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Stochastic Analysis of Differential GPS Surveys for Earth Dam Deformation Monitoring

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Abstract

In GPS measurement, we try to model not just the deterministic part of the measurement but also try to account for their stochastic behavior using the measurement variance-covariance matrix. The variance-covariance matrices are computed as part of a least squares adjustment.

In this study, the results of GPS survey by differential GPS technique were analyzed using the variance – covariance matrix of the adjusted unknown (coordinates). The GPS derived data were processed using Double Differencing (DD) technique and this enabled the removal of most of the errors in the GPS measurement. The variance – covariance matrix for three GPS receivers observing the four satellites for single difference and double difference were formulated. Using the model for 3-D differential observation, the variance-covariance of the 20 baselines in the Dam deformation monitoring network at Ikpoba Dam were computed as part of a least square solution of the observation equation using the Leica Ski pro and Move 3 software.

Standard derivation in Northings, Eastings and Elevation were computed for each observation station in the network. The computed standard deviations along with the baseline length were used to determine linear accuracy and accuracy of the baselines in parts per Million PPM.

The results revealed that the maximum standard deviation in Northings Eastings and Elevation were 1.42mm, 0.86mm and 0.57mm respectively. All the adjusted baselines satisfied 1st order class I accuracy standard which is the standard required for high precision measurements such as in Dam deformation monitoring.

Keywords: Differential GPS, Pseudo range, Carrier phase, Doppler frequency, Stochastic model.

1.0 Introduction

In differential GPS, one of the receivers known as the monitoring or reference receiver is in a precisely known position while the other receivers known as rovers are in unknown position. These receivers also receive signals from the same satellites at the same time as the reference receiver.

The reference receivers make code based GPS Pseudo range measurements but because the monitoring station know its precise position, it can determine the biases (errors) in the measurements. For each satellite in view of the monitoring station, these biases are computed by differentiating the pseudo range measurement and the satellite to reference station geometric range ([1] and [11]). These biases contain error incurred in the pseudo range measurement process for example Ionospheric and Tropospheric delay, receiver noise and the recent clock offset from system time ([4] and [7]).

To provide greater accuracy, differential GPS that utilizes phase information of the GPS satellite signal carrier frequencies have been developed. These techniques are based on interferomatric measurements of the satellite carrier frequencies and are referred to as carrier phase frequency. High accuracy application (cm level) in static application can routinely be achieved by processing the received satellite signal Doppler frequencies. This frequency information is

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integrated to form phase measurement that are processed to achieve the above stated accuracy [6].

In dam Deformation monitoring, accuracy in the mm level are desired. In such a case GPS receiver that measure both code and carrier phase are needed. In the case of the Dual frequency receivers both L1 and L2 signals are tracked. L1 wavelength is 19cm and L2 wavelength is 24cm. Integration of the Doppler frequency offsets results in extremely accurate measurement of the advance in signals carrier phase between time epoch. [8].

Using Double differencing technique in the processing of the GPS data removes most of the error sources [4]. The major exemption remains however. They are multipath which can be mitigated but not altogether eliminated and receiver noise which is still present although not as damaging as multipath. One of the technique that has been successfully used in mitigating the remaining sources of errors have been by use of a complementary Kalman filler [1] and [5]. The high accuracy achievable can be used to monitor building sway underground loading, deflection in dams as a result of hydrostatic and thermal stress changes and tectonic plate movement deflection ([3], [10], [14] and [17]). Both code and phase observably are travel time and thus range related.

The stochastic behavior of actual phase measurement noise and atmospheric effects are estimated using the double difference observation. These enables the highest precision by constraining the integer ambiguities and the points coordinates. The stochastic analysis can be separated into two parts. The first examines the actual measurement characteristics, in the absence of external errors such as ionospehric and orbital errors.

