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The Determination of Some Mechanical Properties of Scheffe's Optimized Mound Soil Concrete

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Abstract

The work determined some mechanical properties of fresh and matured concrete. These properties include Slump, Compressive Strength, Static modulus of elasticity and Modulus of rigidity. It applied Scheffe's optimization theory to determine the ratio of the combined constituents of the concrete mix. The results showed that an optimal compressive strength of $45.49\ /\ mm^2$ ***with a mix composition of cement: fine aggregate: mound soil; coarse aggregate: water cement ratio of 1: 1: 0.5: 2: 0.5. The result also show that mound soil concrete is 6.25% denser than plain concrete. The paper recommends mound soil concrete for structures where the density of concrete is of paramount interest.***

Keywords: Concrete, Mound soil, Strength, Density and Modulus of rigidity

Introduction

Concrete is a composite construction material, made up of; coarse aggregate, fine aggregate, cement, water and some times admixtures [1].

Mound soil is soil devoid of any decayed vegetable material. Ecological reports say that Mound soils are built of earth particles which are cemented together forming hard brick-like materials resistant to weathering and difficult to pick [2]. Various materials have been used as admixture in concrete. These have been done to achieve some desired results [3, 4, 5, 6, and 7]. Mound soil has been shown to be a good construction material [8]. It is very common in the tropics where its producers -Termites, are predominant. Concrete is good in compression but poor in tension. Hence in reinforced concrete design, it is assumed that the concrete in the tension zone of the member has failed [9]. Generally, optimization has been used to produce the best results while conserving available resources [10, 11, 12, 13, 14, 15, 16 and 17].

Background theory

Let the objective function be y –the parameter to be optimized, for example compressive strength, y depends on other factors say $x_1, x_2, x_3 ..., x_n$ –the variables [10]. These variables are also subject to some auxiliary conditions such as limits or boundaries, termed constraints. A major objective in concrete is compressive strength which depends primarily on the proportions of the constituent materials. These include; fine aggregate, coarse aggregate, cement, water and sometimes additives or modifiers here represented as x_1, x_2, x_3, x_4 and x_5 respectively. Assuming concrete as a unit mixture,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \qquad (1)$$

Hence, optimizing any function y depending on the proportion of n variables,

$$x_1 + x_2 + x_3 +,.., x_n = 1 \qquad (2)$$

Simplex Lattice

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Simplex is defined as the structural representation of the line or planes joining the assumed positions of the constituents (atoms) of the material [18].

If a mixture has a total of q components and x_i be the proportions of the ith component in the mixture such that,

$$x_i \geq 0(i = 1,2,...q)$$

Since the mixture is a complete whole, i.e., unity,

$$x_1 + x_2 + x_3 + ... + x_q = 1 \qquad \text{or}$$

$$\sum x_i - 1 = 0 \qquad (3)$$

where, $i = 1,2...q$

Thus the factor space is a regular $(q-1)$ dimensional simplex in which, if $q = 2$, we have 2 points of connectivity giving a line lattice. If $q = 3$, a triangular lattice, if $q = 4$, a tetrahedron etc. Taking a whole factor space in the design, we have a (q, m) simplex lattice.

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed *responses*. Mixture properties were described using polynomials assuming that a polynomial function of degree n in the q variables $x_1, x_2,...x_q$, subject to equation (3) and will be called a (q, n) polynomial having a general form

$$y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \sum b_{i1},_{i2}...i_n x_{i1} x_{i2} x_{in} \qquad (4)$$

where, $(1 \leq i \leq q, 1 \leq i \leq j \leq q, 1 \leq i \leq j \leq k \leq q)$ respectively and b is a constant coefficient.

The usable form of equation 4 is

$$\hat{Y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{15} x_1 x_5 +$$
$$+ b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{25} x_2 x_5 + b_{34} x_3 x_4 + b_{35} x_3 x_5 + b_{45} x_4 x_5 + b_{11} x^2{}_1 + b_{22} x^2{}_2 +$$
$$+ b_{33} x^2{}_3 + b_{44} x^2{}_4 + b_{55} x^2{}_5 \qquad (5)$$

Hence, the (q, n) polynomial which in the present work is a $(5,2)$ polynomial is,

$$\hat{Y} = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5 + \alpha_{12} x_1 x_2 + \alpha_{13} x_1 x_3 + \alpha_{14} x_1 x_4 + \alpha_{15} x_1 x_5 +$$
$$+ \alpha_{23} x_2 x_3 + \alpha_{24} x_2 x_4 + \alpha_{25} x_2 x_5 + \alpha_{34} x_3 x_4 + \alpha_{35} x_3 x_5 + \alpha_{45} x_4 x_5 \qquad (6)$$

In compact form,

$$\hat{Y} = \sum \alpha_i x_i + \sum \alpha_{ij} x_i x_j \qquad (6a)$$

where, $1 \leq i \leq q, 1 \leq i \leq j \leq q, 1 \leq i \leq j \leq q$ respectively and α_i are the coefficients of the regression equation.

