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Comparison of Log-Pearson Type III and Gumbel Probability Distributions for Flood Frequency Analysis of Ikpoba River Catchment at Benin City

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Abstract

In the engineering design of hydraulic structures for flood alleviation and storm water management in or near a river, the magnitude of design flood is required. For the purpose of evaluating the flood discharge for various return periods (FFA) for a particular river, several methods are utilized depending on the availability of data and amount of discharge details required and where hydrometric measurements are limited as is the case of the catchment under study, estimates are made by more than one method and engineering judgment used in deciding design values.

In this study, the extreme value Type 1 (Gumbel) and Log-Pearson Type III probability distributions have been alternatively utilized to perform flood frequency analysis on the peak annual series discharge data of Ikpoba River at Benin City for the water years 1989 to 2000. The predicted design floods by alternative models for return periods of 2yrs, 5yrs, 10yrs, 25yrs, 50yrs, 100yrs, 200yrs and 1000yrs were obtained and compared. Our results indicate that the river peak flows can be satisfactorily modeled by any of the two methods of analysis and that at lower return periods of up to 5yrs, the Gumbel distribution predicts lower peak discharge values and for higher return periods of 10 yrs and above, the Log Pearson Type III distribution predicted lower discharge values with a percentage deviation ranging from – 0.62% to -12.78%.

It is however recommended that the design values obtained by the Gumbel distribution be adopted to assure safe design in view of the available limited hydrometric data utilized for the study.

Keywords: Flood frequency analysis, Log Pearson Type III, Gumbel distribution, Peak discharge, return period, Annual series.

1.0 Introduction

Hydrologic phenomena are characterized by great variability, randomness and uncertainty [16] hence precipitation, evaporation, stream flow and other hydrometric quantities of importance in water resources engineering are treated as random variables with associated measures of frequency that represent likelihood, percentage of time or probability. As random variables are quantities that depend on chance, the value or range of values can be predicted only with an associated probability and not with certainty. For example, the risks that the flow capacity of hydraulic structures will be exceeded, water supply systems will fail to meet demands and flood streams will endanger life and property are fundamental to water resources engineering hence frequency analysis methods are essential to hydrologic design and assessments. Hydrologic design of hydraulic structures is based on adopting acceptable levels of risk which are often specified in design criteria manuals developed by various engineering agencies. Thus, while bridges for major highways may be designed to pass a flood with an annual exceedence frequency of one percent (1%) without overtopping the roadway, bridges and culverts for streets with

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lower traffic volumes may be designed based on less stringent criteria. Perhaps a 2% or 4% exceedence frequency design flood. Design criteria for storm sewers, drainage ditches, detention basins and other components of storm water management systems are likewise based on specified exceedence values.

Often, streams and rivers naturally overflow their channels periodically and flood plains are a natural component of a stream system and problems often result because people tend to live and work in flood plains because rich alluvial soils deposited by centuries of periodic flooding results in prime agricultural lands being located in floodplains with cities being developed near rivers to facilitate water supply, waste water disposal, electric energy production and transportation.

In any local region, flooding may range from the inconvenience of street and yards being inundated fairly frequently to rare extreme flood events with recurrence intervals of many years which may cause loss of life and devastating damages making frequency analysis a key aspect of flood mitigation plans [16].

For many problems in water resources engineering, the frequency of occurrence of specified river flows or the length of time for which a particular river flows are expected to be exceeded are often required. Hence, flood frequency analysis entails the estimation of peak discharge which is likely to be equaled or exceeded on average once in a specified period of T years called the T-year event and the peak discharge , Q_T, is said to have a return period or recurrence interval of T years [14]

A frequency relationship or probability distribution function represents the likelihood of occurrence of values of a random variable and frequency relationships are developed based on observed and or simulated data. Thus, a distribution function provides a probabilistic model of the phenomena represented by a particular random variable.

