, 5] with variable and singular coefficients. The solution technique is based, first, on the two-dimensional integral transform as obtained in [6] which was used to remove the singularity in the governing partial differential equation and to reduce it to a sequence of second order differential equation with variable coefficients. This second order differential equation was then simplified using the modification of the asymptotic technique due to Struble [7]. The method of integral transformation and convolution theory were then employed to obtain the analytical solution of the two-dimensional dynamical problem. The main objective of this paper is to establish that the solution obtained by this technique is not only a formal solution but it is the actual solution to the problem [8].

2 CLOSED FORM SOLUTION TECHNIQUE

The governing fourth order partial differential equation of the problem of flexural vibrations under moving loads of isotropic rectangular plate on a Pasternak
foundation is symbolically written in the form

$$L[W(x,y,t)] - Q(x,y,t) = 0 \tag{2.1}$$

where L is the differential operator with variable coefficient, $W(x,y,t)$ is the plate's response displacement, $Q(x,y,t)$ is the transverse load acting on the rectangular plate, x, y are spatial coordinates and t is the time coordinate.

In the first instance, equation (2.1) is treated with the generalized two dimensional integral transform defined by

$$V(j,k,t) = \int_0^{L_x}\int_0^{L_y} W(x,y,t)W_j(x)W_k(y)dydx \tag{2.2}$$

with the inverse given as

$$W(x,y,t) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\overline{m}}{V_j}\frac{\overline{m}}{V_k}V(j,k,t)W_j(x)W_k(y) \tag{2.3}$$

where

$$V_j = \int_0^{L_x}\overline{m}W_j^2(x)dx \tag{2.4}$$

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$$V_k = \int_0^{L_y} \overline{m} W_k^2(y) dy \tag{2.5}$$

The function $W_j(x)$ and $W_k(y)$ are respectively the beam functions in the x and y directions. Since each of these beam functions satisfies all the boundary conditions in its direction, the kernel (the product of these beam functions) in the integral transform satisfies all boundary conditions for rectangular plate problem of practical interest.

In particular, these beam functions can be defined, respectively as

$$W_j(x) = \sin\frac{\lambda_j x}{L_x} + A_j \cos\frac{\lambda_j x}{L_x} + B_j \sinh\frac{\lambda_j x}{L_x} + C_j \cosh\frac{\lambda_j x}{L_x} \tag{2.6}$$

$$W_k(y) = \sin\frac{\lambda_k y}{L_y} + A_k \cos\frac{\lambda_k y}{L_y} + B_k \sinh\frac{\lambda_k y}{L_y} + C_k \cosh\frac{\lambda_k y}{L_y} \tag{2.7}$$

where $A_j, B_j, C_j, A_k, B_k, C_k$ are constants determined by the boundary conditions. λ_j and λ_k are mode frequencies.

In addition, use is made of the property of Dirac Delta function as an even function to express it in a Fourier cosine series namely,

$$\delta(x - v_i t) = \frac{1}{L_x} + \frac{2}{L_x}\sum_{n=1}^{\infty} \cos\frac{n\pi v_i t}{L_x} \cos\frac{n\pi x}{L_x} \tag{2.8}$$

and

$$\delta(y - s) = \frac{1}{L_y} + \frac{2}{L_y}\sum_{m=1}^{\infty} \cos\frac{n\pi y}{L_y} \cos\frac{n\pi s}{L_y} \tag{2.9}$$

where v is the velocity of the moving load.

In order to simplify and solve the resulting sequence of coupled ordinary differential equations, the modified Struble's asymptotic technique is employed. The technique required that the asymptotic solution of the homogeneous part of (2.1) be of the form

$$V(j,k,t) = \Lambda(j,k,t)\cos[\omega_{j,k} t - \phi(j,k,t)] + \mu V_1(j,k,t) + o(\mu^2) \tag{2.10}$$

where $\Lambda(j,k,t)$ and $\phi(j,k,t)$ are slowly time varying functions or equivalently

$$\begin{aligned} &\frac{d\Lambda}{dt}(j,k,t) \to 0(\mu), && \frac{d^2\Lambda}{d^2 t}(j,k,t) \to 0(\mu^2) \\ &\frac{d\phi}{dt}(j,k,t) \to 0(\mu), && \frac{d^2\phi}{d^2 t}(j,k,t) \to 0(\mu^2) \end{aligned} \tag{2.11}$$

where $\rightarrow$ implies "is of ".

