Journal of the Nigerian Association of Mathematical Physics
Volume **18 (May, 2011), pp 19 - 24**
© J. of NAMP

Coefficient Inequalities for Certain Classes of Analytic And Univalent Functions

[1]Oladipo, A.T., [2]Olatunji, S.O. and [3]Adesola, T.O.
Department of Pure and Applied Mathematics,
Ladoke Akintola University of Technology,
P.M.B 4000, Ogbomoso, Oyo State. Nigeria.

Abstract

In this paper, we make use of the new concept of analytic functions introduced in [1] and we derive some coefficient inequalities for functions $f \in A(\omega)$ to be ω- starlike and $\omega -$ convex and $\omega - \lambda -$ spiral-like Starlike functions all of order α.

1.0 Introduction:

Let $A(\omega)$ denote class of functions of the form

$$f(z) = (z-\omega) + \sum_{k=2}^{\infty} a_k (z-\omega) \qquad (1.1)$$

which are analytic in the unit disk $U = \{z : |z| < 1\}$,normalized with $f(\omega) = 0$ and $f'(\omega) - 1 = 0$ and ω is a fixed point in U.

Also we let $S(\omega) \subset A(\omega)$ denote the class of analytic and univalent in U. With $A(\omega)$ and $S(\omega)$ [1] defined the following

$$ST(\omega) = S^*(\omega) = \left\{ \operatorname{Re} \frac{(z-\omega) f'(z)}{f(z)} > 0,\ z \in U \right\}$$

$$CV(\omega) = S^c(\omega) = \left\{ 1 + \operatorname{Re} \frac{(z-\omega) f''(z)}{f'(z)} > 0,\ z \in U \right\},$$

where the two classes above are respectively the classes of $\omega -$ starlike and $\omega -$ convex functions. Authors like [2], [3] studied the above classes using various extension and many interesting results were obtained. The concept defined in (1.1) was also used by [6] to study certain classes of Bazilevic functions and this also serves as part of motivation for the present works.

For the purpose of this work the following Lemma and definitions shall be employed.

Lemma A: [4]. A function $p(z) \in B$ satisfies the following condition $\operatorname{Re}\left[p(z)\right] > 0 \quad (z \in U)$ *if and only if*

$$p(z) \neq \frac{\varsigma - 1}{\varsigma + 1} \qquad (z \in U; \varsigma \in C; |\varsigma| = 1)$$

Proof: It is fairly obvious that the following transformation

$$h = \frac{z-1}{z+1}$$

maps the unit circle ∂U onto the imaginary axis $\operatorname{Re}(h) = 0$. Indeed for all ς such that $(|\varsigma| < 1, \varsigma \in C)$ we set

$$h \neq \frac{\varsigma - 1}{\varsigma + 1} \qquad (\varsigma \in C; |\varsigma| = 1)$$

Then,

$$|\varsigma| = \left| \frac{1+h}{1-h} \right| = 1$$

which implies that

$$\operatorname{Re}(h) = \operatorname{Re}\left(\frac{\varsigma - 1}{\varsigma + 1} \right) = 0 \qquad (\varsigma \in C; |\varsigma| = 1)$$

Corresponding author: ***Oladipo, A.T.: E-mail:*** atlab_3@yahoo.com Tel. +2348127374376

Moreover, by noting $p(0)=1$ *and* $p(z)\in B$ we know that

$$p(z)\neq\frac{\varsigma-1}{\varsigma+1}\qquad (z\in U\ ,\ \varsigma\in C;\ |\varsigma|=1)$$

Hence the proof.

Definition A: A function $f(z)$ defined as in (1.1) is said to be $\omega-$starlike of order α if and only if

$$\operatorname{Re}\frac{(z-\omega)f'(z)}{f(z)}>\alpha,\qquad 0\le\alpha<1,\ \ z\in U \tag{1.2}$$

and this class of functions is denoted by $S^*(\omega,\alpha)$ and ω is a fixed point in U.

Definition B: A function $f(z)$ defined as in (1.1) is said to be $\omega-$convex of order α if and only if

$$\operatorname{Re}\left\{1+\frac{(z-\omega)f''(z)}{f'(z)}\right\}>\alpha,\qquad 0\le\alpha<1,\ \ z\in U \tag{1.3}$$

This class of functions is denoted by $S^c(\omega,\alpha)$ and ω is a fixed point in U.

