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Parameter Estimation And Hypothesis Testing In A Two Epoch Dam Deformation Measurement

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Abstract

As a result of the processing of GPS measurements the estimate for the coordinates unknown is accompanied by a measure of the quality of the estimator. In as much as the model used in the estimation holds true, the quality is described by precision. In practice however, one is never sure that the model employed is adequate. In such cases a statistical testing procedure is used to determine the validity of the model so as to detect problem misspecification.

In this study the absolute standard ellipse was used for presenting the precision of station coordinates. The standard ellipse represented the propagation of random errors through the mathematical model into the coordinates of the monitoring stations. The error ellipse consisting of semi major axis A, semi minor axis B and the angle Ø between the semi majoe axis and the y-North axis of the coordinate system were computed for the nineteen reference and rover monitoring stations in the network. Statistical tests were performed in each of the estimated parameters using the F, W and t- tests to determine their significance. In the tests, the validity of the null hypothesis Ho, the model used in the estimation was opposed against an alternative hypothesis H_1. If any parameters were found to be statistically insignificant, they were eliminated and a new solution recomputed. Also computed along with the least square solution and statistical testing were the minimum detectable Bias (MDB) and the Bias to Noise Ratio (BNR). All tests and adjustments were carried out using MOVE 3 software along with the LEICA SKI Pro 2.1

From the results of the tests, only observation to Rover station RF 8 failed both the W – tests and t – tests and was therefore regarded as an outlier and the results rejected. The test results showed that there were no model errors present in the observation after rejection of outliers; and no systematic errors were present in the results.

Keywords: Minimum Detectable Bias, Outliers, Bias to Noise Ratio, Error Ellipse, Cycle Slip

1.0 Introduction

The purpose of a survey measurement such as Differential GPS is to provide geometric information. This information usually concern coordinates but should also comprise of the quality of the coordinate estimator [16].

Salzman [11] described quality assurance as consisting of three steps which correspond to the steps before, during and after carrying out (time Varying) measurement.

In the design phase of the system, optimization with respect to quality take place by using measures on precision and reliability. These measures can be computed prior to operation of the system. In these computation, the null hypothesis usually is the default mathematical model. In order to meet the specification on quality in terms of precision and reliability, the measurement system can be defined and a statistical testing carried out.

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In the next step which is the operational phase, a statistical testing procedure is carried out parallel with the estimation. The null hypothesis Ho is opposed to alternate hypothesis H_1. After carrying out the testing procedure, one can be sure to a certain degree about the validity of the model used. This degree is referred to as reliability [12].

At the design stage, quality is described a priori based on the assumed model used. After the operational phase, quality is described a posteriori. The quality measures are computed using the null hypothesis. The check can however be carried out when the null hypothesis has been rejected in favour of an alternate hypothesis [2] and [3]. The null hypothesis, the mathematical model used in the estimation may not be adequate. We have to assess the validity of the model and detect possible misspecification. The null hypothesis therefore have to be confronted with alternate hypothesis. As a result of misspecifications, model errors occur which may be classified as outliers and cycle slips. An outlier is one erroneous observation in a sequence of observation [16]. The error occurs only once, when a slip occurs the observation get biased by a constant error. Specifying an outlier hypothesis for each observation available and testing the null hypothesis against them is called data snooping. [1] and [16]. Measures for quality refer to precision and reliability. The minimal percentage bias is a measure for internal reliability.

The minimum detectable Bias gives for a one-dimensional alternative hypothesis the size of a model error that can be detected with a probability γ by the slippage test. The MDB for an outlier in the code is expressed in metres and for slip in the phase, it is expressed in cycles [15]. It is to be noted that measures on precision are based on the variance – covariance Matrix of the estimator. Measures on reliability are primarily the Minimal Detectable Bias MDB for internal reliability and the Bias to Noise Ratio (BNR) which establishes the significance relative to external reliability.

1.0 QUALITY CONTROL

2.1 Precision: In order to present the precision of each station whose coordinates have been determined the standard ellipse is used. The standard ellipse is a 2-D equivalent of standard deviation.

