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Effects of Perturbations on the Location of Collinear Points In The Restricted Three-Body Problem with Triaxial Primaries

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Abstract

In this paper, the effect of small perturbations in the coriolis and the centrifugal forces on the location of collinear points in the restricted three-body problem has been examined when both primaries are triaxial rigid bodies with one of the axes as the axis of symmetry and its equatorial plane coinciding with the plane of motion. It is seen that the positions of the collinear points are affected significantly by the change in the centrifugal force and triaxiality of the primaries.

Keywords: collinear points, perturbations, triaxial primaries and restricted three body problem.

1.0 INTRODUCTION

The restricted three - body problem is well known and very important for the dynamics of binary and multiple stars and also planetary systems. It possesses five equilibrium points. Three are collinear with the primaries and other two are in equilateral triangular configuration with the primaries. An infinitesimal body can be at rest in a rotating coordinate frame, at these points, where the gravitational and centrifugal forces just balance each other.

The collinear points are unstable, while the triangular points are stable for the mass ratio $\mu < 0.03852$ [11]. Their stability occurs in spite of the fact that the potential energy has a maximum rather than a minimum at these (triangular) points. The stability is actually achieved through the influence of the Coriolis force, because the coordinate system is rotating [13].

The effect of small perturbations in the coriolis and the centrifugal forces in the restricted three - body problem has been discussed by [1], [2], [3], [7], [8], [12] and others.

The bodies in the classical restricted three – body problem are strictly spherical in shape, but several heavenly bodies such as Saturn and Jupiter are sufficiently oblate, Pluto and its moon Charon are exactly frozen in triaxial rigid configuration. The minor planets and meteoroids have irregular shapes. The lack of sphericity, triaxiality or oblateness of the celestial bodies causes large perturbations from a two - body orbit. The motions of artificial Earth satellites are examples of this. This motivates researchers to include the shapes of the bodies in their study [1], [2], [4], [5], [9] and [10].

This paper has therefore studied the effect of triaxiality of the primaries together with small perturbation given in the coriolis and the centrifugal forces on the location of the collinear points.

2. EQUATIONS OF MOTION

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Let m_1 , m_2 be the masses of the bigger and smaller primaries respectively and m the mass of the infinitesimal body. The distance between the primaries and the sum of their primaries are taken as unity. The unit of time is so chosen as to make the gravitational constant unity.

Let (x,y) be the coordinates of the infinitesimal body , where the x-axis is taken along the line joining the primaries and y is taken on the orbital plane of motion of the primaries and is perpendicular to the x-axis. As the motion of primaries is known, we have only to find the motion of the infinitesimal body by following the notations and terminology of [5] and [11]. The equations of motion of the infinitesimal mass in a dimensionless barycentric synodic co-ordinate system are

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial\Omega}{\partial x} \\ \ddot{y} + 2n\dot{x} &= \frac{\partial\Omega}{\partial y} \end{aligned} \qquad (1)$$

where

$$\Omega = \frac{n^2}{2}\left(x^2+y^2\right)+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{1-\mu}{2r_1^3}\left(2\sigma_1-\sigma_2\right)-\frac{3\left(1-\mu\right)}{2r_1^5}\left(\sigma_1-\sigma_2\right)y^2$$

$$+\frac{\mu}{2r_2^3}\left(2\sigma_1'-\sigma_2'\right)-\frac{3\mu}{2r_2^5}\left(\sigma_1'-\sigma_2'\right)y^2$$

$$r_1^2 = \left(x-\mu\right)^2+y^2 \quad , \quad r_2^2 = \left(x+1-\mu\right)^2+y^2$$

and μ is the ratio of the mass of the smaller primary to total mass of the primaries and
$0 < \mu \le ½$. n is the mean motion of the primaries. r_1 and r_2 are distances of the infinitesimal mass from the primaries which are triaxial rigid bodies. The triaxiality of the primaries are characterized by parameters (σ_1, σ_2) and (σ_1', σ_2') , respectively.