Expression for $W(x,y,t)$ is obtained via the method of integral transformation and convolution theory for simply supported end conditions as

$$W(x,y,t) = 4\mu g \sum_{j=i}^{\infty}\sum_{k=1}^{\infty} \frac{[\sin\frac{j\pi vt}{L_x} - \frac{j\pi v}{\gamma_{j,k} L_x}\sin\gamma'_{j,k} t]\sin\frac{k\pi s}{L_y}\sin n\frac{j\pi x}{L_x}\sin n\frac{j\pi y}{L_y}}{\gamma'^2_{j,k} - f_j^2} \tag{2.12}$$

where

$$\gamma'_{jk} = \omega^*_{jk}\left\{1 - \frac{\mu}{2}\left[1 + \sin^2\frac{k\pi s}{L_y} + \frac{v^2 j^2 \pi^2}{\omega^{*2} L_x^2}\left(1 + 4\sin^2\frac{k\pi s}{L_y}\right)\right]\right\} \tag{2.13}$$

$$\omega^*_{jk} = k_1\left(\frac{j^4\pi^4}{L_x^4} + \frac{2 j^2 k^2 \pi^2}{L_x^2 L_y^2} + \frac{k^4\pi^4}{L_y^4}\right) + D_2 + D_3\left(\frac{j^2\pi^2}{L_x^2} + \frac{k^2\pi^2}{L_y^2}\right) \tag{2.14}$$

$$\Gamma_0 = \frac{M}{\overline{m} L_x L_y} = \mu + o(\mu^2) \tag{2.15}$$

where μ is a parameter much less than one

$$f_i = \frac{j\pi v}{L_x}, \qquad k_1 = \frac{D}{\overline{m}}, \qquad D_2 = \frac{K}{\overline{m}}, \qquad D_3 = \frac{G}{\overline{m}} \tag{2.16}$$

K = foundation Stiffness, G = Shear modulus, D = Bending rigidity, $\overline{m}$ = mass per unit area of the plate.
Furthermore, expression for W(x, y, t) for clamped-clamped end conditions is obtained as

$$W(x,y,t) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\overline{m}}{V_j^*}\frac{\overline{m}}{V_k^*}\frac{D^*}{2\gamma_{bjk}(\Omega_j^4-\gamma_{bjk}^4)}\{(\Omega_j^2-\gamma_{bjk}^2)\{\gamma_{bjk}[\cosh\Omega_j t-\cos\gamma_{bjk}]- \sigma_j[\gamma_{bjk}\sinh\Omega_j t-\Omega_j\sin\gamma_{bjk}t]\}+(\Omega_j^2+\gamma_{bjk}^2)\{\gamma_{bjk}[\cos\Omega_j t-\cos\gamma_{bjk}t]- \sigma_j[\gamma_{bjk}\sinh\Omega_j t-\Omega_j\sin\gamma_{bjk}t]\}\}x[\cosh\frac{\lambda_j x}{L_x}-\cos\frac{\lambda_j x}{L_x}-\sigma_j(\sinh\frac{\lambda_j x}{L_x}- \sin\frac{\lambda_j x}{L_x})][\cosh\frac{\lambda_j x}{L_x}-\cos\frac{\lambda_j x}{L_x}-\sigma_j(\sinh\frac{\lambda_j x}{L_x}-\sin\frac{\lambda_j x}{L_x})] \tag{2.17}$$

where

$$\gamma_{bjk} = \gamma_{ajk}\left\{1-\frac{\overline{m}}{2V_j^*}\frac{\overline{m}}{V_k^*}\left[2\sum_{m=1}^{\infty}\cos\frac{m\pi s}{L_y}X_{a1}(j,j)H_b(k,k)+X_{a1}(j,j)H_a(k,k)\right.\right. - \frac{2v^2\lambda_j^2}{L_x^2\gamma_{ajk}^2}\left.\left.\left(2\sum_{m=1}^{\infty}\cos\frac{m\pi s}{L_y}X_{a3}(j,j)H_b(k,k)+X_{a3}(j,j)H_a(k,k)\right]\right)\right\} \tag{2.18}$$