For a standard ellipse the level of confidence is 0.39. To get a level of confidence of 0.95, we multiply the axis by a factor of 2.5 [17]. Absolute standard ellipse represents the propagation of random errors through the mathematical model into the coordinates of the control and reference points. Relative standard ellipse represents the precision between pairs of stations. The shape of the ellipse is defined by semi major axis A, and semi minor axis B. the orientation of an absolute standard ellipse is defined by the angle θ between the semi-major axis and the Y-North axis of the coordinate system.

Taking 2-D coordinates, the control points may have a standard error greater than $\sigma_{\Delta x}$ and $\sigma_{\Delta y}$ in some prescribed direction θ. This direction is referred to as the semi major axis of the error ellipse ($\pm\sigma$max) and the semi minor axis ($\pm\sigma$min) would be at right angle to it. In such case we can determine $\pm\sigma$max and $\pm\sigma$min as follows [5] and [13].

$$\pm\sigma_{\max}^2 = \frac{1}{2}\left(\sigma_x^2+\sigma_y^2\right)+\left[\frac{1}{4}\left(\sigma_x^2-\sigma_y^2\right)+\sigma_{xy}^2\right]^{1/2} \tag{1}$$

$$\pm\sigma_{\max}^2 = \frac{1}{2}\left(\sigma_x^2+\sigma_y^2\right)-\left[\frac{1}{4}\left(\sigma_x^2-\sigma_y^2\right)+\sigma_{xy}^2\right]^{1/2} \tag{1a}$$

Where $\pm\sigma$max and σmin are the Eigen values of the variance-covariance matrix. The angle θ of the semi-major axis can be computed by the expression.

$$\tan\theta = \frac{\sigma_{xy}}{\sigma_{\max}^2-\sigma_x^2} \tag{2}$$

In 2-D statistics plus or minus one standard deviation ($\pm\sigma$) represents a probability of 68.3%. [8] and [9] give the probability of the joint event falling within the error ellipse as only 39.4%.In this study using variance – covariance matrix of the baselines the semi major axis A and semi minor axis B and the angle θ were computed for the twenty baselines.

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2.2 Reliability

The reliability of the network can be described in terms of its sensitivity to the detection of outliers. Reliability can be expressed as both internal and external reliability. Internal reliability is expressed in terms of Minimal Detectable Bias (MDB) which precedes the size of the smallest possible error which is detectable by a statistical test, with a probability equal to the power of the test. Thus under the assumption that a certain alternative hypothesis holds true instead of the null hypothesis, the size of the model error Δ can be computed such that it can be found with the testing procedure with a certain probability [4] and [5].

Once the level of significance α has been chosen, the power γ follows from the q dimensional $\varkappa^2(q.\lambda)$ distribution and $\gamma \square - 1 = \beta$.

On the other hand, once γ has been fixed the non centrality parameter λ can be computed This reference value is then denoted by $\lambda_{0.}$ The non centrality parameter is related to the model error by ([15], [16]) as

$$\lambda = \nabla^T C_1^T Q^{-1} C_1 \nabla \qquad (3)$$

If we put $\lambda = \lambda_0$ in equation (3) we represent the boundaries of a hyper ellipsoid.

For a one-dimensional model error, q = 1, the ellipsoid collapses to an interval I , where

$$I = \left\| C_1 \nabla \right\|^2 Q_1^{-1} \qquad (4)$$

The size of the minimal detectable bias is computed as

$$|\nabla| = \sqrt{\left(\frac{\lambda_0}{C_1^T Q_1^{-1} C_1} \right)} \qquad (5)$$

Equation (5) describes the normal performance of the testing procedure in finding a model error of the type specified in alternate hypothesis.

Equation (6) is the minimal detectable bias MDB related to so called test statistics W which is given by (Teunissen 1997, Ehiorobo 2008) as:

$$W = \frac{\left(C_t^T Q_t^{-1} t \right)^2}{C_t^T Q_t^{-1} C_t} \qquad (6)$$

The detectable model error in term of the vector of observation is given by

$$\Delta_y = C_y \nabla \qquad (7)$$

Equation (7) allows us to analyze internal reliability which describes to what extent model validation is possible. The external reliability is expressed as the Bias to Noise Ratio and are used to determine the influence of a possible error in the observation on the adjusted coordinates. The MDB and BNR for the twenty baselines were computed along with the least squares adjustment of the observation using leica Ski pro 2.1 software along with MOVE. 3 software and the results are presented in tables V and VI.