The second examines the range dependent errors which can be classified as having geometric and ionospheric effect on the measurement system. Geometric effect equivalently affects all GPS measurements. In contrast the ionospheric effect is dispersive which imply that they are radio frequency dependent and the signs are opposite for the code and phase measurement ([15] and [16])

2.0 STOCHASTIC MODELING OF GPS MEASUREMENT

In processing GPS measurements, we strive to model all of the effects that contribute to the reliability of the results. Such effects may include the offset of the satellite and receiver clocks from GPS system time, atmospheric delay and so on. Some of these effects can be modeled using parameters whose values are constant or changing slowly with time and either we can obtain the values of these parameters from external source such as satellite position coordinates or estimate them from the receiver itself (such as the receiver antennae coordinate) [8].

We refer to these components of the model as deterministic as their values are determined in a fixed and predictable manner. However, a receiver's measurement is also affected by non-deterministic carriers such as thermal noise generated in the receiver and its antennae preamplifier. Also included would be effects that typically cannot be modeled fully such as multipath and the residual errors of the mainly non-deterministic effects.

As these effects have a random nature, producing values governed by the law of probability, we refer to them as Stochastic [12].

Thus in GPS observables we, try to model not just the deterministic parts of the measurement but also try to account for their stochastic behavior using the measurement variance – covariance matrix.

A stochastic variable cannot be described by a single and exact value because there is a certain amount of uncertainty involved in the measurement process. This uncertainty is accounted for by a probability distribution. In survey measurements a normal probability distribution is assumed. Such a distribution is based on the mean μ and the standard deviation σ

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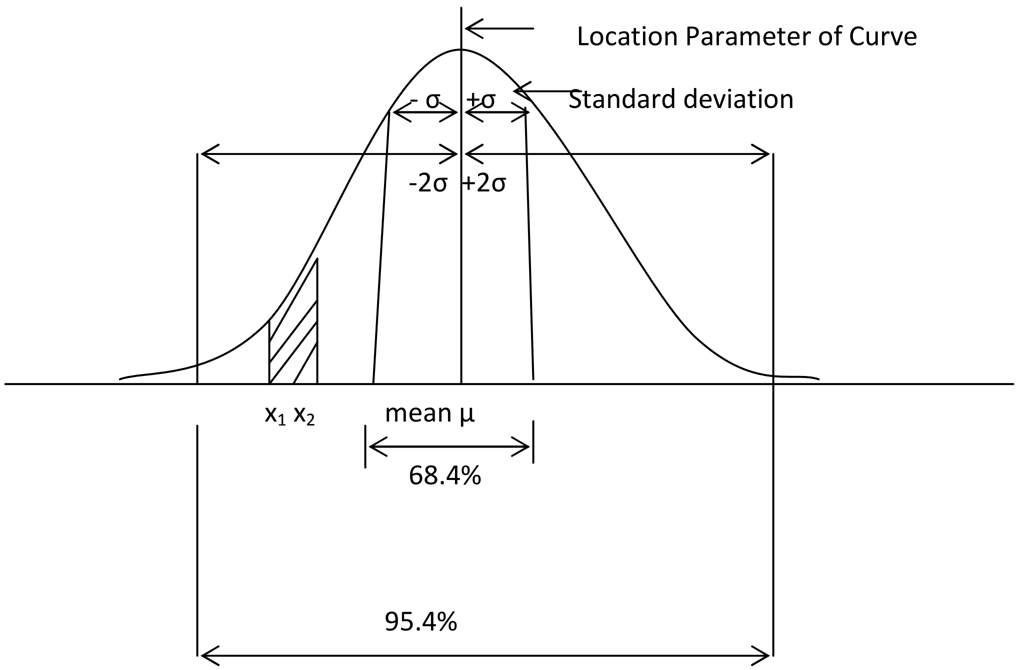

Fig. 1 Normal Probability Distribution Curve

The mean $\boldsymbol{\mu}$ represents the value of the mathematical expectation of the observable. The standard deviation is a measure of the dispersion or spread of the probability. The standard deviation $\boldsymbol{\sigma}$ characterizes the precision of an observation or measurement. By definition there is a 0.684 probability that normally distributed stochastic variable will fall within a window limited by -2$\boldsymbol{\sigma}$ and +2$\boldsymbol{\sigma}$.
In general term, the probability that a stochastic variable takes a value between X1 and X2 is equal to the area enclosed by the curve and the X1 and X2 coordinates (Fig 1) Two or more observables may be correlated. Thus the deviation in one observable will influence the other. The correlation between two observable is given by [2]