$$\frac{\overline{m}}{V_j^*} = \frac{2}{L_x k_{oj}^*}\;; \qquad \frac{\overline{m}}{V_k^*} = \frac{2}{L_y k_{oj}^*} \tag{2.19}$$

$$H_a(k,k) = \int_0^{L_y} W_k(y)dy \tag{2.20}$$

$$H_b(k,k) = \int_0^{L_y}\cos\frac{m\pi y}{L_y}W_k^2(y)dy \tag{2.21}$$

$$X_{a_1}(j,j) = \int_0^{L_x} W_j(x)dx \tag{2.22}$$

$$X_{a_3}(j,j) = \int_0^{L_x} W_j(x)\frac{d^2}{dx^2}W_j(x)dx \tag{2.23}$$

$$\gamma_{ajk} = \upsilon_{sf}\left[1-\frac{\lambda^*}{\upsilon_{sf}^2}\left(\frac{2}{k_{oj}^*k_{ok}^*L_x}\right)\frac{\lambda_j^2}{L_x^2}X_{a_3}(j,j)H_a(k,k)\right] \tag{2.24}$$

$$\upsilon_{sf} = \omega_{j,k}^2 + D_2 + X_{a_3}(j,j)H_a(k,k) \tag{2.25}$$

$$k_{oj}^* = \int_0^{L_x} W_j^2(x)dx \tag{2.26}$$

$$\omega_{jk}^2 = k_1\left(\frac{j^4\pi^4}{L_x^4}+\frac{2j^2k^2\pi^2}{L_x^2L_y^2}+\frac{k^4\pi^4}{L_y^4}\right) \tag{2.27}$$

and $W_j(x)$, $W_k(y)$ and $W_j(vt)$ are corresponding beam functions for clamped end conditions.

THEOREM 1: The series in equation (2.12) is uniformly convergent.
PROOF: Considering the equation (2.12), we have

$$\left|W(x,y,t)\right| = \left|4\mu g\sum_{j=i}^{\infty}\sum_{k=1}^{\infty}\frac{[\sin\frac{j\pi vt}{L_x}-\frac{j\pi v}{\gamma_{j,k}L_x}\sin\gamma_{j,k}t]}{\gamma_{j,k}^2-\frac{j^2\pi^2v^2}{L_x^2}}\sin\frac{k\pi s}{L_y}\sin n\frac{j\pi x}{L_x}\sin n\frac{j\pi y}{L_y}\right| \tag{2.28}$$

$$\left|W(x,y,t)\right| \le 4\mu g\sum_{j=i}^{\infty}\sum_{k=1}^{\infty}\left|\frac{[\sin\frac{j\pi vt}{L_x}-\frac{j\pi v}{\gamma_{j,k}L_x}\sin\gamma_{j,k}t]}{\gamma_{j,k}^2-\frac{j^2\pi^2v^2}{L_x^2}}\sin\frac{k\pi s}{L_y}\sin n\frac{j\pi x}{L_x}\sin n\frac{j\pi y}{L_y}\right| \tag{2.29}$$

It is evident that

$$\left|\gamma_{jk}\right| = \left|\omega_{jk}^*\left\{1-\frac{\mu}{2}\left[1+\sin^2\frac{k\pi s}{L_y}+\frac{v^2j^2\pi^2}{\omega_{jk}^{*2}L_x^2}\left(1+4\sin^2\frac{k\pi s}{L_y}\right)\right]\right\}\right| \tag{2.30}$$

$$\le \omega_{jk}^*\left[1-\frac{\mu}{2}\left(2+\frac{5v^2j^2\pi^2}{\omega_{jk}^{*2}L_x^2}\right)\right] \tag{2.31}$$

$$= \omega_{jk}^*\left[1-QA\right] \tag{2.32}$$

where
$$QA = \frac{\mu}{2}\left[2+\frac{5v^2j^2\pi^2/L_x^2}{k_1\left(L_y^2j^2\pi^2+L_x^2k^2\pi^2/L_x^2L_y^2\right)^2+D^2\left(L_y^2j^2\pi^2+L_x^2k^2\pi^2/L_x^2L_y^2\right)+D_2L_x^2}\right]\omega_{jk}^* \tag{2.33}$$