2.0 STATISTICAL TESTING

In our survey measurements, thee mathematical and stochastic models are based on a set of assumptions. These assumptions or statistical hypothesis will result in different hypothesis. A special set of assumption is referred to as null hypothesis. This hypothesis with reference to our systems of observation imply that [5],

- These are no gross errors (blunders) present in the observation or measurements
- The mathematical model gives a correct description of the relations between the observation and the unknown parameters.
- The chosen stochastic model for the observation appropriately describes the stochastic properties of the observation.

A set of a statistical hypothesis H_0 is an algorithm that leads to a statistical decision concerning the validity of H0. Since the number of sample is small, a definite decision concerning H0 cannot be reached. Thus a decision based on finite samples can be trusted only to a certain degree. This means that such a decision has only a limited confidence attached to it [15]. There are two possible outcomes of the test. Accept Ho or reject Ho. Similarly, there are two possible outcomes of the same test for an alternative hypothesis H1. Since none of the hypothesis may be

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true, the test should at least show which is better. The probability of rejecting Ho when in fact Ho is true is called a type I error [7]. For finite samples one discovers that no Ho is acceptable if $\sigma = 0$ i.e. if there is no risk involved. In survey measurement α is selected between 0,01 and 0.05 Next if H1 is true or when Ho is rejected, the probability of accepting Ho which is false is called a type II error and is related to the power of the test 1 - β in a complementary fashion [7]. The most powerful test is the one that employ the particular alternative hypothesis H1 that yields the smallest type II error β for the same significant level α [6] and [15].

When the probability of false alarm α is specified, the power δ(1- β) is maximized using Newman – Peerson testing principles [6] and [7]. This yields the test statistics t which is a function of the Vector of observation $t_{(\Phi)}$ and the critical region R for the test statistics which is a function of α. Thus the test will comprise [15] and [16].

If $t \in K \Longrightarrow$ reject Ho.

If $t \not\subset K$ accept Ho (Ho not rejected)

The statistical tests used in the study included F-Test, w-test and t-test.

2.1 F – Test

The F- test is the overall model test because it test the model in general. It is a commonly used multi-dimensional test for checking Null-Hypothesis $H_{o.}$ The F test is computed using the equation.

$$F_{comp} = \frac{S^2}{\sigma^2} \tag{5}$$

where S^2 – a-posteriori variance factor which is dependent on the computed residual and the redundancy

σ^2 = a priori variance factor. The F-value is tested against a critical value of F- distribution which is a function of the redundancy and the significant level α If F- computed is less than F critical then we accept H_o. But if F-computed is greater than F critical we reject H_o Thus $F_{comp} < F_{critical}$ – accept H_o. The three sources of rejection of H_o include the presence of gross errors, incorrect mathematical model and incorrect stochastic model. Usually, the information provided by the F-test about acceptance or rejection of H_O is not very specific. Therefore if H_o is rejected, we need to find the cause of the rejection by tracing errors in observation or assumption made in the mathematical and stochastic models. If we suspect that H_O is rejected due to a gross error present in one of the observations, the W-test will be used to detect the outliers. The F and W test are linked by a common value of the power β.This relationship is normally referred to as the B-method of testing. H_o can also be rejected when the mathematical model is incorrect or not refined enough. If this is detected the mathematical model has to be improved in order to prevent an inferior outcome [5]. Finally, another source of rejection of H_o is when the a priori variance- covariance matrix is too optimistic. Such a rejection can easily be remedied by increasing the input standard deviation of the observation [5]. In some cases, a combination of the three sources of rejection discussed above can occur in which case we resort to data snooping utilizing the W-test in order to search for errors in individual observation.

