Journal of the Nigerian Association of Mathematical Physics
***Volume* 17 (November, 2010), pp 363 - 366**

Some Designs From Symmetric Balanced Incomplete Block Design

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Abstract

In this paper the concept of symmetric balanced incomplete block designs was adopted to derive some designs.
A theorem was proposed to aid in the derivations of these designs. Some of these designs were highlighted.

Keywords: Incomplete, block, design, parameter, theorem, symmetric, balanced.

1.0 Introduction

Some of the combinatorial properties of incomplete block designs (IBD) were discussed in [1]. Now, consider a finite set of treatments, ψ and a family $b=\{b_\alpha,\ \alpha\in I\}$ of the subset of ψ. The subsets $b_\alpha{}'s$ are called the blocks, and the pair $(\psi,\ b)$ is called the design on the set ψ. Let $k=\{|b_\alpha|,\ b_\alpha\in b\}$ be the block sizes for $\alpha\in I$, where I is the set of positive integers.

An incomplete block design (IBD) with $v\in\psi$ treatments each replicated r times in blocks of sizes k is said to be balanced if

(i) Each pair of treatments appears λ times in the experiment and no one treatment appears more than once in each block $b_\alpha, \alpha=1,\ 2,\cdots$

(ii) $k<v$ (iii) $bk=vr$ (iv) $\lambda=\dfrac{r(k-1)}{v-1}$ (v) $b>v$ *or* $k<v$

The proofs of (i), (ii), (iii), (iv) and (v) are elaborately shown in [2], [3], [4], [5]. The balanced incomplete block design (BIBD), (ψ, b) that satisfies the conditions above has the parameters $(v,\ b,\ r,\ k,\lambda)$ and incidence matrix

$$N=\begin{pmatrix} n_{11} & n_{12} & \cdots & n_{1b} \\ n_{21} & n_{22} & \cdots & n_{2b} \\ \vdots & \vdots & \cdots & \vdots \\ n_{v1} & n_{v2} & \cdots & n_{vb} \end{pmatrix} \tag{1.1}$$

where the rows represent treatments, the columns represent blocks, and $n_{ij}=1$ or 0 according as the ith treatment does or does not occur in the jth block.

2.0 Symmetric BIBD

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According to [6], if every pair of treatments in a symmetric balanced incomplete block design occurs in λ blocks, then every two blocks of the design have λ treatments in common. Therefore, b alanced incomplete block design is symmetric if $r = k$ or equivalently, $b = v$.

One could derive some new designs with new parameters $\left(v^*, b^*, r^*, k^*, \lambda^*\right)$ from symmetric BIBD such that $v^* = v - r;\ b^* = b - 1;\ r^* = r;\ \ k^* = k - \lambda;$ and $\lambda^* = \lambda$.

Evidently, no two blocks of these new designs can have more than λ treatments in common.

2.1 Theorem

Let $b = \{b_\alpha,\ \alpha = 1,\ 2, \cdots, v\}$ be a set of blocks in a symmetric BIBD such that $b \in \psi$, where ψ is a finite set of treatments. For a given $\alpha; b_1 - b_\alpha,\ \ b_2 - b_\alpha, \cdots, b_v - b_\alpha$ form a new design with parameters $(v - r,\ b - 1,\ r,\ k - \lambda,\ \lambda)$, where "–" is termed block difference.

Proof: For $\alpha = 1,\ 2, \cdots, v$ we have the v blocks $b_1, b_2, \cdots, b_v$. Each of the block differences $b_1 - b_\alpha, b_2 - b_\alpha, \cdots, b_v - b_\alpha$ has $(v-1)$ blocks, and each treatment appears k times in every block resulting to $(v - r)$ blocks left, since $k = r$. Also, each of the blocks $b_\alpha, \alpha = 1, 2, \cdots, v$ has λ treatments that are in common with $(k - \lambda)$ treatments left. Hence, each of the block differences $b_1 - b_\alpha, b_2 - b_\alpha, \cdots, b_v - b_\alpha$, for $\alpha \in I$ contains $(k - \lambda)$ treatments.

Thus, the new design has the parameters $\left(v^*, b^*, r^*, k^*, \lambda^*\right)$ where $v^* = v - r$, $b^* = b - 1,\ r^* = r,\ k^* = k - \lambda^* = \lambda$.

Now, by the theorem, new blocks for the new design are

$$b'_\alpha = \{b_\beta - b_\alpha,\ \alpha \neq \beta\ \ ;\ \ \alpha, \beta = 1,\ 2, \cdots, v\} \tag{1.2}$$

which are derived from the blocks b_α of symmetric BIBD with parameters $(v,\ b,\ r,\ k, \lambda)$.

3.0 Statistical Experiment and Results

Consider an experiment with a finite set of treatments $\psi = \{a,\ b,\ c,\ d,\ e,\ f,\ g,\ h,\ i,\ j,\ k,\ l,\ m,\ n,\ o\}$ such that each pair of treatments appears in every three blocks in the experiment.