Numerous probability distribution functions have been used to model phenomena characterized by significant variability not deterministically explained by physical principles. However, the probability distribution functions commonly used with observed or computed data to develop relationships between random variables and annual exceedence probability P, which is the probability that a specified magnitude will be exceeded at least once in a single year include (i) Normal distribution (ii) Log-Normal distribution (iii) Log Pearson type III distribution and (iv) Extreme value Type I (EVI) or Gumbel distribution. For advising on a design flood for an engineering scheme, it is needful to make estimates of flood magnitude for selection of return periods by more than one method and compare results. In this study, two probability distribution functions namely: (i) Extreme value type 1 (EVI) or Gumbel probability distribution and (ii) Log-Pearson Type III distribution have been used to perform flood frequency analysis (FFA) on maximum annual instantaneous discharge of Ikpoba river at Benin City based on hydrometric measurements carried out by Benin Owena River Basin Development Authority (BORBDA) for the water years 1989 to 2000. The specific objectives of the study include to:

(i) Fit Extreme value type 1 (EVI) or Gumbel probability distribution to the discharge data.
(ii) Fit Log-Pearson Type III probability distribution to the discharge data
(iii) Predict the design floods for the following return periods or estimate i.e. (T = 2, 5, 10, 25, 50, 100, 200 and 1000 yrs) using alternative models
(iv) Compare the predicted design floods obtained by alternative models for the same return periods.
(v) Based on (i), (ii), (iii) and (iv), make necessary recommendations for application of the FFA methods to the Ikpoba river catchment.

The Study Area/Catchment

The study area for this flood frequency study is the Ikpoba River catchment Edo State situated within the Western Littoral hydrological area (HA - 6) of Nigeria [1] which is one of the eight hydrological area into which Nigeria is subdivided. The gauge station at which the hydrometric measurements were made is located along Ikpoba River at Benin City some 160 km East of Lagos. Benin City is located at about 117km away from Benin River which discharges into the Gulf of Guinea.

Important parameters pertaining to the hydrological gauging station are given in Table 1.1

Table 1.1: Ikpoba River Hydrological gauging station parameters

Location of Gauging Station	State	Basin	Latitude	Longitude	Drainage Area (km^2)
Ikpoba River at Benin City	Edo	Ossiomo	6^0 21'N	5^0 39'E	922

Source: BORBDA (2005)

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2.0 Theory Of Extreme Value Type 1 (Gumbel) And Log-Pearson Type III Probability Distribution

2.1 Basic Concepts of Flood Probabilities

If F(x) is the probability of Q < x

Then

$$P(x) = 1 - F(x) \tag{2.1}$$

where P(x) called probability of exceedence is the probability of an annual maximum equaling or exceeding X in a given year since it is the relative proportion of the total number of annual maxima that have equaled or exceed X. If X is equaled or exceeded r times in N years (N – being large), according to[14] and [15] then:

$$P(X) = \frac{r}{N} \tag{2.2}$$

The return period, T (x) is given by

$$T(x) = \frac{N}{r} \tag{2.3}$$

Thus

$$P(x) = \frac{1}{T(x)} \tag{2.4}$$

$$T(x) = \frac{1}{P_{(x)}} = \frac{1}{1-F(x)} \tag{2.5}$$

and

$$F(x) = \frac{T_{(x)} - 1}{T_{(x)}} \tag{2.6}$$

Hence if T(x) = 100 years, P(x) = 0.01 and F(x) = 0.99

When N is not large, empirical relative frequency relations (plotting position formulae) provides reasonably accurate estimates of probabilities for frequent events well within the range covered by observations [16]. Using plotting position formula, the probability of exceedence P(x) is calculated for each value of X according to the formula utilized. According to [14], of the several formulae in use, the best is that due to Gringorten while the most widely used is that due to Weibull though both give similar results.

The Gringorten formula is given by

$$P(x) = \frac{r - 0.44}{N + 0.12} \tag{2.7}$$

Where r is the rank of X and N is the total number of data values. The Weibull formula is given by:

$$P(x) = \frac{r}{N+1} \tag{2.8}$$

However for a full range of values including events with recurrence intervals greater than the number of years of observation, analytical probability distribution functions such as Log Pearson Type III, Gumbel, Normal and Log Normal may be used with observed or computed data to develop relations between a random variable and annual exceedence probability, P, which is the probability that a specified magnitude will be exceeded at least once in a year [14] and [16].

2.2 The Extreme Value Type I (Gumbel) Probability Distribution

The Extreme Value Type I (EV1) or Gumbel probability distribution is based on the theory of extremes. [9] considered that annual flood peaks are extreme values of flood in each of the annual series of recorded floods and hence suggested that extreme value distribution was appropriate for flood analysis since the annual flood could be assumed to be the largest of a sample of 365 possible values each year. The equation of the Gumbel extreme value Type 1 distribution is given in Shaw (1988) as:

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$$F(x) = \exp\left[-e^{-b(x-a)}\right] \quad (2.9)$$