Consequently,

$$\left|W(x,y,t)\right| \le 4\mu g\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1+\frac{j\pi v}{\omega_{jk}^*(1-QA)L_x}}{\omega_{jk}^{*2}(1-QA)^2\left(1-\frac{j^2\pi^2v^2}{L_x^2\omega_{jk}^{*2}(1-QA)^2}\right)} \tag{2.34}$$

In view of equation (2.14), we have

$$= 4\mu g\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left\{\frac{1+\frac{j\pi v}{L_x(1-QA)\left[k_1\left(\frac{j^4\pi^4}{L_x^4}+\frac{2j^2k^2\pi^4}{L_x^2L_y^2}+\frac{k^4\pi^4}{L_y^4}\right)+D_3\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)+D_2\right]^{\frac{1}{2}}}}{(1-QA)^2\left[k_1\left(\frac{j^4\pi^4}{L_x^4}+\frac{2j^2k^2\pi^4}{L_x^2L_y^2}+\frac{k^4\pi^4}{L_y^4}\right)+D_3\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)+D_2\right]-\frac{j^2\pi^2v^2}{L_x^2}}\right\} \tag{2.35}$$

$$= 4\mu g\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left\{\frac{1+\frac{vL_xL_y^2}{\pi(1-QA^*)\left[k_1+D_3\frac{L_x^2L_y^2}{\pi^2}+D_2\frac{L_x^4L_y^4}{\pi^4}\right]^{\frac{1}{2}}}}{\left(\frac{\pi}{L_xL_y}\right)^4(1-QA^*)^2\left[k_1+D_3\frac{L_x^2L_y^2}{\pi^2}+D_2\frac{L_x^4L_y^4}{\pi^4}\right]-\frac{\pi^2v^2}{L_x^2}}\right\}\cdot\frac{1}{\left(j^2+k^2\right)^2} \tag{2.36}$$

$$= 4\mu g \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left\{\frac{1+\dfrac{vL_xL_y^4}{\pi(1-QA^*)\left[k_1+D_3\dfrac{L_x^2L_y^2}{\pi^2}+D_2\dfrac{L_x^4L_y^4}{\pi^4}\right]^{\frac{1}{2}}}}{\left(\dfrac{\pi}{L_xL_y}\right)^4(1-QA^*)^2\left[k_1+D_3\dfrac{L_x^2L_y^2}{\pi^2}+D_2\dfrac{L_x^4L_y^4}{\pi^4}\right]-\left(\dfrac{vL_xL_y^2}{\pi(1-QA^*)}\right)^2}\right\}\cdot\frac{1}{j^4}+\frac{1}{k^4} \tag{2.37}$$

where $$QA^* = \frac{\mu}{2}\left[2+\frac{5v^2L_x^2L_y^4}{\pi^2\left(k_1+D_3\dfrac{L_x^2L_y^2}{\pi^2}+D_2\dfrac{L_x^4L_y^4}{\pi^4}\right)}\right] \tag{2.38}$$

Thus, the series solution (2.12) uniformly converges and rapidly as $\dfrac{1}{j^4}+\dfrac{1}{k^4}$

THEOREM 2: The series in equation (2.17) is convergent.
PROOF: Evidently, equation (2.17) can be rewritten as

$$W(x,y,t) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\overline{m}}{V_j^*}\frac{\overline{m}}{V_k^*}\frac{P_0W_k^*(s)}{2\gamma_{bjk}(\Omega_j^4-\gamma_{bjk}^4)}\{\gamma_{bjk}\,W\ddot{W}_j(vt)-\gamma_{bjk}^3W_j(vt)$$
$$+(\Omega_j^2-\gamma_{bjk}^2)(\sigma_j\Omega_j\sin\gamma_{bjk}t+\gamma_{ajk}\cos\gamma_{ajk}t)+(\Omega_j^2+\gamma_{bjk}^2)(\sigma_j\Omega_j\sin\gamma_{bjk}t$$
$$-\gamma_{bjk}\cos\gamma_{bjk}t)\}W_j(x)W_k(y) \tag{2.39}$$

