Journal of the Nigerian Association of Mathematical Physics
***Volume* 17 (November, 2010), pp 399 - 402**

Determination of the Spatial Variation of the Hole Current Injected into N-Type Material, in ID.

***D.U. Onah, B.N. Osonwa and J.E Ekpe.**
Department of Industrial Physics,
Ebonyi State University, Abakaliki Nigeria

Abstract

The spatial variation of the hole current injected into n-type material was determined by using the current density due to hole diffusion J_h and the current of a positive charge q moving with a velocity v at an electric field E in the continuity equation. The boundary condition imposed in using these equations was that the conduction current density $e\mu_h N_h E$ was neglected in comparison with the diffusion current due to holes. Method of separation of variables was also incorporated in the solution and the hole current in n-type material in ID was determined.

Keywords: hole current, n-type material, continuity equation and separation of variables.

Introduction

Semiconductors are materials in which only a very small portion of the valence electrons are free to move through the crystal lattice [1]. The two modes of conduction in semiconductors are the n-type and p-type conduction. The n-type conduction is by negative electron migration while in p-type, the conduction is by migration of positive holes. Thus current may be considered to be carried entirely by fictitious particles of positive charge which fill all those levels in the band unoccupied by electrons. These particles are called holes [2]. An intrinsic semiconductor is one that has equal number of holes and electrons.

The most important properties of semiconductors are obtained by addition of small impurities to them during manufacture. This deliberate addition of impurities to semiconductors is known as doping. If the impurity added to semiconductor leads to an increase in the number of conduction elections, then conduction will be due to electrons (negative current carriers). Such a semiconductor is the n-type. On the other hand, if the introduced impurity leads to the increase in the number of holes in the crystal structure, the conduction will be caused by holes (positive current carriers) and such a semiconductor is a p-type [1,3].

The continuity equation is a general expression representing mass conservation for a fluid in which the velocity v and density ρ are functions of position [4]. It is an equation which can be used to represent the conservation of charge in an electromagnetic theory. This equation is given by:

$$\frac{\partial \rho}{\partial t} + \nabla . J_h = 0 \qquad (1)$$

where ρ is injected hole density. It expresses the fact that a decrease in charge inside a small volume with time must correspond to a flow of charge out through the surface of the small volume since total charge is conserved [5]. The charge may be positive or negative. Holes are vacant orbitals in band, and in an applied electric and magnetic fields, a hole acts as if it has a positive charge $+e$ [6] Hole current is injected into n-type material from a p-type material only when a forward bias voltage is applied in a p-n junction. This is called hole recombination current and is highly sensitive to the applied voltage. The scope of this paper does not go beyond the hole recombination current. In contrast to the hole recombination current, is the hole generation current which flows from the n-side t o the p-side

*Corresponding author: E-mail; d_onah@tahoo.co.uk. Tel. +2348051060257

of the junction. This current is not sensitive to the applied voltage V in a p-n junction [7]. This later current is not discussed in this paper.

Methods:

The current density due to hole diffusion, the current of positive charge q moving with a velocity v at an electric field E and the continuity equation are respectively given as follows:

$$J_h = -eD_h \frac{\partial \rho}{\partial t} \quad (2)$$

$$J_h = q\text{v}E \quad (3)$$

where, e is the election charge, t is time and D_h is diffusion coefficient of the holes [3,5]. Equations 2 and 3 are to be used in equation 1 to determine the value of $J_h(x,t)$ with the boundary condition that:

$$\frac{\partial \rho}{\partial \boldsymbol{t}} = 0; D_h(x=0) = injected\ hole\ density \quad (4)$$

It should be observed that equation 1 can be interpreted formally as:

$$\frac{\partial \rho}{\partial \boldsymbol{t}} + \frac{\partial J_h}{\partial x} + \frac{\partial J_h}{\partial y} + \frac{\partial J_h}{\partial z} = 0\ in\ 3 - dimensions \quad (5)$$

Or
$$\frac{\partial \rho}{\partial \boldsymbol{t}} + \frac{\partial J_h}{\partial x} = 0\ in\ 1 - dimension \quad (6)$$

It implies that using equation 2 in equation 6, gives

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left[-\text{e}D_h \frac{\partial}{\partial x}\right] = 0 \quad (7)$$

$$\Rightarrow \frac{\partial \rho}{\partial t} - \text{e}D_h \frac{\partial^2 \rho}{\partial x^2} = 0 \quad (8)$$

Therefore
$$\frac{\partial \rho}{\partial t} = \text{e}D_h \frac{\partial^2 \rho}{\partial x^2} \quad (9)$$

This is one-dimensional diffusion.
Applying the method of separation of variables, assuming that:

$$\rho(x,t) = X(x)T(t) \quad (10)$$

where X and T are the respective functions of x and t alone such that:

$$\frac{\partial \rho}{\partial t} = X\frac{dT}{dt} \text{and} \frac{\partial^2 \rho}{\partial x^2} = \frac{T\partial^2 X}{\partial x^2} \quad (11)$$

Substituting equation 11 in equation 9, gives

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{\text{e}D_h\text{T}}\frac{\partial T}{\partial t} \quad (12)$$

Let
$$K^2 = \text{e}D_h, \quad (13)$$

then equation 12: becomes

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{k^2\text{T}}\frac{\partial T}{\partial t} \quad (14)$$