Where F(x) is the probability of an annual maximum Q < x as defined previously in section 2.1, while a and b are two parameters related to the moments of the population of Q values. Defining the first moment (the mean) by μ_Q and the second moment (the variance) by σ_Q^2 the parameters a and b are given by the following expression

$$a = \mu_Q - \frac{\gamma}{b},$$

$$\gamma = 0.5772 \quad (2.10)$$

$$b = \frac{\pi}{\sigma_Q\sqrt{6}} \quad (2.11)$$

In equations (2.10) and (2.11), μ_Q and σ_Q^2 pertain to the whole statistical population of floods at the station. With a finite sample, they can only be estimated from the moment of the data sample [13] and [16] Thus,

$$\hat{\mu}_Q = \bar{Q} = \frac{1}{N}\sum_{i=1}^{n} Q_i \quad (2.12)$$

$$\hat{\sigma}_Q^2 = S_Q^2 = \frac{1}{(N-1)}\sum_{i=1}^{n}\left(Qi - \bar{Q}\right)^2 \quad (2.13)$$

From equation (2.9), F(x) can be found for a specified annual maximum X. once F(x) is known, P(x) = 1 – F(x) is known and therefore the return period $T(x) = \frac{1}{P(x)}$ can be determined.

Equating equations (2.6) and (2.9) for F(x), we have;

$$\exp(-e^{-b(x-a)} = \frac{T(x)-1}{T(x)} \quad (2.14a)$$

And taking logarithms of both sides, we obtain;

$$-b(x-a) = \ln\left[-\ln\frac{T(x)-1}{T(x)}\right] \quad (2.14b)$$

And rearranging, we obtain;

$$X = a - \frac{1}{b}\ln \ \ln\left[\frac{T(x)}{T(x)-1}\right] \quad (2.14c)$$

Substituting for the parameter a and b with the sample mean $\bar{Q}$ and standard deviation S_Q as estimate of population values μ_Q and σ_Q then estimates of X may be found using the following equations [8] and [16]:

$$\hat{X} = \bar{Q} + K(T)S_Q \quad (2.15)$$

$$K(T) = -\frac{\sqrt{6}}{\pi}\left(\gamma + \ln \ \ln\left[\frac{T(x)}{T(x)-1}\right]\right) \quad (2.16)$$

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where γ (euler's constant) = 0.5772

K(T) is called frequency factor. Although it is dependent on the parameters of the probability distribution, K(T) in equation (2.16) a function only the return period T, is specifically for the Gumbel Type 1 distribution and is given in Table 2.1

Table2.1:The T–K (T) relationship for Gumbel distribution(Shaw, 1988)

T	K(T)	T	K(T)	T	K(T)
1	$-\infty$	15	1.64	100	3.14
2	-0.16	20	1.86	200	3.68
3	0.25	25	2.04	400	4.08
4	0.52	30	2.20	600	4.52
5	0.72	40	2.40	800	4.76
6	0.88	50	2.61	1000	4.94
7	1.01	60	2.73		
8	1.12	70	2.88		
9	1.21	80	2.94		
10	1.30	90	3.07		

Thus if an estimate of the annual maximum discharge for a return period of 100 years is required, then

T(x) = 100 yrs, K (T) = 3.14 and

$$Q_{100} = \overline{Q} + 3.14S_Q$$

With the mean and standard deviation of a sample of annual maximum flows and assuming the Gumbel distribution for the data, estimate of peak flow for any required return period may be obtained from the equation (2.17) as:

$$Q_T = \overline{Q} + K(T)S_Q \qquad (2.17)$$

Using the appropriate K (T) values obtained from Table 2.1.

2.3 Log Pearson Type III Distribution

The Log Pearson type III distribution is one of the numerous probability distribution functions which is used to model phenomena characterized by significant variability not deterministically explained by physical principles. The probability density function (PDF) for the distribution is given in [10] as:

$$F(x) = \frac{\lambda^{\beta}\left(X - X_0\right)^{\beta}\beta^{-1}e^{-x(x-x_0)}}{\Gamma(\beta)} x \geq x_0 \qquad (2.18)$$

Where $\overline{X}$ = mean, $\Gamma =$ (gamma function)

y = reduced variate

$$y = \frac{x - x_0}{\beta} \qquad (2.19)$$

$$\overline{X} = X_0 + \beta\gamma \qquad (2.20)$$

$$\beta = \frac{\sqrt{\nu}}{\sqrt{\gamma}} \qquad (2.21)$$

$$\gamma = \left(\frac{2}{G}\right)^2 \qquad (2.22)$$

Where X = mean, β = standard deviation

V= variance, G = Skewness

Because annual flood series are rarely normally distribution [15], a histogram of such series is usually Skewed, that is, the mean values does not coincide with the mode (the value of variate with largest frequency), Pearson