We introduce small perturbations in the coriolis and centrifugal forces with the help of the parameters φ and ψ , respectively such that $\varphi = 1+\varepsilon;\ \varepsilon << 1$, $\psi = 1+\varepsilon';\ \varepsilon' << 1$. The unperturbed value of each is unity. Hence, the equations of motion of the infinitesimal mass in the perturbed restricted three-body problem with triaxial primaries are:

$$\begin{aligned} \ddot{x} - 2n\varphi\dot{y} &= \frac{\partial\Omega}{\partial x} \\ \ddot{y} + 2n\varphi\dot{x} &= \frac{\partial\Omega}{\partial y} \end{aligned} \qquad (2)$$

where

$$\Omega = \frac{n^2\psi}{2}\left(x^2+y^2\right)+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{1-\mu}{2r_1^3}\left(2\sigma_1-\sigma_2\right)-\frac{3\left(1-\mu\right)}{2r_1^5}\left(\sigma_1-\sigma_2\right)y^2$$
$$+\frac{\mu}{2r_2^3}\left(2\sigma_1'-\sigma_2'\right)-\frac{3\mu}{2r_2^5}\left(\sigma_1'-\sigma_2'\right)y^2 \qquad (3)$$

with σ_i, σ_i' (i = 1, 2) << 1,

The mean motion, n , is given by

$$n^2 = 1+\frac{3}{2}\left(2\sigma_1-\sigma_2\right)+\frac{3}{2}\left(2\sigma_1'-\sigma_2'\right) \qquad \text{(Sharma et al, 2001)} \qquad (4)$$

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3. Location of Collinear Points

The collinear points are the solutions of the equations $\Omega_x = 0$ and $\Omega_y = 0$, y=0 , that is, the collinear points lie on the x-axis (line joining the two primaries). To obtain their abscissae, we denote the expression $\left[\Omega_x\right]_{y=0}$ by $f(x)$. Then

$$f(x) = n^2\psi x - \frac{1-\mu}{r_1^3}(x-\mu) - \frac{\mu}{r_2^3}(x+1-\mu) - \frac{3(1-\mu)}{2r_1^5}(2\sigma_1 - \sigma_2)(x-\mu)$$

$$-\frac{3\mu}{2r_2^5}\left(2\sigma_1' - \sigma_2'\right)(x+1-\mu) \tag{5}$$

where $r_1 = |x - \mu|$ and $r_2 = |x + 1 - \mu|$

The abscissae of the collinear points are the roots of the equation $f(x)$ =0.

$$f'(x) = n^2\psi + 2\frac{(1-\mu)}{|x-\mu|^3} + \frac{2\mu}{|x+1-\mu|^3} + \frac{6(1-\mu)(2\sigma_1-\sigma_2)}{|x-\mu|^5} + \frac{6\mu(2\sigma_1'-\sigma_2')}{|x+1-\mu|^5} > 0$$

$$\because 0 < \mu \le \frac{1}{2}, \sigma_i, \sigma_i'(i=1,2) << 1 \text{ and } \psi > 1$$

Now, for $x = \pm\infty$, it is obvious that $f'(x) = n^2\psi > 0$

for $x = \mu - 1$, $f'(x) = n^2\psi + 2(1-\mu) + \infty + 6(1-\mu)(2\sigma_1-\sigma_2) + \infty = \infty$

for $x = \mu$, $f'(x) = n^2\psi + \infty + 2\mu + \infty + 6\mu(2\sigma_1' - \sigma_2') = \infty$

Since $f'(x) > 0$ in each of the open intervals $(-\infty, \mu - 1)$, $(\mu - 1, \mu)$ and (μ, ∞), it follows that the function f is strictly increasing in each of them.