The blocks $b_\alpha,\ \alpha = 1,\ 2, \cdots, 15$ for this experiment are obtained by combinatorial principles. That is

$$\begin{aligned}
b_1 &= \{a,\ b,\ c,\ d,\ e,\ f,\ g\} \\
b_2 &= \{b,\ d,\ f,\ h,\ j,\ l,\ n\} \\
b_3 &= \{b,\ e,\ f,\ i,\ k,\ l,\ o\} \\
b_4 &= \{b,\ d,\ g,\ h,\ k,\ m,\ o\} \\
b_5 &= \{b,\ e,\ g,\ i,\ j,\ m,\ n\} \\
b_6 &= \{c,\ d,\ g,\ i,\ j,\ l,\ o\} \\
b_7 &= \{c,\ d,\ f,\ i,\ k,\ m,\ n\} \\
b_8 &= \{a,\ b,\ c,\ l,\ m,\ n,\ o\} \\
b_9 &= \{a,\ d,\ e,\ h,\ i,\ l,\ m\} \\
b_{10} &= \{a,\ d,\ e,\ j,\ k,\ n,\ o\}
\end{aligned} \tag{1.3a}$$

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$$
\begin{aligned}
b_{11} &= \{a,\ f,\ g,\ h,\ i,\ n,\ o\} \\
b_{12} &= \{c,\ e,\ f,\ h,\ j,\ m,\ o\} \\
b_{13} &= \{a,\ b,\ c,\ h,\ i,\ j,\ k\} \\
b_{14} &= \{a,\ f,\ g,\ j,\ k,\ l,\ m\} \\
b_{15} &= \{c,\ e,\ g,\ h,\ k,\ l,\ n\}
\end{aligned} \tag{1.3b}
$$

The b_α , $\alpha = 1,\ 2, \cdots, 15$ are the blocks of symmetric balanced incomplete block design with the parameters $(v = 15,\ b = 15,\ r = 7,\ k = 7,\ \lambda = 3)$

We now obtain new designs by implementing (1.2) in (1.3a) and (1.3b) respectively.

New Designs:

1. BIBD (8, 14, 7, 4, 3) with blocks

$$
\begin{aligned}
b'_1 &= \{h,\ j,\ l,\ n\} \\
b'_2 &= \{i,\ k,\ l,\ o\} \\
b'_3 &= \{h,\ k,\ m,\ o\} \\
b'_4 &= \{i,\ j,\ m,\ n\} \\
b'_5 &= \{i,\ j,\ l,\ o\} \\
b'_6 &= \{i,\ k,\ m,\ n\} \\
b'_7 &= \{l,\ m,\ n,\ o\} \\
b'_8 &= \{h,\ i,\ l,\ m\} \\
b'_9 &= \{j,\ k,\ n,\ o\} \\
b'_{10} &= \{h,\ i,\ n,\ o\} \\
b'_{11} &= \{h,\ j,\ m,\ o\} \\
b'_{12} &= \{h,\ i,\ j,\ k\} \\
b'_{13} &= \{j,\ k,\ l,\ m\} \\
b'_{14} &= \{h,\ k,\ l, n\}
\end{aligned}
$$

2. BIBD (8, 14, 7, 4, 3) with blocks

$$
\begin{aligned}
b'_1 &= \{f,\ j,\ m,\ o\} \\
b'_2 &= \{d,\ f,\ i,\ m\} \\
b'_3 &= \{d,\ i,\ j,\ o\} \\
b'_4 &= \{a,\ b,\ m,\ o\} \\
b'_5 &= \{a,\ d,\ i,\ m\} \\
b'_6 &= \{a,\ d,\ j,\ o\} \\
b'_7 &= \{a,\ f,\ i,\ o\} \\
b'_8 &= \{a,\ f,\ j,\ m\} \\
b'_9 &= \{a,\ b,\ i,\ j\} \\
b'_{10} &= \{a,\ b,\ d,\ f\} \\
b'_{11} &= \{b,\ d,\ f,\ j\}
\end{aligned}
$$

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$$b'_{12} = \{b,\ d,\ m,\ o\}$$
$$b'_{13} = \{b,\ f,\ i,\ o\}$$
$$b'_{14} = \{b,\ i,\ j, m\}$$

3. BIBD (8, 14, 7, 4, 3) with blocks

$$b'_{1} = \{a,\ c,\ e,\ g\}$$
$$b'_{2} = \{e,\ i,\ k,\ o\}$$
$$b'_{3} = \{g,\ k,\ m,\ o\}$$
$$b'_{4} = \{e,\ g,\ i,\ m\}$$
$$b'_{5} = \{c,\ g,\ i,\ o\}$$
$$b'_{6} = \{c,\ i,\ k,\ m\}$$
$$b'_{7} = \{a,\ c,\ m,\ o\}$$
$$b'_{8} = \{a,\ e,\ i,\ m\}$$
$$b'_{9} = \{a,\ e,\ k,\ o\}$$
$$b'_{10} = \{a,\ g,\ i,\ o\}$$
$$b'_{11} = \{c,\ e,\ m,\ o\}$$
$$b'_{12} = \{a,\ c,\ i,\ k\}$$
$$b'_{13} = \{a,\ g,\ k,\ m\}$$
$$b'_{14} = \{c,\ e,\ g,\ k\}$$

4.0 Conclusion

Interestingly, these new designs are balanced incomplete block designs with the same configuration (8, 14, 7, 4, 3) but different treatment placement in the bocks b'_{α}. In otherwords, treatment's arrangement in the blocks of any of the design derived differ from one configuration to the other. The new designs are not symmetric. Similarly, other eleven new designs could be possibly obtained from (13a) and (1.3b) by using (1.2).

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