The LHS and RHS of equations 14 are constants because of variables being separated and hence, we write;

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{k^2\text{T}}\frac{\partial T}{\partial t} = -\lambda^2\ (constant\ of\ separation) \quad (15)$$

*Corresponding author: E-mail; d_onah@tahoo.co.uk. Tel. +2348051060257

Hence, $$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\lambda^2 \quad (16)$$

$$\Rightarrow \frac{\partial^2 X}{\partial x^2} + \lambda^2 X = 0 \quad (17)$$

$$\Rightarrow X(x) = A\cos\lambda x + B\sin\lambda x \quad (18)$$

Also $$\frac{1}{K^2 \mathrm{T}}\frac{\partial T}{\partial t} = -\lambda^2 \quad (19)$$

$$\Rightarrow \frac{\partial T}{\partial t} + k^2\lambda^2 T = 0 \quad (20)$$

$$\Rightarrow T(t) = Ce^{-\lambda^2 k^2 t} \quad (21)$$

The solution of equation 10 is then

$$\rho(x,t) = X(x)T(t) = (A\cos\lambda x + B\sin\lambda x)\left(Ce^{-\lambda^2 k^2 t}\right) \quad (22)$$

Thus

$$\rho(x,t) = (A\cos\lambda x + B\sin\lambda x)e^{-\lambda^2 k^2 t} \quad (23)$$

The constant C can be absorbed, so that we now apply the boundary conditions.

$$\frac{\partial\rho}{\partial t} = -\lambda^2 k^2 (Acos\lambda x + B\sin\lambda x\,)e^{-\lambda^2 k^2 t} \quad (24)$$

Hence $$\frac{\partial\rho}{\partial t}|x = 0 = -\lambda^2 k^2 (A)\, e^{-\lambda^2 k^2 t} = 0, \Rightarrow A = 0 \quad (25)$$

Thus $$\rho(x,t) = Be^{-\lambda^2 k^2 t}\sin\lambda x \quad (26)$$

$$\frac{\partial\rho}{\partial t} = -\lambda^2 k^2 e^{-\lambda^2 k^2 t}\sin\lambda x \quad (27)$$

Recalling that equation 6 is given as

$$\frac{\partial\rho}{\partial \boldsymbol{t}} + \frac{\partial J_h}{\partial x} = 0 \quad (28)$$

We now substitute equations 27 and 3 into equation 6, and get

$$-\lambda^2 k^2 Be^{-\lambda^2 k^2 t}\sin\lambda x + q\mathrm{v}\frac{\partial \mathrm{E}}{\partial \boldsymbol{x}} = 0 \quad (29)$$

$$\Rightarrow q\mathrm{v}\frac{\partial \mathrm{E}}{\partial x} = \lambda^2 k^2 Be^{-\lambda^2 k^2 t}\sin\lambda x \quad (30)$$

So that $$\frac{\partial \mathrm{E}}{\partial \boldsymbol{x}} = \frac{\lambda^2 k^2}{q\mathrm{v}} Be^{-\lambda^2 k^2 t}\sin\lambda x \quad (31)$$

$$\Rightarrow E(x,t) = -\frac{\lambda k^2}{q\mathrm{v}} Be^{-\lambda^2 k^2 t}\cos\lambda x \quad (32)$$

Therefore, $$J_h = q\mathrm{v}\mathrm{E} = -\lambda k^2 Be^{-\lambda^2 k^2 t}\cos\lambda x \quad (33)$$

*Corresponding author: E-mail; d_onah@tahoo.co.uk. Tel. +2348051060257

$$\Rightarrow J_h = -\lambda e D_h B e^{-\lambda^2 k^2 t} \cos \lambda x \qquad (34)$$

where $\lambda, B,$ and K are constants.

CONCLUSION

The hole current injected into n-type material is a function of an applied electric field or voltage. A hole is a vacant orbital in a band, and it acts as if it has a positive electronic charge $+e$, in the applied electronic and magnetic fields. Continuity equation is a general expression representing mass conservation, but it can also be extended to represent charge conservation in an electromagnetic theory. Using the continuity equation and the separation of variables, the spatial variation of the hole current injected into the n-type material was determined to be $J_h = \lambda e D_h B e^{-\lambda^2 k^2 t} \cos \lambda x$ where λ, B and K are constants while D_h is hole diffusion coefficient and t is time.

References

[1] Bennet G.A.G (1980) Electricity and Modern Physics. ELBS and Edward Arnold, London.

[2] Ashcroft N.W. and Mermin N.D (1976) Solid State Physics Holt, Rinehart and Winston, New York.

[3] Solymar L. and Walsh D. (1988) Lectures on the Electrical Properties of Materials. Oxford University Press, Oxford.

[4] Tritton D.J. (1982) Physical Fluid Dynamics. ELBS and Van Nostrand Reinhold (UK) Co. Ltd. Wokingham.

[5] Jackson J.D (1975). Classical Electrodynamics John Wiley and Sons, Inc. London.

[6] Kittel C. (1985). Solid State Physics. Wiley Eastern Ltd. New Delhi

[7] Puri R.K. and Babbar V.K. (2000) Solid State Physics and Electronics. S. Chaud and Company Ltd. New Delhi.

*Corresponding author: E-mail; d_onah@tahoo.co.uk. Tel. +2348051060257