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Generalized Photogravitational Restricted Three-Body Problem and Coplanar Libration Points

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Abstract

The formula for the locations of coplanar libration points of the restricted three-body problem when the primaries are oblate spheroid and radiating was established.

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1. Introduction

Photogravitation is the resultant of the radiation pressure and the gravitation on the restricted three-body problem. Oblateness is the degree of non-sphericity of the primaries.
The classical restricted three-body problem was generalized in various forms by considering either the Photogravitational effect or oblateness of the primaries or a combination of both the Photogravitation and oblateness to define new problem ([1], [2], [6] and [7]).
The classical restricted three-body problem is known to have five libration points $(L_1, \ldots, L_5)$.
The study of the Photogravitational effect ([4] and [5]) showed the existence of the coplanar libration points $(L_6, \ldots, L_9)$ in the xz-plane. Radzievisky ([5]) showed that the coplanar points L_6 and L_7 are located in the xz-plane symmetrically with respect to the x-axis along the curve which starts at one of the primaries and asymptotically approaches the z-axis. A chronological review of Photogravitational effects on the locations of and stability of libration points in the restricted three-body problem was compiled by [3] and was found that the Photogravitational effects give rise to some new aspects of the problem. Zheng [8] the Photogravitational effect on restricted three-body problem in which both primaries are radiating and obtained a calculating formula for locating the coplanar libration points.
In the present study, using Zheng's method, we consider both primaries oblate spheroid and radiating study the location of coplanar libration points in the generalized Photogravitational restricted three-body problem.
The study is significant because bodies in solar and stellar systems move under the gravitation in which some are emitters of radiation as well as oblate spheroid.

2. Equations of motion

Let m_1 and m_2 be the masses of the bigger and smaller primaries respectively. m is the mass of the infinitesimal body. We assume that both primaries are oblate spheroid and radiating as well. Let A_1 and A_2 denote the oblateness

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coefficients of the bigger and smaller primaries respectively such that 0 < A_i <<1, (i =1, 2). We denote the respective radiation factors for the bigger and smaller primaries as $q_i (i=1,2)$ such that 0 < $1-q_i$ <<1, (i =1, 2).

Let (x, y) be the coordinates of the infinitesimal body in the orbital plane. Following the notations and terminology of [9], the equations of motion of the infinitesimal body in the dimensionless barycentric-synodic coordinate system are [1]

$$\ddot{x}-2n\dot{y}=\Omega_x \quad , \ \ddot{y}+2n\dot{x}=\Omega_y \quad , \ \ddot{z}=\Omega_z \tag{2.1}$$

where the potential is

$$\Omega=\frac{1}{2}n^2(x^2+y^2)+\frac{1-\mu}{r_1}q_1+\frac{\mu}{r_2}q_2+\frac{1-\mu}{2r_1^3}A_1q_1+\frac{\mu}{2r_2^3}A_2q_2 \ , \tag{2.2}$$

the distances of the primaries from the infinitesimal body are

$$\begin{aligned} r_1^2 &= (x-\mu)^2+y^2+z^2 \\ r_2^2 &= (x+1-\mu)^2+y^2+z^2 \end{aligned} \tag{2.3}$$

n is the perturbed mean motion of the primaries and is given by

$$n^2=1+\frac{3}{2}(A_1+A_2) \tag{2.4}$$

$$\Omega_x=\Omega_y=\Omega_z=0$$

give

$$n^2x-\frac{(1-\mu)(x-\mu)}{r_1^3}q_1-\frac{\mu(x+1-\mu)}{r_2^3}q_2-\frac{3}{2}\frac{(1-\mu)(x-\mu)}{r_1^5}A_1q_1-\frac{3}{2}\frac{\mu(x+1-\mu)}{r_2^5}A_2q_2=0$$

$$n^2y-\frac{(1-\mu)y}{r_1^3}q_1-\frac{\mu y}{r_2^3}q_2-\frac{3}{2}\frac{(1-\mu)y}{r_1^5}A_1q_1-\frac{3}{2}\frac{\mu y}{r_2^3}A_2q_2=0 \tag{2.5}$$

$$\frac{(1-\mu)z}{r_1^3}q_1+\frac{\mu z}{r_2^3}q_2+\frac{3}{2}\frac{(1-\mu)z}{r_1^5}A_1q_1+\frac{3}{2}\frac{\mu z}{r_2^5}A_2q_2=0\,.$$

The libration point corresponding to $y=0, z=0$ is the collinear point and is obtainable from equation (2.5). There three collinear points L_1, L_2 and L_3. L_2 is located between the primaries, L_1 comes before the second primary while L_3 is located after the first primary.