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{1}$$

The coefficient takes on values of between -1 and +1

$$-1 \leq \rho \leq 1$$

If the observation are uncorrelated then we can conclude that $\rho = 0$. The Vector elements Δx, Δy, Δz of a GPS baseline are correlated variables and the correlation can be expressed in a 3 X 3 matrix to give a symmetric matrix with a combination of standard deviation and correlation coefficient. This matrix may be expressed as

$$\text{Des and Adj } \rho \text{ – Matrix} = \begin{bmatrix} \sigma_{\Delta x} & & \\ \rho_{\Delta x \Delta y} & \sigma_{\Delta y} & \\ \rho_{\Delta x \Delta z} & \rho_{\Delta y \Delta z} & \sigma_{\Delta z} \end{bmatrix} \tag{2}$$

In equation (2) the stochastic model consist of a choice for the probability distribution of the observables. Thus for each observable standard deviation σ, is chosen based on the measurement process and experience. For this study, an apriori standard deviation of 10mm was chosen for the adjustment of the observation.

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3.0 VARIANCE – COVARIANCE OF GPS MEASUREMENTS

In differential GPS single observation, the survey accuracy is provided in the network Adjustment by the form of variance-covariance matrix $\sigma^2_{\Delta xy}$ which is given as

$$\sigma^2_{\Delta xy} = \begin{bmatrix} \sigma^2_{\Delta x} & \rho_{xy} & \rho_{y^2} \\ \rho_{xy} & \sigma^2_{\Delta y} & \rho_{yz} \\ \rho_{xz} & \rho_{yz} & \sigma^2_{\Delta z} \end{bmatrix} \tag{3}$$

In equation (3), the diagonal elements represent the error variances along the primary axis (usually in the Northings, Eastings and Ups). The off diagonal elements represent error correlation in these axis. In the case of double difference carrier phase measurement, the accuracy is given by the equation

$$\sigma_{\Delta\nabla\phi'} = \frac{1}{\lambda}\left(\sigma'^2_{\Delta x} - 2\rho_{\Delta x\Delta y} + \sigma'^2_{\Delta y}\right) \tag{4}$$

In (4)

λ is wavelength

$\rho_{\Delta x\Delta y}$ =Covariance between satellite i and j

The covariance of any two carrier phase measurements between receiver p and q and satellite I and j are expressed as [13].

$$\sigma^{pq}_{ij} = E\left[\begin{matrix}\left[\left(\frac{1}{\lambda}\left[\left(\Delta t^{p}_{sat\ i} - \Delta t^{i}_{rec\ p}\right) + \delta T^{p}_{i} - \delta\ell^{p}_{i} + m^{p}_{i} + n^{p}_{i}\right]\right)\right] \\ \left[\left(\frac{1}{\lambda}\left[\left(\Delta t^{q}_{sat\ j} - \Delta t^{j}_{rec\ q}\right) + \delta T^{q}_{j} - \delta\ell^{q}_{j} + m^{q}_{j} + n^{q}_{j}\right]\right)\right]\end{matrix}\right] \tag{5}$$

In equation (5) δ T and δ l are tropospheric and ionospheric models while n and m are noise and multipath errors respectively. Equation (5) can further be simplified by observing that error sources are uncorrelated among themselves. Thus the equation is rewritten as;

$$\sigma^{pq}_{ij} = \frac{1}{\lambda^2}E\left[\left(\Delta t^{p}_{sat\ i}\ \Delta t^{q}_{sat\ j} + \Delta t^{i}_{rec\ p}\ \Delta t^{j}_{rec\ q}\right) + \delta T^{p}_{i}\delta T^{q}_{j} + \delta\ell^{p}_{i}\delta\ell^{q}_{j} + m^{p}_{i}m^{q}_{j} + n^{p}_{i}n^{q}_{j}\right]$$

$$= \sigma^{pq}_{ij}(\Delta t_{sat}) + \sigma^{pq}_{ij}(\Delta t_{rec}) + \sigma^{pq}_{ij}(\delta T) + \sigma^{pq}_{ij}(\delta\ell) + \sigma^{pq}_{ij}(m) + \sigma^{pq}_{ij}(n) \tag{6}$$