Also let $P(\omega)\subset P$ (the class of Caratheodory functions) which are analytic, with $p(\omega)=0$ and $\operatorname{Re}p(z)>0$ and of the form

$$P(z)=1+\sum_{n=2}^{\infty}B_n(z-\omega)^n,\qquad n=1,\qquad z\in U \tag{1.4}$$

where

$$|B_n|\le\frac{2}{(1+d)(1-d)^n}\qquad n\ge1,\qquad |\omega|=d$$

and ω is a fixed point in U [5].

2. Coefficient inequalities

First, we shall derive the following lemma which shall play a major role in all our next results.

Lemma 2.1: A function $f\in A(\omega)$ is in the class $S^*(\omega,\alpha)$ if and only if

$$1+\sum_{n=2}^{\infty}A_n(z-\omega)^{n-1}\neq0 \tag{2.1}$$

where
$$A_n=\frac{n+1-2\alpha+(n-1)\varsigma}{2-2\alpha}a_n$$

and ω is a fixed point in 𝕌.

Proof: From (1.1),(1.2) and (1.4), let us set

$$p_\omega(z)=\frac{\dfrac{(z-\omega)f'(z)}{f(z)}-\alpha}{1-\alpha}\qquad f(z)\in S^*(\omega,\alpha).$$

We find that $p_\omega(z)\in B$ and $\operatorname{Re}[p(z)]>0$

Using Lemma A, we have

$$\frac{\dfrac{(z-\omega)f'(z)}{f(z)}-\alpha}{1-\alpha}\neq\frac{\varsigma-1}{\varsigma+1}\qquad(z\in U;\varsigma\in C;|\varsigma|=1) \tag{2.2}$$

which readily yields

$$(\varsigma+1)(z-\omega)f'(z)+(1-2\alpha-\varsigma)f(z)\neq0\qquad (f(z)\in S^*(\omega,\alpha),z\in\mathbb{U},\ \varsigma\in\mathbb{C}).$$

Thus we have

$$(\varsigma+1)\left[(z-\omega)+\sum_{n=2}^{\infty}na_n(z-\omega)^n\right]+(1-2\alpha-\varsigma)\left[(z-\omega)+\sum_{n=2}^{\infty}a_n(z-\omega)^n\right]\neq0$$

that is

$$(\varsigma+1)(z-\omega)+(\varsigma+1)\sum_{n=2}^{\infty}na_n(z-\omega)^n+(1-2\alpha-\varsigma)(z-\omega)+(1-2\alpha-\varsigma)\sum_{n=2}^{\infty}a_n(z-\omega)^n\neq0$$

which also gives
$$2(1-\alpha)(z-\omega)\left[1+\sum_{n=2}^{\infty}\left[\frac{1+\varsigma(n-1)+n-2\alpha}{2(1-\alpha)}a_n(z-\omega)^{n-1}\right]\right]\neq0 \tag{2.3}$$

dividing both sides of (2.3) by $2(1-\alpha)(z-\omega)$ to obtain

$$1+\sum_{n=2}^{\infty}\frac{1+n-2\alpha+(n-1)\varsigma}{2(1-\alpha)}a_n(z-\omega)^{n-1}\neq 0 \qquad (z\in U, \varsigma\in \mathbb{C}, |\varsigma|=1)$$

and ω is a fixed point U. This completes the proof of Lemma 2.1.
Using Lemma 2.1 we state and proof the following

Theorem 2.1: If $f(z)$ satisfies the following condition

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left(\left|\sum_{k=1}^{n}\left[\sum_{j=1}^{k}(-1)^{k-j}(j+1-2\alpha)\binom{\beta}{k-j}a_j\right]\binom{\gamma}{n-k}\right| + \left|\sum_{k=1}^{n}\left[\sum_{j=1}^{k}(-1)^{k-1}(j-1)\binom{\beta}{k-j}a_j\right]\binom{\gamma}{n-k}\right|\right)\leq 2(1-\alpha) \qquad (2.4)$$

then $f\in S^*(\omega,\alpha)$ and ω is a fixed point in U.

Proof: We first note that

$$(1-(z-\omega))^{\beta}\neq 0 \text{ and } (1+(z-\omega))^{\gamma}\neq 0$$

and ω is a fixed point in U.