Since $W_j(x)$ are the beam functions in the direction of x of the rectangular plate, they are bounded, that is

$$\left|W_j(x)\right| \le \eta_1 < \infty \tag{2.40}$$

Also,

$$\left|W_j'(x)\right| \le \eta_1^a < \infty \tag{2.41}$$

$$\left|W_j''(x)\right| \le \eta_1^b < \infty \tag{2.42}$$

$$\left|W_j'''(x)\right| \le \eta_{1r}^c < \infty \tag{2.43}$$

Similarly, $W_k(y)$ is also bounded, that is

$$\left|W_k(y)\right| \le \eta_0 < \infty \tag{2.44}$$

$$\left|W_k'(y)\right| \le \eta_0^a < \infty \tag{2.45}$$

$$\left|W_k''(y)\right| \le \eta_0^b < \infty \tag{2.46}$$

$$\left|W_k'''(y)\right| \le \eta_{0r}^c < \infty \tag{2.47}$$

Furthermore, $W_k^*(s),\, W_j(vt),\, \dot{W}_j(vt),\, \ddot{W}_j(vt),\, and\, \dddot{W}_j(vt),$ are bounded and we have

$$\left|W_j(vt)\right| \le \eta_1^* < \infty\,; \qquad \left|\dot{W}_j(vt)\right| \le \eta_2 < \infty\,; \tag{2.48}$$

$$\left|\ddot{W}_j(vt)\right| \le \eta_3 < \infty\,; \qquad \left|\dddot{W}_j(vt)\right| \le \eta_4 < \infty \tag{2.49}$$

Now, $$\left|W_k^*(s)\right| \le \eta_0^d < \infty\,; \tag{2.50}$$

$$\left|k_{ok}^{*}\right| \le \int_{0}^{L}\left|W_k^2(y)\right|dy \le \eta_0^2 L_y < \infty \tag{2.51}$$

Thus,

$$\left|\frac{1}{k_{ok}^{*}}\right| \le \eta_0^c < \infty. \tag{2.52}$$

Similarly,

$$\left|\frac{1}{k_{oj}^{*}}\right| \le \eta_1^c < \infty. \tag{2.53}$$

In view of inequalities (2.40) –(2.53),

$$\left|W(x,y,t)\right| \le \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left|\frac{\overline{m}}{V_j^*}\frac{\overline{m}}{V_k^*}\frac{P_0 W_k^*(s)}{2\gamma_{bjk}(\Omega_j^4-\gamma_{bjk}^4)}\left\{\omega_{j,k}\ddot{W}_j(vt)\right.\right.$$
$$-\gamma_{bjk}^3 W_j(vt) + (\Omega_j^2-\gamma_{bjk}^2)(\sigma_j\Omega_j \sin\gamma_{bjk}t - \gamma_{bjk}\cos\gamma_{bjk}t)$$
$$\left.\left.+(\Omega_j^2+\gamma_{bjk})(\sigma_j\Omega_j \sin\gamma_{bjk}t - \gamma_{bjk}\cos\gamma_{bjk}t)\right\}W_j(x)W_k(y)\right| \tag{2.54}$$

$$\left|W(x,y,t)\right| \le \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{2P_0\eta_0^c\eta_1^c\eta_0^d}{L_xL_y\gamma_{bjk}(\Omega_j^4-\gamma_{ajk}^4)}\left\{\gamma_{bjk}\eta_3 - \gamma_{ajk}^4\eta_1 + 2\Omega_j^3\sigma_j - 2\Omega_j^2\gamma_{bjk}\right\}\eta_1\eta_0 \tag{2.55}$$

Consider the first series on the right hand side of the above inequality in (2.55), that is

$$k_a = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{\eta_3}{\Omega_j^4-\gamma_{bjk}^4} \tag{2.56}$$

