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Numerical Solutions of Generalized Burger's-Huxley Equation by Modified Variational Iteration Method

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Abstract

Numerical solutions of the generalized Burger's-Huxley are obtained using a Modified Variational Iteration Method (MVIM) with minimal computational efforts. The computed results with this technique have been compared with other results. The present method is seen to be a very reliable alternative method to some existing techniques for such nonlinear problems.

Keywords: Burger's-Huxley, modified variational iteration method, lagrange multiplier, Taylor's series, partial differential equation.

1.0 Introduction

Behaviors of many physical systems encountered in models of reaction mechanisms, convection effects and diffusion transports give rise to the generalized Burger's-Huxley equation.
Many researchers have used various numerical approaches for the solution of generalized Burger's-Huxley equation. Mittal & Ram [6] studied the Differential Quadrature Method (DQM) for the Burger's-Huxley equation, [10] presented Exp-Function for solving Huxley equation, [2] studied Exp-Function method to the generalized Burger's-Huxley equation. Also, [3] solved generalized Burger's-Huxley and generalized Burger-Fisher equations using Adomian Decomposition Method (ADM) and [1] presented analytical treatment of generalized Burger's-Huxley equation by Homotopy Analysis Method (HAM).
Unlike some previous methods that used various transformations and several iterations, we present a new Modified Variational Iteration Method (MVIM) for the numerical solutions of both linear and nonlinear partial differential equations.

2.0 Modified Variational Iteration Method (MVIM)

To illustrate the basic concept of the MVIM, we consider the following general nonlinear partial differential equation:

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = g(x,t) \tag{2.1}$$

where L is a linear time derivative operator, R is a linear operator which has partial derivative with respect to x, N is a nonlinear operator and g is an inhomogeneous term. According to MVIM, we can construct a correct functional as follows:

$$u_0(x,t) = u(x,0) + g_1(x)t \tag{2.2}$$

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$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda \left[Lu_n + R\tilde{u}_n + N\tilde{u}_n - g\right] d\tau \qquad (2.3)$$

where $g_1(x)$ can be evaluated by substituting $u(x,0)$ in (2.1) and at $t = 0$.

λ is a Lagrange multiplier which can be identified optimally via modified variational iteration method. The subscript n

denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation i.e, $\delta\tilde{u}_n = 0$.

3.0 MVIM for the solution of generalized Burger's-Huxley equation

The following generalized Burger's-Huxley equation problems arising in various field of science is considered.

$$\frac{\partial u}{\partial t} + \alpha u^{\delta}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1-u^{\delta})(u^{\delta}-\gamma),\ 0 \le x \le 1,\ t \ge 0 \qquad (3.1)$$

$$u(x,0) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh(A_1 x)\right)^{\frac{1}{\delta}} \qquad (3.2)$$

$$u(0,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh(-A_1 A_2 t)\right)^{\frac{1}{\delta}},\ t \ge 0 \qquad (3.3)$$

where

$$u(1,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh(A_1(1 - A_2 t))\right)^{\frac{1}{\delta}},\ t \ge 0 \qquad (3.4)$$

$$A_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1+\delta)}}{4(1+\delta)}\gamma \qquad (3.5)$$

$$A_2 = \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(-\alpha + \sqrt{\alpha^2 + 4\beta(1+\delta)})}{2(1+\delta)}\gamma \qquad (3.6)$$

α, β, γ *and* δ are parameters that $\beta \ge 0, \delta > 0$. If $\beta = 0$, (3.1) reduces to Burger's equation and when $\alpha = 0$, it reduces to Huxley equation which describes the nerve pulse propagation in the nerve fibres and wall motion in liquid crystal.

From [3] the exact solution is given as :

Applying (2.2) in (3.1), we have the following:

$$u(x,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}Tanh(A_1(x - A_2 t))\right)^{\frac{1}{\delta}},\ t \ge 0 \qquad (3.7)$$

4.0 Results and Discussion

Figure 1 shows the graph of MVIM and Exact solution against t for $\alpha = 1, \beta = 1, \gamma = 0.001$ *and* $\delta = 1$.

Figure 2 shows the graph of MVIM and Exact solution against t for $\alpha = 1, \beta = 1, \gamma = 0.001$ *and* $\delta = 2$.

Figure 3 shows the graph of MVIM and Exact solution against t for $\alpha = 1, \beta = 1, \gamma = 0.001$ *and* $\delta = 3$.

Figure 4 shows the graph of MVIM and Exact solution against t for $\alpha = 1, \beta = 1, \gamma = 0.001$ *and* $\delta = 10000$.