In (6) the covariance of the individual errors were introduced. For three receivers p, q, r, observing to three satellites i, j, k, nine observation equations can be formed and the covariance for the nine observations is written as follows:

$$C_n = \begin{bmatrix}
\sigma^{p^2}_{1} & \sigma^{p}_{12} & \sigma^{p}_{13} & \sigma^{pq}_{1} & \sigma^{pq}_{12} & \sigma^{pq}_{13} & \sigma^{pr}_{1} & \sigma^{pr}_{12} & \sigma^{pr}_{13} \\
\sigma^{p}_{21} & \sigma^{p^2}_{2} & \sigma^{p}_{23} & \sigma^{pq}_{21} & \sigma^{pq}_{2} & \sigma^{pq}_{23} & \sigma^{pr}_{21} & \sigma^{pr}_{2} & \sigma^{pr}_{23} \\
\sigma^{p}_{31} & \sigma^{p}_{32} & \sigma^{p^2}_{3} & \sigma^{pq}_{31} & \sigma^{pq}_{32} & \sigma^{pq}_{3} & \sigma^{pr}_{31} & \sigma^{pr}_{32} & \sigma^{pr}_{3} \\
\sigma^{qp}_{1} & \sigma^{qp}_{12} & \sigma^{qp}_{13} & \sigma^{q^2}_{1} & \sigma^{q}_{12} & \sigma^{q}_{13} & \sigma^{pr}_{1} & \sigma^{pr}_{12} & \sigma^{pr}_{13} \\
\sigma^{qp}_{21} & \sigma^{qp}_{2} & \sigma^{qp}_{23} & \sigma^{q}_{21} & \sigma^{q^2}_{2} & \sigma^{q}_{23} & \sigma^{pr}_{21} & \sigma^{pr}_{2} & \sigma^{pr}_{23} \\
\sigma^{qp}_{31} & \sigma^{qp}_{32} & \sigma^{qp}_{3} & \sigma^{q}_{31} & \sigma^{q}_{32} & \sigma^{q^2}_{3} & \sigma^{pr}_{31} & \sigma^{pr}_{32} & \sigma^{pr}_{3} \\
\sigma^{rp}_{1} & \sigma^{rp}_{12} & \sigma^{rp}_{13} & \sigma^{rq}_{1} & \sigma^{rq}_{12} & \sigma^{rq}_{13} & \sigma^{r^2}_{1} & \sigma^{r}_{12} & \sigma^{r}_{13} \\
\sigma^{rp}_{21} & \sigma^{rp}_{2} & \sigma^{rp}_{23} & \sigma^{rq}_{21} & \sigma^{rq}_{2} & \sigma^{rq}_{23} & \sigma^{r}_{21} & \sigma^{r^2}_{2} & \sigma^{r}_{23} \\
\sigma^{rp}_{31} & \sigma^{rp}_{32} & \sigma^{rp}_{3} & \sigma^{rq}_{31} & \sigma^{rq}_{32} & \sigma^{rq}_{3} & \sigma^{r}_{31} & \sigma^{r}_{32} & \sigma^{r^2}_{3}
\end{bmatrix} \tag{7}$$

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The matrix in equation (7) is Symmetric and only four basic forms of variances exist. These include σ_i^p, σ_i^{pq}, σ_{ij}^p σ_{ij}^{pq}. In addition, elements of the form σ_{ij}^p are equal to their corresponding elements of the form σ_{ji}^p, σ_i^{pq} equal element of the form σ_{ji}^{qp}.