Hence, if the following inequality

$$\left(1+\sum_{n=2}^{\infty}A_n(z-\omega)^{n-1}\right)(1-(z-\omega))^{\beta}(1+(z-\omega))^{\gamma}\neq 0 \qquad (z\in U, \beta;\gamma\in R) \qquad (2.5)$$

and ω is a fixed point in U holds true, then we have

$$\left(1+\sum_{n=2}^{\infty}A_n(z-\omega)^{n-1}\right)\neq 0$$

which is the relation (2.1) of Lemma 2.1. It is easily seen that (2.4) is equivalent to

$$\left(1+\sum_{n=2}^{\infty}A_n(z-\omega)^{n-1}\right)\left(\sum_{n=0}^{\infty}(-1)^n b_n(z-\omega)^n\right)\left(\sum_{n=0}^{\infty}c_n(z-\omega)^n\right)\neq 0 \qquad (2.6)$$

where for convinience we write

$$b_n=\binom{\beta}{n} \quad \text{and} \quad c_n=\binom{\gamma}{n}$$

Considering the Cauchy product of the first two factors of (2.6), we have

$$\left(1+\sum_{n=2}^{\infty}B_n(z-\omega)^{n-1}\right)\left(\sum_{n=0}^{\infty}c_n(z-\omega)^n\right)\neq 0 \qquad (2.7)$$

Where $B_n=\sum_{j=1}^{n}(-1)^{n-j}A_j b_{n-j}$.

Furthermore, applying the same method, the Cauchy product for the above factors gives

$$1+\sum_{n=2}^{\infty}\left(\sum_{k=1}^{n}B_k c_{n-k}\right)(z-\omega)^{n-1}\neq 0,$$

or equivalently, that

$$1+\sum_{n=2}^{\infty}\left[\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}A_j b_{k-j}\right)c_{n-k}\right](z-\omega)^{n-1}\neq 0.$$

Thus, if $f(z)\in A(\omega)$ satisfies the following inequality,

$$\sum_{n=2}^{\infty}\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}A_j b_{k-j}\right)c_{n-k}\right|(r+d)^{n-1}\leq 1, \qquad |z-\omega|=r+d.$$

that is , if $\frac{1}{2(1-\alpha)}\sum_{n=2}^{\infty}(r+d)^{n-1}\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j+1-2\alpha)+(j-1)\varsigma\right)a_j b_{k-j}c_{n-k}\right|$

$$\leq \frac{1}{2(1-\alpha)}\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j+1-2\alpha)a_j b_{k-j}\right)c_{n-k}\right|+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j-1)b_{k-j}a_j\right)c_{n-k}\right|\right]\leq 1$$

for $0\leq\alpha<1, \varsigma\in C, |\varsigma|=1$.

then $f(z)\in S^*(\omega,\alpha)$. Hence the proof.

Corollary 2.1: If $f(z)\in A(\omega)$ satisfies the following condition:.

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j+1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j-1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right]\leq 2 \quad (2.8)$$

$(\beta\in R, \gamma\in R)$,

then $f(z)\in S^*(\omega)$.

Proof: This is obvious from Theorem 2.1 when $\alpha=0$

Theorem 2.2: If $f(z)\in A(\omega)$ satisfies the following condition

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\, j(j+1-2\alpha)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\, j(j-1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right]\leq 2(1-\alpha)$$

$$for \quad 0\leq\alpha<1,\ \beta,\gamma\in R$$

then $f(z)\in S^*(\omega,\alpha)$ and ω is a fixed point in U.

Proof: Since $(z-\omega)f'(z)$ belongs to the class $S^*(\omega,\alpha)$ if and only if

we replace a_j in Theorem 2.1 by ja_j, Theorem 2.2 is readily proved.

Corollary 2.2: If $f(z)\in A(\omega)$ satisfies the following condition

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\, j(j+1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\, j(j-1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right]\leq 2$$

$(\beta\in R, \gamma\in R)$ then $f(z)\in S^c(\omega)$ and ω is a fixed point in U.

Proof: By setting $\alpha=0$ in Theorem 2.2, the result follows immediately.

With various choices of the parameter involved we could obtain various existing classes of coefficient inequalities and some new ones.

3.Coefficient inequalities for functions in the class $SP(\omega,\lambda,\alpha)$

In this section, we consider the subclass $SP(\omega,\lambda,\alpha)$ of $A(\omega)$, which consist of functions $f(z)\in A(\omega)$ if and only if the following inequality holds true:

$$\mathrm{Re}\left[e^{i\lambda}\left(\frac{(z-\omega)f'(z)}{f(z)}-\alpha\right)\right]>0 \qquad \left(z\in U;\ 0\leq\alpha<1;\ -\frac{\pi}{2}<\lambda<\frac{\pi}{2}\right) \qquad (3.1)$$

and ω is a fixed point in U

For the purpose of the next result, we shall derive the following Lemma.

Lemma 2.2: A function $f(z)\in A(\omega)$ is in the class $SP(\omega,\lambda,\alpha)$ if and only if

$$1+\sum_{n=2}^{\infty}L_n(z-\omega)^{n-1}\neq 0 \qquad (3.2)$$

where $L_n=\dfrac{n-1+2(1-\alpha)e^{-i\lambda}\cos\lambda+(n-1)\varsigma}{2(1-\alpha)e^{-i\lambda}\cos\lambda}a_n$

and ω is a fixed point in U.