Let the response function to the pure components (x_i) be denoted by (y_i) and the response to a 1:1 binary mixture of components i and j be y_{ij} , From Eq. 6,

$$\sum \alpha_i x_i = \sum y_i x_i \qquad (7)$$

Where, $i = 1$ to 5

The general equations for evaluating a_i and a_{ij} are found to be of the form

$$y_i = \alpha_i \qquad (8)$$

$$\alpha_{ij} = 4_{ij} - 2y_i - 2y_j \qquad (9)$$

The number of α_{ij} values given as [19],

$$q(q-1)/2! = 5(5+1)/2! = 15$$

The design matrix as shown in Table 1 or $x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, x_4^{(4)}$ and $x_5^{(1)}$ for the ith experimental points are referred to as Pseudo-Components. For any actual component Z, the pseudo-component (x) is given by [19],

$$X = AZ \qquad (10)$$

where A is the inverse of Z matrix and

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$$Z = BX^T \qquad (11)$$

Where B is the inverse of Z matrix and X^T is the transpose of matrix X .

Methodology

Crushed granite from Ifon was used, the maximum size of which was 14mm. The grading and properties of the coarse aggregate conformed to BS882.

Okhuahe River Sand (OKRS) was used. It consisted of quartzite with the grading and properties conforming to BS882 and belongs to zone 3 of the ASHTO classification. Mound soil from Iyeke-Ogba area in Edo State of Nigeria was used. Mound soils are classified as SC (clayey-sand) in the Unified Soil Classification System.

As specified by BS3148:1980, potable water was used.

The materials for the experiment were sourced and transferred to the laboratory where they were allowed to dry. The mound soil was pulverized using wooden Mortar and Pestle. Sampling was carried out using the quartering method.

The Pseudo-components of the mixes were designed following the background theory and from it, the real or actual variables were developed.

The compressive strength (f_c) were obtained from the ratio

$$f_c = \frac{Maximumload}{Cross-\sec tionalArea} = \tfrac{P}{A} \qquad (12)$$

Three cubes were tested for each point and the average taken as the compressive strength of thee point.

Static modulus of elasticity (E_c) have been obtained using the relationship

$$E_c = 1.7\rho^2 f_{cu}^{0.33} \times 10^{-6} \qquad (13)$$

Modulus of rigidity (G_c) have been obtained using the relationship

$$G_c = \frac{E_c}{2(\mu + 1)} \qquad (14)$$

where, Poisson's ratio μ , was obtained as lateral strain /Axial strain (20).

Results

The results of both the theoretical and the experimental parts of the work are present. The extra control points have been introduced to check the statistical accuracy of the methodology [1].

Table 1: Design Matrix for Scheffe's (5, 2) Lattice (Pseudo and Real components)

Pseudo-Components						Response	Actual Variables				
No.	X_1	X_2	X_3	X_4	X_5	Comp.	Z_1	Z_2	Z_3	Z_4	Z_5
1	1	0	0	0	0	Y_1	1	1	0.5	2	0.5
2	0	1	0	0	0	Y_2	1	2	1.5	5	0.55
3	0	0	1	0	0	Y_3	1	1.5	0.25	3	0.325
4	0	0	0	1	0	Y_4	1	3	1	6	0.6