For double –difference observation, the variance covariance matrix can be determined using propagation of matrics with the aid of a suitable B matrix. Thus for a set of observation made by three receivers to three satellites, the variance-covariance from the double difference equation is [8].

$$C_{\Delta\nabla} = BC_nB^T = \begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} . C_n . \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

C_n in equation (8) is obtained from (7). Equation (8) is also symmetric and fully populated.

In this study, the variance-covariance matrix of the observation were computed as part of the least squares solution for the 20 baselines observed using the relationship [8].

$$C_{\Delta\nabla} = \begin{bmatrix} \sigma_{\Delta x}^2 & \sigma_{\Delta y\Delta y} & \sigma_{\Delta y\Delta z} \\ & \sigma_{\Delta y}^2 & \sigma_{\Delta y\Delta x} \\ & & \sigma_{\Delta z}^2 \end{bmatrix} \quad (9)$$

The survey accuracy was provided in the network adjustment and based on a posterior standard deviation of σ_0 the accuracies in Northings, Eastings and Ups were computed using the relationship [18].

$$\sigma_x = \sigma_0\sqrt{\sigma_{\Delta x}} \quad (10)$$

$$\sigma_y = \sigma_0\sqrt{\sigma_{\Delta y}} \quad (11)$$

$$\sigma_z = \sigma_0\sqrt{\sigma_{\Delta z}} \quad (12)$$

Where $$\sigma_0 = \sqrt{\frac{V^T PV}{m-u}} \quad (13)$$

V - Residual
P - Weight of Observation
m - Number of observations
n - Number of parameters

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The horizontal network accuracy and position accuracy were also computed. All computations and adjustments were carried out using the leica Ski-pro2.1 and MOVE.3 software.

4.0 RESULTS AND DISCUSSIONS

A layout of the baselines between the reference and rover stations is shown in Fig. 1. The computed variance-covariance matrix for the 20 baselines is shown in Table I. Fig. 3 – 6 presents the error variances in Northings, Eastings, Ups and horizontal position respectively. From the table and figures, the maximum error in position fixing of 1.7mm occurred in RF 7 while the best fix of 0.56mm occurred in the baseline DEFM7SI– CFG113B. The results in general meet with 1st order control specification with a minimum linear accuracy of 1:100,000.

The error variances are shown as the diagonal elements presented for each baseline in Table 1 while the off diagonal elements represent error correlation in these axes. Also computed for each fixed point were the a posteriori standard errors. Using equation (10) – (13), the standard deviation in Northings, Eastings and Ups together with those for the horizontal position were computed for the 20 baselines and the results are presented in Table II are the accuracies in parts per million and linear accuracy for the 20 baselines computed as a ratio of the errors in position to the length of baselines.

TABLE I: Variance-Covariance Matrix of Baselines from Single epoch GPS mission for monitoring points on Dam crest

1	RF2-CFG113B	a posteriori rms = 0.8146 q_X q_y q_z q_x $+2.986140 \times 10^{-6}$ $+1.090451 \times 10^{-6}$ $+6.457660 \times 10^{-7}$ q_y $+1.1000190 \times 10^{-6}$ $+3.383370 \times 10^{-7}$ q_z $+4.337240 \times 10^{-7}$
2	RF10-CFG113B	a posteriori rms = 0.4902 q_X q_y q_z q_x $+ 3.1899050 \times 10^{-6}$ $+1.586400 \times 10^{-8}$ $+1.021820 \times 10^{-7}$ q_y $+1.3708530 \times 10^{-6}$ $+ 1.806800 \times 10^{-8}$ q_z $+2.9145900 \times 10^{-7}$
3	BMB1-CFG113B	A posteriori rms = 0.5681 q_X q_y q_z q_x $+2.0278170 \times 10^{-6}$ $+2.440000 \times 10^{-9}$ $+5.411500 \times 10^{-8}$ q_y $+2.737000 \times 10^{-7}$ $+ 4.974200 \times 10^{-8}$ q_z $+3.2097200 \times 10^{-7}$
4	RF8-CFG113B	a posteriori rms = 0.5023 q_X q_y q_z q_x $+ 3.5592330 \times 10^{-6}$ -9.016700×10^{-8} $+2.2515200 \times 10^{-7}$ q_y $+1.3995300 \times 10^{-6}$ $+ 9.006100 \times 10^{-8}$ q_z $+3.1575000 \times 10^{-7}$
6	RF01-CFG113B	a posteriori rms = 0.4618 q_X q_y q_z q_x $+ 1.608896 \times 10^{-6}$ $+4.813490 \times 10^{-7}$ $+1.068800 \times 10^{-8}$ q_y $+9.205660 \times 10^{-7}$ $+ 4834400 \times 10^{-7}$ q_z $+3.6452900 \times 10^{-7}$
7	RF2-CFG113B	a posteriori rms = 0.8417 q_X q_y q_z q_x $+ 1.9177207 \times 10^{-3}$ $-3.1655952 \times 10^{-3}$ $+3.773089 \times 10^{-4}$ q_y $+5.5279899 \times 10^{-3}$ $+ 6.1315593 \times 10^{-4}$ q_z $+1.592182 \times 10^{-4}$
8	RF9-CFG113B	a posteriori rms = 0.5015 q_X q_y q_z