The libration points corresponding to $y \neq 0, z=0$ are the triangular points. From the first and second of equations (2.5) we obtain

$$n^2-\frac{q_1}{r_1^3}-\frac{3}{2}\frac{A_1q_1}{r_1^5}=0 \text{ and } n^2-\frac{q_2}{r_2^3}-\frac{3}{2}\frac{A_2q_2}{r_2^5}=0 \tag{2.6}$$

so that

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$$\frac{q_1}{r_1^3}+\frac{3}{2}\frac{A_1 q_1}{r_1^5}=\frac{q_2}{r_2^3}-\frac{3}{2}\frac{A_2 q_2}{r_2^5} \tag{2.7}$$

$$1-\frac{1-\mu}{r_1^3}q_1-\frac{\mu}{r_2^3}q_2-\frac{3}{2}\frac{(1-\mu)}{r_1^5}A_1 q_1-\frac{3}{2}\frac{\mu}{r_2^5}A_2 q_2=0\,. \tag{2.8}$$

We obtain r_1 and r_2 from equations (6). Let $\alpha_i (i=1,2)$ be very small perturbations so that $r_1=1+\alpha_1$ and $r_2=1+\alpha_2$. Then using equations (6) $\alpha_i (i=1,2)$ are obtained, consequently r_1 and r_2 are also obtained and from equations (2.3) the coordinates of the positions of the triangular libration points are given as

$$x=\mu-\frac{1}{2}+\frac{1}{3}(1-q_1)-\frac{1}{3}(1-q_2)-\frac{1}{2}A_1+\frac{1}{2}A_2 \tag{2.9}$$

$$y=\pm\frac{\sqrt{3}}{2}\left\{1-\frac{2}{3}(A_1+A_2)-\frac{2}{9}(1-q_1)-\frac{2}{9}(1-q_2)+\frac{1}{3}(A_1+A_2)\right\} \tag{2.10}.$$

These are the triangular libration points in which (x, y) is L_4 and $(x,-y)$ is L_5.

For the existence of real solutions of equations (2.5), the third equation of (2.5) implies either the signs of q_1 and q_2 are opposite or $q_1=q_2=0$ since A_1, A_2 are non-negative.
The implication of the first case is that the radiation pressure force of one of the primaries exceeds its gravitational attraction so that the system depends on radiation pressure force. The second means that the radiation pressure forces balance the gravitational attraction, which is not our interest.
When the signs q_1 are opposite to that of q_2, the solutions for $y=0$ can be obtained from the second equation of (2.5). That is, in the $xz-plane$ there exists the libration points called the co-planar libration points.
Substituting equation (2.9) into (2.5) we have

$$\left[1-\left(\frac{r_1}{r_2}\right)^2\right]r_2^7+n^2(2\mu-1)r_2^5-2q_2\mu r_2^2-3A_2\mu=0$$

and by setting $k=\frac{r_1}{r_2}=1-\frac{1}{3}(1-q_1)+\frac{1}{3}(1-q_2)+\frac{1}{2}A_1-\frac{1}{2}A_2$, we have

$$(1-k^2)r_2^7+n^2(2\mu-1)r_2^5-2q_2\mu r_2^2-3A_2\mu=0\,, \tag{2.11}$$

where r_2 is the distance between the smaller primary and the coplanar libration point and satisfies the equation.
The (x,z) coordinates of the coplanar libration points are given by equation(2.3) and the first of equation (2.5) as

$$x=-\left(\frac{q_2\mu}{r_2^3}+\frac{3}{2}\frac{A_2\mu}{r_2^5}\right) \tag{2.12}$$

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$$z = \pm\sqrt{r_2^3 - \frac{1}{4}\left[1 + \left(1 - k^2\right) r_2^2\right]^2} \qquad (2.13)$$

Knowing the values of r_2 from equation (2.11) for given values q_1, q_2, A_1, A_2 and μ the locations of the coplanar libration points can be obtained from equations (2.12) and (2.13).

3. Conclusion

Equations (2.12) and (2.13) give the positions of the coplanar libration points for the restricted three-body problem when the bigger primary is radiating and the smaller primary is oblate spheroid as well as radiating. It is evident from the two equations that the coplanar libration points are located in the xz-plane symmetrically with respect to the x-axis. Equation (2.11) is a formula for calculating the distance of the infinitesimal body from the second primary, which can then be used in equations (2.12) and (2.13).

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