Proof: Let us set

$$p_\omega(z)=\frac{e^{i\lambda}\left(\frac{(z-\omega)f'(z)}{f(z)}-\alpha\right)-i(1-\alpha)\sin\lambda}{(1-\alpha)\cos\lambda}$$

$$(z\in U;\gamma\in R;\varsigma\in C;|\varsigma|=1).$$

We see that $p(z)\in B$ and $\operatorname{Re}[p(z)]>0$ and ω is a fixed point in U.

It follows from Lemma 2.1 that

$$\frac{e^{i\lambda}\left(\frac{(z-\omega)f'(z)}{f(z)}-\alpha\right)-i(1-\alpha)\sin\lambda}{(1-\alpha)\cos\lambda}\neq\frac{\varsigma-1}{\varsigma+1} \tag{3.3}$$

$$(z\in U;\gamma\in R;\varsigma\in C;|\varsigma|=1)$$

We need not consider Lemma 2.1 for the case when $z=\omega$, because (3.3) implies that

$$p(\omega)\neq\frac{\varsigma-1}{\varsigma+1},\qquad(\varsigma\in C,|\varsigma|=1).$$

It also follows from (3.3) that

$$\frac{e^{i\lambda}[(z-\omega)f'(z)-\alpha f(z)]-i(1-\alpha)f(z)\sin\lambda}{(1-\alpha)\cos\lambda}\neq\frac{\varsigma-1}{\varsigma+1}f(z)$$

$$(z\in U;\lambda\in R;\varsigma\in C;|\varsigma|=1)$$

which readily yields

$$(\varsigma+1)\left[e^{i\lambda}[(z-\omega)f'(z)-\alpha f(z)]-i(1-\alpha)f(z)\sin\lambda\right]\neq(\varsigma-1)(1-\alpha)f(z)\cos\lambda$$

$$(z\in U;\lambda\in R;\varsigma\in C;|\varsigma|=1)$$

or equivalently that

$$(\varsigma+1)e^{i\lambda}(z-\omega)f'(z)-\alpha e^{i\lambda}f(z)-\varsigma\alpha e^{i\lambda}f(z)-i(1-\alpha)f(z)\sin\lambda-i\varsigma(1-\alpha)f(z)\sin\lambda$$
$$\neq\varsigma(1-\alpha)f(z)\cos\lambda-(1-\alpha)f(z)\cos\lambda \tag{3.4}$$

which implies, from (3.4) that

$$(\varsigma+1)e^{i\lambda}(z-\omega)f'(z)-\alpha e^{i\lambda}f(z)-\varsigma\alpha e^{i\lambda}f(z)-\varsigma(1-\alpha)e^{i\lambda}f(z)+(1-\alpha)e^{-i\lambda}f(z)\neq 0$$

that is

$$(\varsigma+1)e^{i\lambda}(z-\omega)f'(z)+\left(e^{-i\lambda}-2\alpha\cos\lambda-\varsigma e^{i\lambda}\right)f(z)\neq 0\qquad(z\in U;\lambda\in R;\varsigma\in C;|\varsigma|=1)$$

Using simple transformation, especially through (1.1) we have the following

$$(\varsigma+1)e^{i\lambda}\left((z-\omega)+\sum_{n=2}^{\infty}na_n(z-\omega)^n\right)+\left(e^{-i\lambda}-2\alpha\cos\lambda-\varsigma e^{i\lambda}\right)\left((z-\omega)+\sum_{n=2}^{\infty}a_n(z-\omega)^n\right)\neq 0$$

or equivalently

$$2(1-\alpha)(z-\omega)\cos\lambda\left(1+\sum_{n=2}^{\infty}\frac{n+e^{-2i\lambda}-2\alpha e^{-i\lambda}\cos\lambda+(n-1)\varsigma}{2(1-\alpha)e^{-i\lambda}\cos\lambda}a_n(z-\omega)^{n-1}\right)\neq 0 \tag{3.5}$$

,

dividing both sides of (3.5) by $2(1-\alpha)(z-\omega)\cos\lambda\neq 0$ and noting that

$e^{-2i\lambda}=-1+2e^{-i\lambda}\cos\lambda,$ we obtain

$$1+\sum_{n=2}^{\infty}\frac{n-1+2(1-\alpha)e^{-i\lambda}\cos\lambda+(n-1)\varsigma}{2(1-\alpha)e^{-i\lambda}\cos\lambda}a_n(z-\omega)^{n-1}\neq 0$$

$$\left(0\le\alpha<1,-\frac{\pi}{2}<\lambda<\frac{\pi}{2},\varsigma\in C;|\varsigma|=1\right)$$

and this completes the proof of Lemma 2.2.