Figure 5 shows the graph of MVIM solution against t for $\alpha = 0, \beta = 1, \gamma = 0.001$ *and* $\delta = 10000$.

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Figure 6 shows the graph of MVIM solution against t for $\alpha = 1,\ \beta = 1,\ \gamma = 0.001\ and\ \delta = 10000$.
Figure 5 and 6 shows that the generalized Huxley and generalized Burger's- Huxley approaches the same steady state as δ increases.

$$u_0(x,t) = \left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta} + \begin{pmatrix} \left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta\tanh(Ax)^2 A^2 \\ +\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta^2 A^2\tanh(Ax)^2 \\ +\alpha\left(\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\right)^{\delta}\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta A\tanh(Ax) \\ -2\tanh(Ax)\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta^2 A^2 - \\ \alpha\left(\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\right)^{\delta}\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta A + \\ \beta\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\left(\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\right)^{\delta}\gamma \\ +\beta\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\left(\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\right)^{\delta} + \\ \left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta^2 A^2 - \left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\delta A^2 \\ -\beta\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\gamma \\ -\beta\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\left(\left(\left(\frac{\gamma}{2}+\frac{1}{2}\gamma\tanh(Ax)\right)^{\delta}\right)^{\delta}\right)^{2} \end{pmatrix} t \quad .....(3.8)$$

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\begin{matrix} \dfrac{\partial u_n(x,\tau)}{\partial t} + \alpha\, u_0(x,\tau)^{\delta}\dfrac{\partial u_n(x,\tau)}{\partial x} - \dfrac{\partial^2 u_n(x,\tau)}{\partial x^2} - \\ \beta u_n(x,\tau)(1-u_n(x,\tau)^{\delta})(u_n(x,\tau)^{\delta}-\gamma) \end{matrix}\right] d\tau \quad .....(3.9)$$

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Using Maple software, we obtained the following graphical results from (3.9) when $n = 1$:

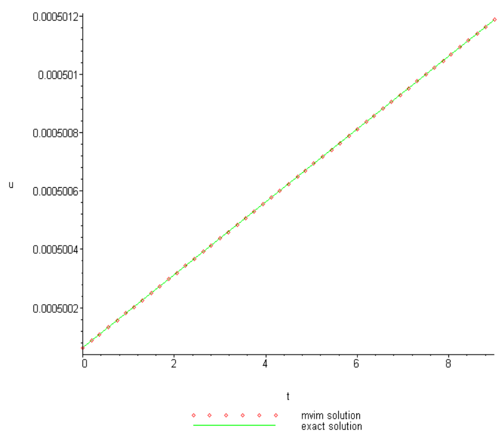

Figure 1: Exact solution and MVIM solution against t when $\delta = 1$

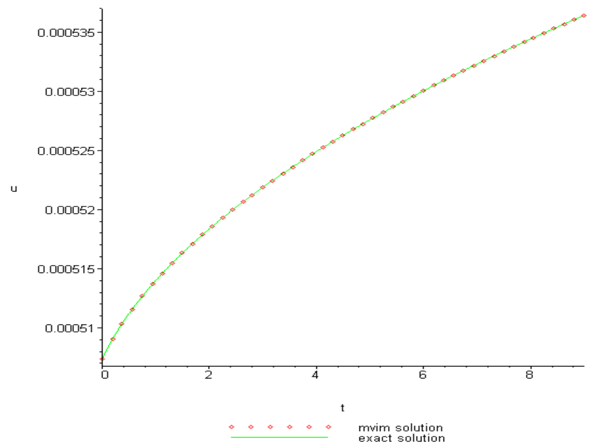

Figure 2: Exact solution and MVIM solution against t when $\delta = 2$

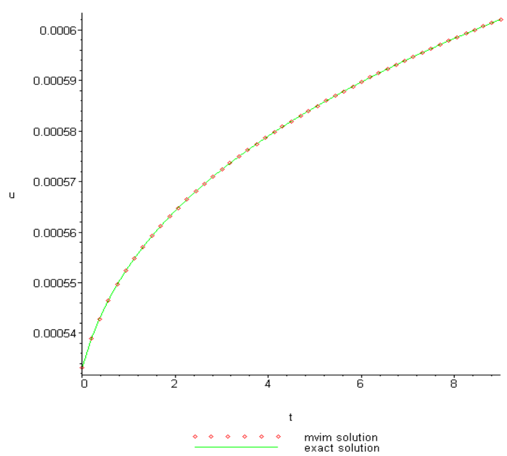

Figure 3: Exact solution and MVIM solution against t when $\delta = 3$

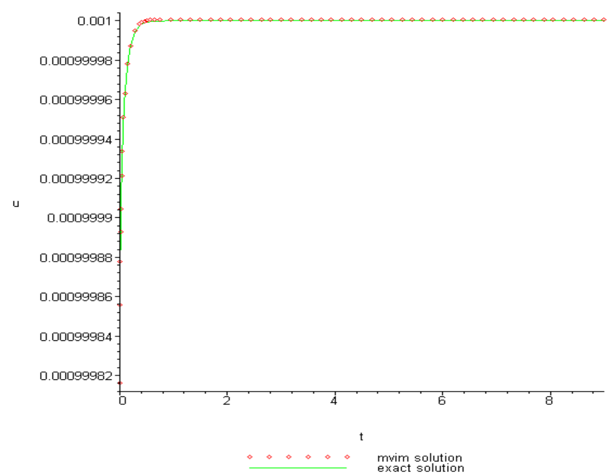

Figure 4: Exact solution and MVIM solution against t when $\delta = 10000$

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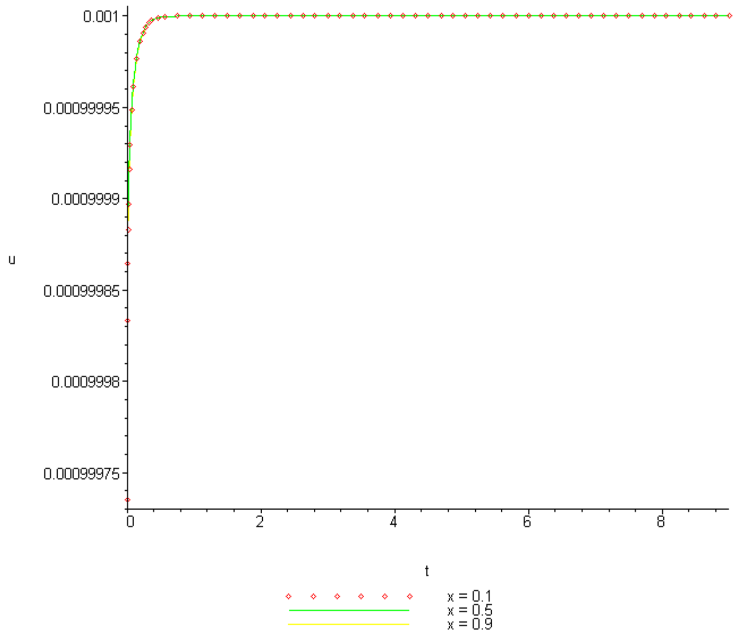

Figure 5: MVIM solution against t for generalized Huxley equation when $\delta = 10000$

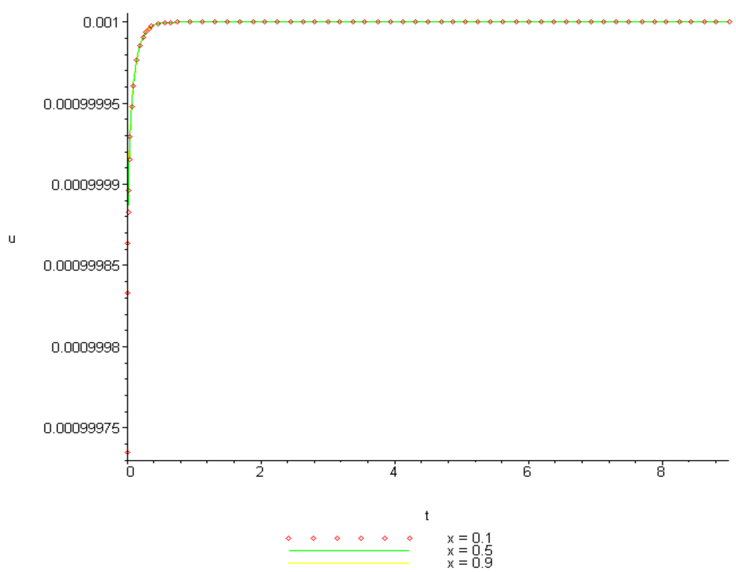

Figure 6: MVIM **solution against t for generalized Burger's-Huxley equation when** $\delta = 10000$

5.0 Conclusion

In this research, we have proposed an improved algorithm for the solution of generalized Burger's-Huxley Equation with highly very small error. One iteration is enough.

For concrete problems where an exact solution does not exist, the method is very good choice to achieve a high degree of accuracy.

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This results reveals that MVIM is straightforward, concise and a promoting tool for solving a class of nonlinear evolution equations.

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