2.2 W – Test

A rejection of the F-test does not directly lead to the source of the rejection itself. In case the null-hypothesis is rejected, other hypothesis must be formulated which describe a possible error, or a combination of errors. There is an infinite number of hypotheses which can be formulated as an alternative for the null hypothesis. The more complex these hypotheses become, the more difficult they will be to interprete. A simple, but effective hypothesis is the alternative hypothesis H_1 which is based on the assumption that there is an outlier present in one single observation while all others are assumed to be correct. The one dimensional test associated with this hypothesis is the W-Test. A strong rejection of the F-test can often be traced back to a gross error or blunder in just one observation. There is a conventional alternative hypothesis for each observation which implies that each individual observation is tested. The process of testing each observation in the network by a W-test is called Data snooping. The size of the least squares correction alone is not always a very precise indication when checking the observation for outliers [1] and [16]. A better test quantity, though only suited for uncorrelated observation is the least squares correction divided by its standard deviation [10]. For correlated observation such as the three elements in each of the measured base line dx, dy, dz, the complete weight matrix of the observation must be considered . This condition is fulfilled by the test quantity W of the W-test which has a standard normal distribution and is most sensitive for an error in one of the observations. The critical value W_{crit}. depends on the choice of the significant level α_o.If W> W_{crit} , the W-test is rejected and there is a probability of 1- α_o that the corresponding observation

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holds an outlier. On the other hand there is a probability α_o that the observation does not hold an outlier, which means the rejection is unjustified. Table I presents an overview of the α_o values and the corresponding critical values.

Table I: Significant level /critical value for W-test.

Situation	W_1	W_2	W_3
Significant level α_o	0.001	0.010	0.050
Critical value W-test	3.29	2.58	1.96

In Geodetic measurements, α_o between 0.001 and 0.05 are commonly used. An α_o = 0.001 means one false rejection in every 1000 measurements. Thus we can presume this to be a very comfortable choice. Essential for the B- method of testing is that an outlier is detected with the same probability by both the F-test and the W- test (Mikhail 1976). For this purpose, the power β of both tests is fixed on a level of 0.80 as was done in this research. The level of significance α_o of the W-test is also fixed which leaves the level of significance α of the F-test to be determined .Having α_o and β fixed, α depends strongly on the redundancy in the network. For large scale networks with many observations, and a considerable amount of redundancy, it is difficult for the F-test to react to a single outlier. The F-test being an overall model test, is not sensitive enough for this task. As a consequence of the link between the F-test and the W-test by which the power is forced at 0.80, the level of significance α of the F-test is increased. Consequently, no matter the outcome of the F-test, it will be necessary to carry out Data snooping [5].

2.3 t – Test

The W- test earlier discussed is a I –dimensional test used to check alternate hypothesis. These hypothesis assumes that there is just one observation erroneous at the time. This so called Data snooping works very well for single observation such as direction, distances, height difference etc. For GPS baseline such as encountered in this research, it is not sufficient to test dx, dy, dz elements of the vector separately. it is essential that the baseline be tested as a whole. For this purpose, we use the t-test. The T-test is a 3-D or 2-D test. The t-test is also linked to the F-Test by the B-method of testing and has the same power as the other two methods of testing earlier discussed. However, the t-test has its own level of significance and its own critical values as shown in Tables II and III.

Table II: Significant level /critical value of 2-D t-test based on α_o of the W-Test

Situation	t_1	t_2	t_3
Significant level α_o	0.001	0.010	0.050
Significant level α (2-D)	0.003	0.022	0.089
Critical value T-Test	5.91	3.81	2.42

Table III: Significant level critical value of 3-D t- test based on α_o of W test

Situation	t_1	t_2	t_3
Significant level α_o	0.001	0.010	0.050
Significant level α (3-D)	0.005	0.037	0.129
Critical value T-test	4.24	2.83	1.89

The T- test is useful when testing known stations. The data snooping will test for an outlier due to erroneous entry. The deformation of a station might not be detected by the data snooping when the deformation shifts decomposes in Easting , Northing and H-direction are relatively small. A different hypothesis may be formulated for testing deformation influencing both the X-Easting, Y Northing and H- coordinate The 3-D t- test is better equipped to test in the complete coordinate to trace deformation, although it will not be able to trace the exact direction in which the station has moved [15].

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3.0 PRECISION ESTIMATION AND STATISTICAL TESTING OF GPS OBSERVATION DATA

3.1 Prescision Of Adjusted Unknown

For presenting the precision of station coordinates the absolute standard ellipse were used (Koch 1988). These standard ellipse represents propagation of random errors through the mathematical model into the coordinates of the monitoring and control points. The error ellipse parameters consisting of semi major axis A, Semi minor axis B and the angle $\emptyset$ were computed for 20 baselines in the GPS measurements from the control point to the monitoring station for a 95% confidence level.

3.2 Statistical Testing

Statistical tests were performed on each estimated parameter to determine their significance.