With the aid of Lemma 2.2, we shall state and prove the following

Theorem 3.1: If $f(z)\in A(\omega)$ satisfies the following condition

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\left(j-\alpha+(1-\alpha)e^{-2i\lambda}\right)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right.$$

$$+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j-1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\Bigg] \le 2(1-\alpha)\cos\lambda \qquad (3.6)$$

$$\left(0 \le \alpha < 1, -\frac{\pi}{2} < \lambda < \frac{\pi}{2}, \beta \in R; \gamma \in R\right)$$

then $f(z) \in SP(\omega, \lambda, \alpha)$.

Proof: Applying the same method as in Theorem 2.1, we see that $f(z)$ is in the class $SP(\omega, \lambda, \alpha)$ if

$$\sum_{n=2}^{\infty}\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j} L_j b_{k-j}\right)c_{n-k}\right|(r+d)^{n-1} \le 1 \qquad (3.7)$$

where b_n, c_n are as earlier defined and the coefficients L_n *is as* given in Lemma 2.2. It follows from inequality (3.7) that

$$\frac{1}{\left|2(1-\alpha)e^{-i\lambda}\cos\lambda\right|}\sum_{n=2}^{\infty}(r+d)^{n-1}\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\left(j-1+2(1-\alpha)e^{-i\lambda}\cos\lambda\right)+\varsigma(j-1)a_j b_{k-j}\right)c_{n-k}\right| \le$$

$$\frac{1}{2(1-\alpha)\cos\lambda}\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\left(j+\alpha+(1-\alpha)(-1+2e^{-i\lambda}\cos\lambda\right)+\varsigma(j-1)a_j b_{k-j}\right)c_{n-k}\right|\right.$$

$$\left.+|\varsigma|\left|\sum_{k=1}^{n}\left[\sum_{j=1}^{k}(-1)^{k-j}(j-1)a_j b_{k-j}\right]c_{n-k}\right|\right] \le 1$$

$$\left(0 \le \alpha < 1, -\frac{\pi}{2} < \lambda < \frac{\pi}{2}, \varsigma \in C; |\varsigma| = 1\right)$$

which implies that if $f(z)$ satisfies the hypothesis (3.6) of Theorem 3.1, then $f(z) \in SP(\omega, \lambda, \alpha)$, and ω.is a fixed point in U . This completes the proof of the Theorem.

Corollary 3.1: If $f(z) \in A(\omega)$ satisfies the following condition:

$$\sum_{n=2}^{\infty}(r+d)^{n-1}\left[\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}\left(j+e^{-2i\lambda}\right)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right.$$

$$\left.+\left|\sum_{k=1}^{n}\left(\sum_{j=1}^{k}(-1)^{k-j}(j-1)\binom{\beta}{k-j}a_j\right)\binom{\gamma}{n-k}\right|\right] \le 2\cos\lambda \qquad (3.8)$$

$$\left(0 \le \alpha < 1; \beta \in R; \gamma \in R; -\frac{\pi}{2} < \lambda < \frac{\pi}{2}\right)$$

Then, $f(z) \in SP(\omega, \lambda, 0) = SP(\omega, \lambda)$ and ω.is a fixed point in U .

Proof: By setting $\alpha = 0$ in Theorem 3.1, the result follows immediately.

References

[1] S.Kanas and F. Ronning, Uniformly starlike convex functions and other related classes of univalent functions, Ann.Univ. Mariae Curie-Sklodowska Sect. A, 53 (1999), 95-105.

[2] M. Acu and S.Owa, On some subclasses of univalent functions, J. inequalities in Pure and Applied Mathematics, 6 (2005), No 3, Art. 70, 1-6.

[3] A.T. Oladipo, On subclasses of analytic and univalent functions, Advances in Applied Mathematical Analysis Vol 4, No 1 (2009) 87-93.

[4] T.Hayami, S.Owa, and H.M. Strivastava, Coefficient inequalities for certain classes of analytic and univalent functions, Journal of inequalities in Pure and Applied Mathematics, 8 (4): 95, 2007 1-21.

[5] J.K. Wald, On starlike functions, Ph.D Thesis, University of Delaware, New Ark, Delaware (1978).

[6] A.T. Oladipo and T. O. Opoola, On coefficient bounds of a subclass of univalent functions. Journal of the Nigerian Association of Mathematical Physis. Vol. 16 2010 395-400.