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5	0	0	0	0	1	Y_5	1	2.5	2	1.5	0.5
6	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	Y_{12}	1	1.5	1	3.5	0.525
7	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	Y_{13}	1	1.25	0.375	2.5	0.5
8	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	Y_{14}	1	1.25	0.75	4	0.55
9	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	Y_{15}	1	2.25	1.25	1.75	0.5
10	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	Y_{23}	1	1.75	0.875	4	0.538
11	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	Y_{24}	1	2.5	1.25	5.5	0.575
12	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	Y_{25}	1	2.25	1.75	3.25	0.525
13	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	Y_{34}	1	2.25	0.625	4.5	0.563
14	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	Y_{35}	1	2	1.125	2.25	0.513
15	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	Y_{35}	1	2.75	1.5	3.75	0.55
Control											
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	C_1	1	1.375	0.688	3	0.514
2	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	C_2	1	1.625	0.813	4	0.544
3	0	$\frac{1}{4}$	0	0	$\frac{3}{4}$	C_3	1	2.375	1.875	2.375	0.503
4	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	C_4	1	2.125	1.063	3.5	0.538
5	$\frac{1}{8}$	0	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$	C_5	1	1.875	0.813	2.875	0.525
6	$\frac{1}{4}$	0	$\frac{3}{4}$	0	0	C_6	1	1.375	0.312	2.75	0.644
7	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	C_7	1	2	0.938	2.125	0.531

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8	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	C_8	1	2	1.05	2.3	0.535

Legend:

X_1 = Fraction of Ordinary Portland cement (OPC), X_2 = Fraction fine aggregate (Okhuahe river Sand),
X_3 = Fraction of Mound Soil, X_4 = Fraction of coarse aggregate, X_5 = Water cement ratio

Table 2: Sample Results

No.	Replications	Failure Load (kN)	dy $(\times 10^{-2} mm)$	Dx $(\times 10^{-2} mm)$	Wet Weight (kg)	Dry Weight (kg)
1	A	1040	180	96	8.670	8.819
	B	1050	170	30	7.988	8.398
	C	980	340	320	8.552	8.681
2	A	900	158	95	8.404	8.666
	B	850	270	602	8.435	8.638
	C	1100	175	252	8.843	8.594
3	A	850	299	300	8.564	8.768
	B	930	165	250	8.588	8.704
	C	970	194	110	8.543	8.672
4	A	940	131	15	8.552	8.575
	B	1000	188	105	8.208	8.515
	C	865	162	120	8.247	8.460
5	A	830	156	130	8.420	8.398
	B	730	142	70	8.283	8.414
	C	785	139	15	8.478	8.501
6	A	1030	142	90	8.233	8.606
	B	890	233	500	8.552	8.841

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	B	890	233	500	8.552	8.841
	C	1065	144	20	8.398	8.644
7	A	955	168	125	8.617	8.834
	B	945	151	115	8.293	8.405
	C	945	224	215	8.998	8.589
8	A	975	178	158	8.255	8.676
	B	850	232	575	7.893	8.684
	C	990	139	30	8.247	8.696
9	A	900	268	305	8.800	8.707
	B	950	180	245	8.088	8.585
	C	935	152	65	8.552	8.588
10	A	710	171	45	8.504	8.732
	B	700	173	210	8.545	8.585
	C	720	156	75	8.524	8.631
11	A	710	228	295	8.564	8.732
	B	700	176	37	8.388	8.585
	C	720	118	10	8.543	8.631
12	A	735	147	115	8.152	8.686
	B	640	161	120	7.908	8.866
	C	665	163	110	8.160	8.660
13	A	1040	131	80	8.720	8.771
	B	980	171	158	8.483	8.549
	C	940	140	200	8.478	8.630
14	A	1010	167	120	8.533	8.517
	B	975	154	465	8.852	8.336
	C	1050	150	105	8.598	8.289
15	A	620	154	85	8.470	8.715
	B	650	114	30	7.988	8.408
	C	630	164	145	8.452	8.535

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Control						
1	A	780	129	65	8.504	8.685
	B	750	219	205	8.445	8.741
	C	800	137	10	8.324	8.709
2	A	810	161	30	8.564	8.713
	B	770	140	35	8.448	8.601
	C	775	183	65	8.543	8.583
3	A	810	130	17	8.552	8.571
	B	800	160	55	8.508	8.688
	C	800	137	45	8.247	8.409
4	A	680	257	365	8.520	8.558
	B	670	129	25	8.383	8.519
	C	745	157	100	8.378	8.549
5	A	660	186	730	8.533	8.664
	B	730	167	100	8.752	8.836
	C	760	74	50	8.492	8.713
6	A	815	161	52	8.517	8.647
	B	900	142	10	8.193	8.466
	C	830	148	58	8.390	8.576
7	A	685	199	200	8.355	8.632
	B	740	111	15	7.893	8.367
	C	765	99	15	8.247	8.233

Table 3: Compressive Strength, Poisson's Ratio, Young's Modulus and Modulus of rigidity