If any parameter was found to be statistically insignificant, they were eliminated and a new solution recomputed. The statistical testing presented in this study was carried out together with the least squares adjustment and were based on analysis of the least square residuals.

The tests parameters used in the testing were as follows:

Alfa (α) Multidimensional	= 1.000	
Alfa o (αo) One dimensional	=	0.0500
Beta (β)	=	0.80
Critical W-test =	1. 69	
Critical Value t – test (3-D)	=	1.89
Critical Value t-test (2-d) =	2.42	
Critical Value F-Test	=	3.24
F – Test	=	12.622 (rejected)

Results based on a posterior variance factor. The result of the various tests carried out for parameter estimates are presented in Table V while Table VI presents the estimated errors for observation with rejected W and t Tests.

4.0 RESULTS AND DISCUSSIONS

The results of the computed semi major axis A, semi minor axis B and their Azimuth $\emptyset$ are as presented in Table IV.

The results of the parameter testing for the combined adjustment for first and second epoch measurement are presented in table V. In the table, the minimal detectable bias (MDB) Residual Vectors (RED), Bias to Noise Ratio (BNR), W and T-tests results, obtained after an adjustment are presented. The gaps in the table under the minimal Detectable Bias (MDB), the Residual (RED), Bias to Noise Ratio BNR and W-test indicates that these baselines did not contain model errors.

The results for baselines CFG 113B – RF1, CFG113B – DEFM7SI, CFG113B-DEFM8SI, CFG 113B – DEFM5SI and

CFG113B – RF8 contain model errors, whose values are indicated in the MDB, RED BNR, and W-test columns of Table V

Further analysis indicated that the model errors in the first four baselines were within acceptable limit and only in the case of the Baseline CFG113B – RF8 are the errors above the acceptable limit.

From table VI, the computed values for W is 2.96 and for t is 2.93 while the critical values for W is 1.96 and for t is 1.89. Additionally, the estimated errors for the observation with rejected W and T-tests occurred in RF 8 with a maximum of 0.128m. This is quite high compared to the acceptable value of 0.050m. Thus we can conclude that the point RF8 contains systematic errors and therefore can be regarded as an outlier and was therefore rejected.

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Table IV: Absolute Standard Ellipse for Differential GPS Measurements

STATION	SEMI-MAJOR AXIS (A)	SEMI-MINOR AXIS (B)	$\frac{A}{B}$	Φ Azimuth
DEFM 1 SI	0.0066	0.0066	1.0	90°
DEFM 6SI	0.0068	0.0068	1.0	90°
DEFMISI	0.0068	0.0068	1.0	90°
DEFM10SI	0.0068	0.0068	1.0	90°
DEFM11SI	0.0068	0.0068	1.0	90°
DEFM3SI	0.0073	0.0073	1.0	90°
DEFM4SI	0.0074	0.0074	1.0	90°
DEFM5SI	0.0074	0.0074	1.0	90°
DEFM8SI	0.0074	0.0074	1.0	90°
CFG113B				°
DEFM9SI	0.0074	0.0074	1.0	90°
RF 1	0.0068	0.0068	1.0	90°
RF 2	0.0069	0.0069	1.0	90°
RF 8	0.0072	0.0072	1.0	90°
RF 7	0.0071	0.0071	1.0	90°
RF 10	0.0074	0.0074	1.0	90°
RF 4	0.0071	0.0071	1.0	90°
BMB 1	0.0074	0.0074	1.0	90°
CFG113B	0.0000	0.0000	0.0	0
RF9	0.0073	0.0073	1.0	90°

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Table V. **STATISTICAL TESTS OF OBSERVATIONS FOR COMBINED 1ST AND 2ND EPOCH DATA**

	Station	Target	MDB	Red	BNR	W-test	T-test
DX	CFG113B	O1SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	06SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	07si					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	10SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	11si					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	4si					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	RF 01	0.1185m	33	4.0	1.83	1.13
DY			0.1185 m	33	4.0	0.05	
DZ			0.1185 m	33	4.0	0.19	
DX	CFG113B	BMB 01					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	DEFM02					Free obs
DY							Free obs
DZ							Free obs