Also,

$$f(\mu-2) = (\mu-1)\left(\frac{7}{4}+\varepsilon'\right) - \varepsilon' - \frac{3}{16}(31-15\mu)\sigma_1 + \frac{3}{32}(31-15\mu)\sigma_2 - 6(1-\mu)\sigma_1' + 3(1-\mu)\sigma_2' < 0$$

$$\because 0 < \mu \le \frac{1}{2}, \psi = 1+\varepsilon', \varepsilon', \sigma_i, \sigma_i'(i=1,2) << 1$$

Further, for $x = 0$, we have

$$f(0) = \frac{[1-2\mu][(1-\mu)^2+\mu^2+\mu(1-\mu)]}{\mu^2(1-\mu)^2} + \frac{3}{2\mu^4(1-\mu)^4}\left[(1-\mu)^5(2\sigma_1-2\sigma_2) - \mu^5(2\sigma_1'-2\sigma_2')\right] > 0$$

$$\because 0 < \mu \le \frac{1}{2}, \psi = 1+\varepsilon', \varepsilon', \sigma_i, \sigma_i'(i=1,2) << 1$$

Next, for $x = \mu + 1$, we get

$$f(\mu+1) = \mu\left(\frac{7}{4}+\varepsilon'\right) + \varepsilon' + \frac{3}{16}(15+17\mu)\sigma_1 + \frac{3}{32}(-15-17\mu)\sigma_2 + \frac{3}{16}(16+15\mu)\sigma_1' + \frac{3}{32}(-16-15\mu)\sigma_2' > 0$$

$$\because 0 < \mu \le \frac{1}{2}, \psi = 1+\varepsilon', \varepsilon', \sigma_i, \sigma_i'(i=1,2) << 1$$

Hence, there are only three real roots of the equation *f(x)* = 0 with one lying in each of the open intervals $(\mu-2, \mu-1)$, $(\mu-1, 0)$ and $(\mu, \mu+1)$. These three roots correspond to the three collinear points L_1, L_2 and L_3. Eq. (5) indicates that their locations are affected by the centrifugal force and the triaxiality of the primaries.

To locate the positions of the three collinear points , we start with the first point denoted by L_1 which is to the left of the second primary. To find its position we consider the following in equation (5)

$$n^2\psi x - \frac{1-\mu}{r_1^3}(x-\mu) - \frac{\mu}{r_2^3}(x+1-\mu) - \frac{3(1-\mu)}{2r_1^5}(2\sigma_1-\sigma_2)(x-\mu)$$

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$$-\frac{3\mu}{2r_2^{\,5}}\left(2\sigma_1{}'-\sigma_2{}'\right)(x+1-\mu)=0$$

$r_1=(\mu-x)$, $r_2=(\mu-1-x)$, which when substituted in the above equation, results in

$$n^2\psi\,x+\frac{1-\mu}{|x-\mu|^2}+\frac{\mu}{|x+1-\mu|^3}+\frac{3(1-\mu)}{2|x-\mu|^4}(2\sigma_1-\sigma_2)+\frac{3\mu}{2|x+1-\mu|^4}\left(2\sigma_1{}'-\sigma_2{}'\right)=0 \qquad (6)$$

Let ρ_1 be the distance of L_1 from m_2, then

$r_2=\mu-1-x_1=\rho_1$, $x_1=\mu-1-\rho_1$, $r_1=\mu-x_1$, putting r_1, r_2 and x_1 in equation (6) we obtain

$$n^2\psi\,(\mu-1-\rho_1)+\frac{(1-\mu)}{(1+\rho_1)^2}+\frac{\mu}{\rho_1^{\,2}}+\frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2(1+\rho_1)^4}+\frac{3\mu\left(2\sigma_1{}'-\sigma_2{}'\right)}{2\rho_1^{\,4}}=0 \qquad (7)$$