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		q_x	+ 2.9683830 x 10^{-6}	-7.818600 x10^{-8}	+1.8253700 x10^{-7}
		q_y		+1.1652540 x 10^{-6}	+6.942300x10^{-8}
		q_z			+2.6213500 x10^{-7}

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TABLE II ACCURACY STANDARDS FOR 3-D DIFFERENTIAL GPS SURVEY AT IKPOBA DAM 2008

			σ_0	σ_x	σ_y	σ_z	$\sigma_H = \sqrt{\sigma_x^2 + \sigma_y^2}$	$\sigma_\rho = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$			
S/NO	ROVER ID	REF ID	AP RMS						DISTANCE	Accuracy IN PPM	Linear Accuracy
1	DEFM 1 SI	CFG113B	0.6645	0.86	0.64	0.37	1.072	1.1341	1584.4455	6.76×10^{-4}	1:1,478,027
2	DEFM 2 SI	CFG113B	0.4129	0.6	0.31	0.24	0.6754	0.7167	1670.4497	4.043×10^{-4}	1:2,473,274
3	DEFM 3 SI	CFG113B	0.4489	1.18	0.47	0.46	1.2702	1.3509	2348.8115	5.408×10^{-4}	1:1,849,166
4	DEFM 4 SI	CFG113B	0.4955	0.81	0.37	0.32	0.8905	0.9463	2361.6424	3.771×10^{-4}	1:2,652,040
5	DEFM 5 SI	CFG113B	0.5815	0.79	0.34	0.31	0.8601	0.9142	2437.5191	3.529×10^{-4}	1:2,833,995
6	DEFM 6 SI	CFG113B	0.4711	0.5	0.3	0.22	0.5831	0.6232	1791.4017	3.255×10^{-4}	1:3,072,203
7	DEFM 7 SI	CFG114B	0.3799	0.49	0.2	0.18	0.5292	0.559	1751.9536	3.021×10^{-4}	1:3,310,569
8	DEFM 8 SI	CFG113B	0.5941	0.88	0.3	0.33	0.9297	0.9866	2477.3642	3.753×10^{-4}	1:2,664,692
9	DEFM 9 SI	CFG113B	0.684	0.98	0.35	0.39	1.0406	1.1113	2424.5803	4.292×10^{-4}	1:2,329,982
10	DEFM10SI	CFG113B	0.3932	0.98	0.31	0.33	1.0279	1.0795	1829.7721	5.618×10^{-4}	1:1,780,107
11	DEFM11SI	CFG113B	0.3879	0.93	0.3	0.31	0.9772	1.0252	1773.8696	5.509×10^{-4}	1:1,815,257
12	BMB 1	CFG113B	0.5681	0.81	0.3	0.33	0.8638	0.9247	2400.2482	3.599×10^{-4}	1:2,778,708
13	RF 1	CFG113B	0.4618	0.59	0.44	0.28	0.736	0.7875	1810.4184	4.065×10^{-4}	1:2,459,807
14	RF 2	CFG113B	0.8146	1.42	0.85	0.54	1.655	1.7408	1879.4902	8.806×10^{-4}	1:1,135,643
15	RF 4	CFG113B									
16	RF 7	CFG113B	0.8705	1.56	0.86	0.57	1.7813	1.8703	2089.3239	8.526×10^{-4}	1:1,173,920
17	RF 8	CFG113B	0.5023	0.95	0.59	0.28	1.1183	1.1528	2165.7972	5.163×10^{-4}	1:1,936,687
18	RF 9	CFG113B	0.5015	0.86	0.54	0.26	1.0155	1.0482	2260.5345	4.492×10^{-4}	1:2,226,031
19	RF 10	CFG113B	0.4902	0.88	0.57	0.26	1.0485	1.0802	2355.6598	4.451×10^{-4}	1:2,246,695