It is noted that

$$\left|\gamma_{bjk}^2\right| \le \left|\omega_{jk}^2 m_{a1} + m_{b1}\right| < \infty \tag{2.57}$$

$$\le \omega_{jk}^2 \text{ since } m_{a1} > 0 \text{ and } m_{b1} > 0. \tag{2.58}$$

where

$$m_{a1} = m_a\left[1-6\mu\eta_0^c\eta_1^c\left(\eta_0^2\eta_1^2 - \frac{v^2\lambda\eta_1\eta_3\eta_0^2}{L_x^2\left[m_ak_1\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)^2+m_b\right]^2}\right)\right]^2 \tag{2.59}$$

$$m_{b1} = m_b\left[1-6\mu\eta_0^c\eta_1^c\left(\eta_0^2\eta_1^2 - \frac{v^2\lambda\eta_1\eta_3\eta_0^2}{L_x^2\left[m_ak_1\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)^2+m_b\right]^2}\right)\right]^2 \tag{2.60}$$

$$m_a = (1-Q_a)^2 \tag{2.61}$$

$$m_b = D_2 m_a \tag{2.62}$$

$$Q_a = \frac{2\lambda^*\eta_0^c\eta_1^c\left(\frac{\lambda^2}{L_x^2}\eta_0^2\eta_3 + \eta_0\eta_1^2\eta_0^2\right)}{\mathrm{k}_1\left(\frac{\mathrm{j}^4\pi^4}{L_x^4}+\frac{2j^2k^2\pi^4}{L_x^2L_y^2}+\frac{k^4\pi^4}{L_y^4}\right)+\mathrm{D}_2} \tag{2.63}$$

$$\sigma_j = -\frac{1}{A_j} = \frac{\cosh\lambda_j - \cos\lambda_j}{\sinh\lambda_j - \sin\lambda_j} \tag{2.64}$$

In view of the above, in order to show that γ_{bjk} is convergent, it is just necessary to show that ω_{jk} is convergent.

The natural circular frequencies of a rectangular plate, $\omega_{j,k}$ are known to be real and hence form a countable set. Except possibly for a finite number of $|\omega_{j,k}|$, generally

$$|\omega_{j+1,k+1}| \;>\; |\omega_{j,k}|. \tag{2.65}$$

Thus, using ratio test [9]

$$\lim_{\substack{k\to\infty\\ j\to\infty}} \left| \frac{1}{\omega_{j+1,k+1}} \Big/ \frac{1}{\omega_{j,k}} \right| = c_a \;<\; 1. \tag{2.66}$$

Hence $\sum \left|\frac{1}{\omega_{j,k}}\right|$ is convergent. In view of argument in theorem 1, the series $\sum\left(\frac{1}{\omega_{j,k}}\right)^r,\ r\geq 1$ is absolutely convergent. Similar argument presented here also holds for $\sum\left(\frac{1}{\Omega_j}\right)^r,\ r\geq 1$, where

$$\Omega_j = \frac{\lambda_j v}{L_x} \tag{2.67}$$

and λ_j are the mode frequencies of a uniform beam. These are similarly known to be real and countable. Thus,

$$|\lambda_j| \leq \lambda \langle \infty \tag{2.68}$$

In view of these results, provided that $\Omega_j \neq \omega_{j,k}$, it is either that $\frac{1}{\Omega_j^4}$ dominates series (2.56) or $\frac{1}{\omega_{j,k}^4}$ dominates. In either case, series k_a is convergent.

In the same sense, the second , third and fourth series, that is

$$k_b = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \frac{\gamma_{bjk}^2 \eta_1^*}{\Omega_j^4 - \gamma_{bjk}^4} \tag{2.69}$$

$$k_c = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \frac{2\Omega_j^3 \sigma_j}{\omega_{j,k}\left(\Omega_j^4 - \gamma_{bjk}^4\right)} \tag{2.70}$$

and

$$k_d = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \frac{2\Omega_j^2}{\Omega_j^4 - \gamma_{bjk}^4} \tag{2.71}$$

are convergent since the constant σ_j in the beam function is bounded. Hence the series in equation (2.17) is convergent.

Consequently, the analytical solutions of two illustrative examples of this class of problems have been shown to converge.

CONCLUSION

This paper has established the convergence of analytical solutions obtained for the initial boundary value moving mass problem of rectangular plates resting on Pasternak foundation thereby establishing the robustness of the analytical solution technique. Accordingly, the closed form solutions to the dynamical systems are not mere formal solutions but are actual solutions to the moving mass problems.

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