Let k = $\left[1+\frac{3}{2}(2\sigma_1-\sigma_2)+\frac{3}{2}\left(2\sigma_1{}'-\sigma_2{}'\right)\right]\psi$, so that equation (7) becomes

$$\rho_1^{\,9}+(5-\mu)\rho_1^{\,8}+(10-4\mu)\rho_1^{\,7}+\frac{1}{k}\left[(10-6\mu)k-1\right]\rho_1^{\,6}$$

$$+\frac{1}{k}\left[(5-4\mu)k-2(1+\mu)\right]\rho_1^{\,5}+\frac{1}{k}\left[(1-\mu)k-(1+5\mu)-\frac{3}{2}(1-\mu)(2\sigma_1-\sigma_2)-\frac{3}{2}\mu\left(2\sigma_1{}'-\sigma_2{}'\right)\right]\rho_1^{\,4}$$

$$-\frac{1}{k}\left[4\mu+6\mu\left(2\sigma_1{}'-\sigma_2{}'\right)\right]\rho_1^{\,3}-\frac{1}{k}\left[\mu+9\mu\left(2\sigma_1{}'-\sigma_2{}'\right)\right]\rho_1^{\,2}-\frac{1}{k}\left[6\mu\left(2\sigma_1{}'-\sigma_2{}'\right)\right]\rho_1-\frac{1}{k}\left[\frac{3}{2}\mu\left(2\sigma_1{}'-\sigma_2{}'\right)\right]=0$$

This is an algebraic equation of the ninth order in ρ_1 with parameter μ which shows only one change of sign. Therefore according to Descartes sign rule there exist one positive root and the value of this root depends upon μ. Solving the equation for ρ_1 (using the small parameter method) we find that there is one real root for μ = 0. Now equation (7) can be written as

$$(1-\mu)\left[k(1+\rho_1)-(1+\rho_1)^{-2}-\frac{3}{2}(2\sigma_1-\sigma_2)(1+\rho_1)^{-4}\right]=\mu\left[\rho_1^{\,-2}+\frac{3}{2}\left(2\sigma_1{}'-\sigma_2{}'\right)\rho_1^{\,-4}-k\rho_1\right]$$

$$\frac{\mu}{1-\mu}=\frac{\rho_1^{\,4}\left[k(1+\rho_1)^5-(1+\rho_1)^2-\frac{3}{2}(2\sigma_1-\sigma_2)\right]}{(1+\rho_1)^4\left[\rho_1^{\,2}-k\rho_1^{\,5}+\frac{3}{2}\left(2\sigma_1{}'-\sigma_2{}'\right)\right]} \qquad (8)$$

In order to obtain a series solution for ρ in powers of the quantity $\upsilon^4=\frac{\mu}{1-\mu}$, we assume that

$$\rho_1=c_1\upsilon+c_2\upsilon^2+c_3\upsilon^3+c_4\upsilon^4+\ldots\ldots\ldots\ldots+c_9\upsilon^9+\ldots\ldots\ldots \qquad (9)$$

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which when substituted in equation (8), we obtain the first four coefficients as follows:

$$c_1 = \left\{ \frac{3\left(2\sigma_1{}' - \sigma_1{}'\right)}{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_1)\right]} \right\}^{\frac{1}{4}} \tag{10}$$

$$c_2 = \frac{-\{k + 2 + 6(2\sigma_1 - \sigma_2)\}\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{1}{2}}}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{3}{2}}} \tag{11}$$

$$c_3 = \frac{-(10k - 1)\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{3}{4}}}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{7}{4}}} + \frac{\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{3}{4}}(25k - 10)\left(k + 2 + 6(2\sigma_1 - \sigma_2)\right)}{4\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{11}{4}}}$$

$$+ \frac{3\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]^2\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{3}{4}}}{8\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{11}{4}}} + \frac{1 + 9\left(2\sigma_1{}' - \sigma_2{}'\right)}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{3}{4}}\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{1}{4}}}$$

$$- \frac{\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{3}{4}}}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{\frac{7}{4}}} \tag{12}$$

$$c_4 = \frac{-5k\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{2}} - \frac{3(10k - 1)\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{3}}$$

$$+ \frac{(25k - 10)\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{4\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{3}} - \frac{(25k - 10)\left[1 + 9\left(2\sigma_1{}' - \sigma_2{}'\right)\right]}{4\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{2}}$$