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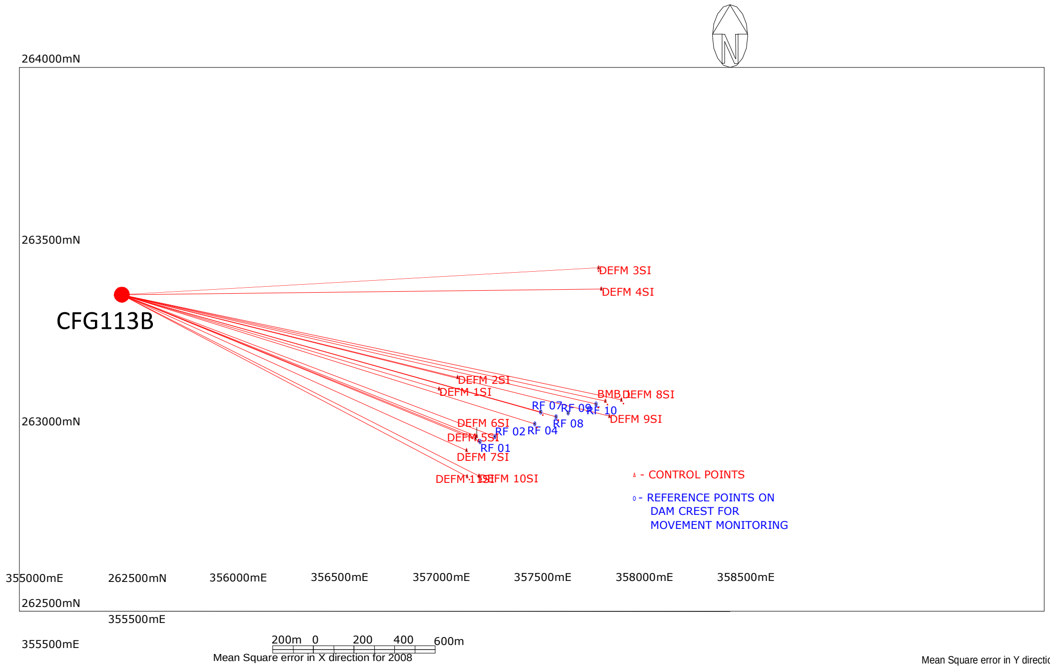
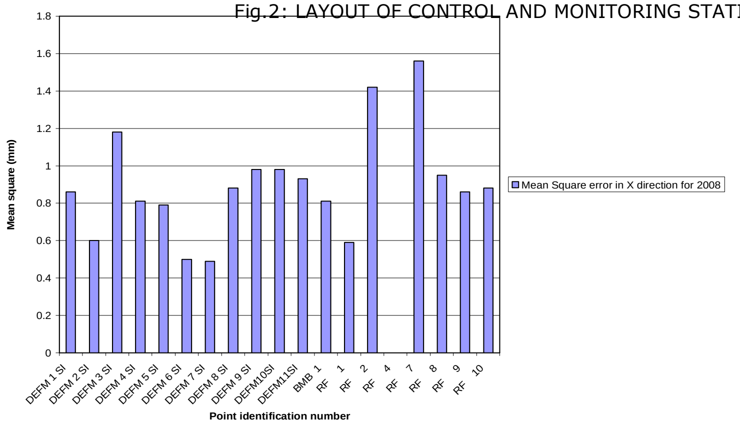