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DX	CFG113B	DEFM06					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	DEFM07					Free obs
DY							Free obs
DZ							Free obs
DX	CFG113B	DEFM 10					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM11					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM 3SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM 4SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM 5SI					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM 7sI	0.1185 m	33	4.0	-1.83	Free obs
DY			0.1185 m	33	4.0	-0.05	Free obs
DZ			0.1185 m	33	4.0	-0.19	Free obs
DX	CFG 113B	DEFM 8SI	0.1191 m	33	4.0	-1.06	Free obs
DY			0.1191 m	33	4.0	0.06	Free obs
DZ			0.1191 m	33	4.0	0.01	Free obs
DX	CFG 113B	DEFM 9SI					Free obs
DY							Free obs
DZ							Free obs

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DX	CFG 113B	DEM 1SI					Free obs
DY							Free obs
DZ							
DX	CFG 113B	RF 01	0.1216 m	34	3.9	2.96**	
DY			0.1216 m	34	3.9	-0.01	
DZ			0.1216 m	34	3.9	0.19	
DX	CFG 113B	RF 02					
DY							
DZ							
DX	CFG 113B	RF 02 (3)					
DY							
DZ							
DX	CFG 113B	RF 07					
DY							
DZ							
DX	CFG 113B	RF 08					
DY							
DZ							
DX	CFG 113B	RF 10					
DY							
DZ							
DX	CFG 113B	defm 9si					Free obs
DY							1.13
DZ							
DX	CFG 113B	Rrf 9					
DY							0.37
DZ							
DX	CFG 113B	3si					
DY							Free obs
DZ							Free obs

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DX	CFG 113B	5si	0.1191 m	33	4.0	1.06	Free obs
DY			0.1191 m	33	4.0	-0.06	Free obs
DZ			0.1191 m	33	4.0	-0.01	Free obs
DX	CFG 113B	5si	0.1216 m	34	4.0	-2.96**	Free obs
DY			0.1216 m	34	4.0	0.01	2.93**
DZ			0.1216 m	34	4.0	-0.19	
DX	CFG 113B	8si					
DY							Free obs
DZ							Free obs
DX	CFG 113B	DEFM981					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	RF 02					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	RF 08					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	RF 07					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	RF 10					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	RF 4					Free obs
DY							Free obs
DZ							Free obs
DX	CFG 113B	BMB					Free obs
DY							Free obs
DZ							Free obs

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DX DY DZ	CFG 113B	Rf 09					Free obs Free obs

TABLE VI Estimated Errors For Observation With Rejected W And T-Test

ESTIMATED ERRORS FOR OBSERVATION WITH REJECTED W – TEST (max 10)

RECORD	STATION	TARGET	W-Test	Fact	Est. Err
25 DX	CFG 113B	RF 08	2.96	1.5	0.1286 m
35 DX	cfg 113b	RF 08	-2.96	1.5	-0.1286m

ESTIMATED ERRORS FOR OBSERVATIONS WITH REJECTED ANTENNA HGT W – TESTS (max 10)

Record	Station	Target	W-Test	Fact	MDB	Est. ant. Err
35 DX	cfg 113b	RF 08	-2.95	1.5	0.1216 m	-0.1281m
25 DX	CFG 113B	RF 08	2.95	1.5	-0.1216m	0.1281 m

ESTIMATED ERRORS FOR OBSERVATION WITH REJECTED W – TEST (max 10)

RECORD	STATION	Target	W-Test	Fact	Est. Err
25 DX DY DZ	CFG 113B	**RF 08**	2.93	1.2	0.1286 m -0.0003 m 0.00081 m
35 DX DY DZ	CFG 113B	RF 08	-2.96	1.5	-0.1286m 0.0003 m -0.0081 m

5.0 CONCLUSIONS

Proper Stochastic modeling of GPS observation is crucial for both positional accuracy and the accuracy of the statistical estimates returned by an adjustment. Computation for internal reliability showed that the minimum detectable bias were small except for the coordinates of RF 8. The study revealed that rigorous and judicious application of hypothesis can uncover previously unseen issues in an otherwise established problem. This is clearly demonstrated in the use of f-test, W-test and t-test for model validation and detection of outliers in the measurement results.

The statistical tests revealed that observation to RF 8 contain error and was therefore regarded as outlier. The results for RF 8 were therefore rejected. Finally, it can be concluded that both the mathematical and stochastic models chosen were good.

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