$$- \frac{3(25k - 10)\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]^2\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{16\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{4}} - \frac{\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}^{\frac{1}{2}}}{2\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{2}}$$

$$- \frac{(25k - 10)^2\left[k + 2 + 6(2\sigma_1 - \sigma_2)\right]\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{8\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}^{4}} + \frac{(25k - 10)(10k - 1)\left\{3\left(2\sigma_1{}' - \sigma_2{}'\right)\right\}}{4\left\{2\left[k - 1 - \frac{3}{2}(2\sigma_1 - \sigma_2)\right]\right\}}$$

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$$-\frac{(25k-10)\left[k+2+6(2\sigma_1-\sigma_2)\right]^2\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{8\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^4}+\frac{\left[k+2+6(2\sigma_1-\sigma_2)\right]^3\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{8\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^4}$$

$$-\frac{3\left[k+2+6(2\sigma_1-\sigma_2)\right]^2\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{4\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^3}+\frac{9\left[k+2+6(2\sigma_1-\sigma_2)\right]^3\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{16\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^4}$$

$$+\frac{3\left[k+2+6(2\sigma_1-\sigma_2)\right]\left\{1+9\left(2\sigma_1'-\sigma_2'\right)\right\}}{4\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^2}+\frac{3\left[k+2+6(2\sigma_1-\sigma_2)\right]^2\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{8\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^3}$$

$$+\frac{3\left[k+2+6(2\sigma_1-\sigma_2)\right]^2(25k-10)\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{8\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^4}+\frac{1+9\left(2\sigma_1'-\sigma_2'\right)}{2\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}}$$

$$-\frac{\left\{1+9\left(2\sigma_1'-\sigma_2'\right)\right\}\left[k+2+6(2\sigma_1-\sigma_2)\right]}{2\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^2}+\frac{2+3\left(2\sigma_1'-\sigma_2'\right)}{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]}$$

$$-\frac{3\left[k+2+6(2\sigma_1-\sigma_2)\right](10k-1)\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{4\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^3}-\frac{(10k-1)\left\{3\left(2\sigma_1'-\sigma_2'\right)\right\}}{2\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^2}$$

$$+\frac{(25k-10)\left[k+2+6(2\sigma_1-\sigma_2)\right]3\left(2\sigma_1'-\sigma_2'\right)}{4\left\{2\left[k-1-\frac{3}{2}(2\sigma_1-\sigma_2)\right]\right\}^3} \qquad (13)$$

We substitute the values of the first four coefficients C_1, C_2, C_3 and C_4 into equation (9) using the fact that

$$\frac{\mu}{1-\mu}=\mu\left(\frac{1}{1-\mu}\right)=\mu\left(1+\mu+\mu^2+......\right)\approx\mu$$

and obtain the abscissa of the first collinear point L_1 given by $x_1=\mu-1-\rho_1$

This shows that the abscissa of L_1 depend upon the triaxial nature of both primaries and affected by a small perturbation in the centrifugal force but independent of the small perturbation in the coriolis force.

Similarly we can find the positions of L_2 and L_3.

4.0 Conclusion

By taking small perturbations in the coriolis and the centrifugal forces in the restricted three body problem when the primaries are triaxial rigid bodies, we have seen that there are three collinear points which lie on the line joining the primaries. The positions of these points are affected by the triaxiality of the primaries and the change in the centrifugal force. The change in the Coriolis force is not capable to influence them. The mean motion depends on the triaxiality of the primaries.

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In the absence of perturbations in the coriolis and centrifugal forces, the results obtained coincide with those of [5]. Ignoring perturbations in the coriolis and centrifugal forces and further assuming that the primaries are non-triaxial, then the results obtained reduce to those of the classical case as worked out by [11]. If the primaries are spherical ($\sigma_1 = \sigma_2 = \sigma_1' = \sigma_2' = 0$), the results fully coincide with those of [3].

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