Fig.2: LAYOUT OF CONTROL AND MONITORING STATIONS



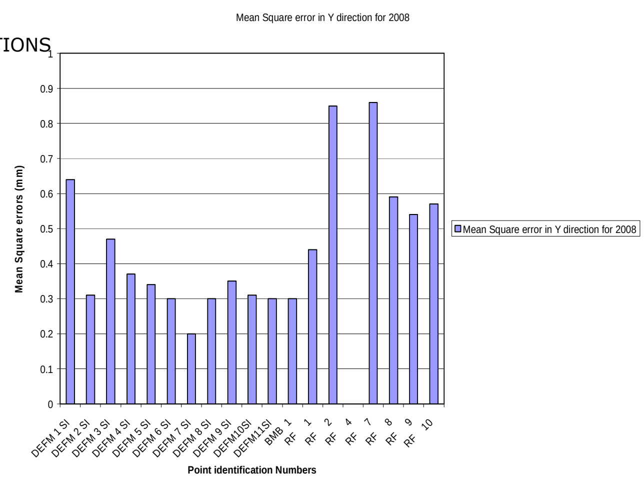


Fig. 3: Standard errors in X Coordinates

Fig. 4: Standard errors in Y Coordinates

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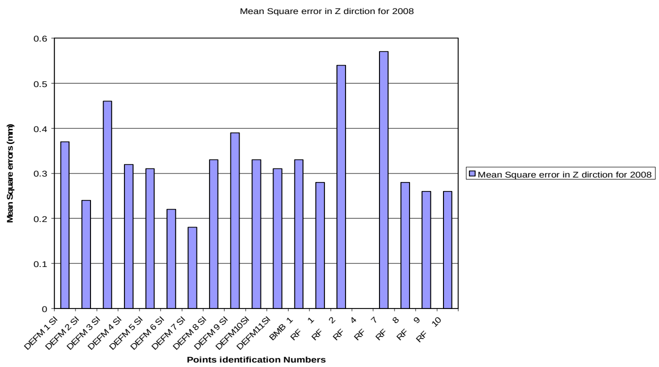

Fig. 5: Standard error in Elevation for monitoring and reference points

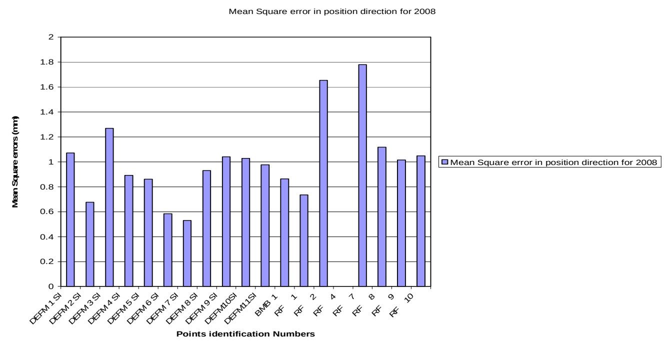

Fig. 6: Standard errors in Horizontal Position for monitoring and reference points

5.0 CONCLUSIONS

The results obtained in this study are quite good. This is perhaps due to the fact that using Differential GPS code and carrier phase measurement techniques significantly reduces the biases or errors influencing GPS measurement accuracy.

Proper stochastic modeling of the GPS observation is crucial for both position accuracy and the accuracy of the statistical estimates ***returned*** by the least squares adjustment. For position data, the maximum standard deviation in Northings, Eastings and Elevation were 1.42mm, 0.86mm and 0.57mm respectively.

In order to have good results for GPS measurements, it is recommended that measured data should be examined for systematic effects not captured by the functional model and subsequent analysis will help choose an a prori standard error for the adjustment.

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