No.	Replications	f_c (N/m^2)	Average f_c (N/m^2)	ρ (kN/m^3)	μ	E $(\times 10^{-5})$ N/m^2	G $(\times 10^{-5})$ kN/m^3
1	A	46.22	45.49	2.56	0.55	3.93	1.27
	B	46.67					
	C	43.56					

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2	A	40.00	42.22	2.56	1.25	3.83	0.85
	B	37.78					
	C	48.89					
3	A	37.78	40.75	2.56	0.94	3.79	0.98
	B	41.33					
	C	43.11					
4	A	41.78	41.55	2.58	0.47	3.87	1.32
	B	44.44					
	C	38.44					
5	A	36.89	34.74	2.50	0.48	3.43	1.16
	B	32.44					
	C	34.89					
6	A	45.78	44.22	2.58	0.97	3.95	1.00
	B	39.56					
	C	47.33					
7	A	42.44	42.15	2.55	0.82	3.80	1.04
	B	42.00					
	C	42.00					
8	A	43.33	41.70	2.57	1.19	3.85	0.88
	B	37.77					
	C	44.00					
9	A	40.00	41.26	2.55	0.98	3.77	0.95
	B	42.22					
	C	41.56					
10	A	31.55	31.55	2.56	0.65	3.48	1.05
	B	31.11					
	C	32.00					
11	A	31.55					
	B	31.11					

	C	32.00					
12	A	32.67	30.22	2.59	0.73	3.51	1.01
	B	28.44					
	C	29.56					
13	A	46.22	43.85	2.56	0.99	3.88	9.75
	B	43.56					
	C	41.78					
14	A	44.89	44.96	2.49	1.48	3.70	7.46
	B	43.33					
	C	46.67					
15	A	27.56	28.50	2.50	0.57	3.21	1.02
	B	28.89					
	C	28.00					

No.	Replications	f_c (N/m^2)	Average f_c (N/m^2)	ρ kN/m^3	μ	E $(\times 10^{-5})$ N/m^2	G $(\times 10^{-5})$ kN/m^3
1	A	34.66	34.51	2.57	0.72	3.61	1.05
	B	33.33					
	C	35.55					
2	A	36.00	34.89	2.56	0.26	3.60	1.43
	B	34.22					
	C	34.44					
3	A	36.00	35.70	2.53	0.27	3.54	1.39
	B	35.55					
	C	35.55					
4	A	30.22					
	B	29.77					

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	C	33.11	31.03	2.53	0.75	3.38	0.97
5	A	29.33	31.85	2.59	1.73	3.57	0.65
	B	32.44					
	C	33.77					
6	A	36.22	37.70	2.54	0.26	3.63	1.44
	B	40.00					
	C	36.89					
7	A	30.44	32.44	2.49	0.43	3.32	1.16
	B	32.89					
	C	34.00					

Discussion

Table 1 shows the Pseudo, X_n and Real, Z_n corresponding components of the designed mixes. Where n =1…5. Table 2 presents the results of compressive strength, density, Poisson's ratio, static modulus of elasticity and modulus of rigidity. Table s 2 and 3 showed that the maximum compressive strength of 45.49 N/mm^2 was achieved with a mix composition of
1:1:0.5:2:0.5of the real or actual variables. This was closely followed by of 1:1.5:1:3.5:0.525 with a compressive strength of 44.22 N/mm^2 and then 1:2:1.125:2.25:0.513 of 43.85 N/mm^2. Table 2 also showed that the mix which gave the least compressive strength was 1:2.75:1.5:3.75:0.55 with a compressive strength of 28.50 N/mm^2. The low compressive strength of this mix may be attributable to the high concentration of aggregates, as shown in Table 1, No. 15.

Table 3 showed that MSC designed using Scheffe's theory has an average density of 25.5 kN/m^3, Poisson's ratio of 0.84, static modulus of elasticity, 3.67x 10^{-5}N/m^2 and a modulus of rigidity of 2.09x 10^{-5} N/m^2. The relatively high density of MSC when compared with normal dense concrete of 24.0 kN/m^3 [21] is due to the SC nature of the mound soil. The result showed that MSC is 6.25% denser than normal concrete. Hence, MSC is recommended where high density is a necessity in concrete such as in radio active and x-ray laboratories. The work showed that concrete mixes in which admixtures will be required can be viewed and designed as a five component mix from the beginning with the admixture as the fifth component instead of using the rule of thumb.

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