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devised a measure of Skewness $\left(\frac{\text{mean - mode}}{\sigma}\right)$ and developed a family of curves to describe degrees of Skewness. The coefficient of Skew (G) is defined mathematically in [13] as:

$$G = \frac{N\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2}{(N-1)(N-2)\sigma^3} \tag{2.23}$$

where σ = standard deviation

The Log Pearson type III distribution has 3 parameters that includes, Skew coefficient (equation 2.23), mean $\left(\overline{X}\right)$ and standard deviation σ, ([2], [3] and [16]) and is represented by the general equation;

$$X = \overline{X} + K\sigma \tag{2.24}$$

where K = frequency factor obtained from Tables

The model parameters; mean $\left(\overline{X}\right)$, standard deviation (σ) and the Skew coefficient (G) are computed from N observation using the following formulae [16].

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{2.25}$$

$$\sigma = \left[\frac{1}{(N-1)}\sum (x_i - \overline{x})^2\right]^{½} \tag{2.26}$$

$$G = \frac{N\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^3}{(N-1)(N-2)\sigma^3} \tag{2.27}$$

The Log Pearson Type III distribution of X is equivalent to applying the Log Pearson type III distribution to the transformed random variable log X and it is represented by the equation below ([11], [12] and [16])

$$\text{Logx} = \overline{\text{logx}} + K\sigma_{\text{logx}} \tag{2.28}$$

with $\overline{\text{logx}}, \sigma_{\log x}$ and G computed using the formulae

$$\text{logx} = \frac{\sum \text{logx}_i}{N} \tag{2.29}$$

$$\sigma_{\log x} = \left[\frac{\sum(\log x_i - \overline{\log x})^2}{(N-1)}\right]^{0.5} \tag{2.30}$$

$$G = \frac{N\sum\left(\text{logx}_i - \overline{\text{logx}}\right)^3}{(N-1)(N-2)\sigma_{\text{logx}}{}^3} \tag{2.31}$$

where N is the number of observation of X, the flood of some specified probability, $\overline{\log x}$ is the average of the log x discharge values.

3.0 Methodology

The daily discharge data of Ikpoba river at Benin City from 1989 to 2000 obtained from the records of hydrometric measurements carried out by Benin-Owena River Basin Development Authority (BORBDA) were obtained and subjected to flood frequency analysis (FFA) utilizing two different methods namely: Log-Pearson Type III and Extreme value Type 1 (Gumbel) probability distribution methods.

To satisfy the assumption of independence and identical distribution of data, the maximum of discharge which is the largest instantaneous peak flow occurring at anytime during the year were selected [6] in order to obtain annual

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series data and in order to ensure that annual peaks are independent of one another, water year rather than calendar year was utilized for the analysis [14]. Estimates of the recurrence interval (T) were obtained using the Weibull formula given in the literature e.g. [11], [15] and [16] as:

$$T = \frac{n+1}{m} \tag{3.1}$$

where n is the number of years of record and m is the rank.

In applying the extreme value type 1 (Gumbel) probability distribution method of analysis to the observed data, the following steps were followed [7] and [11]:

(i) The annual flood series data (X) were assembled

(ii) The mean $(\overline{Q})$ and standard deviation (σ) of the flood series were computed using equations 2.12 and 2.13 respectively.

(iii) Several return periods (T_j) i.e. T=2, 5, 10, 25, 50, 100, 200 and 1000 years and their corresponding exceedence probabilities (P) were selected, $P_j = \frac{1}{T_j}$

(iv) The frequency factor, K(T) for the selected return periods (T_j) were obtained from Tables 2.1

(v) Assuming Gumbel distribution for the observed data, the peak flow (Q_T) for the required return period (T) was estimated from the following equation [14];

$$Q_T = \overline{Q} + K(T)\,\sigma \tag{3.4}$$

(vi) Plots of the estimated peak discharge for various return periods, and reduced variates were made for the fitted data

(vii) In view of the short data series, confidence limits about the fitted straight line relationships between the annual maxima and the reduced variate were constructed [14]. The first step being the calculation of the standard error (SE) of estimate for a peak discharge (Q) in terms of return period. For the Gumbel probability distribution, expression for standard error is given by:

$$SE\ (Q_T) = \frac{\sigma}{\sqrt{N}}\left[1+1.14K(T)+1.10(K(T)^2)\right]^{1/2} \tag{3.5}$$

where N is the number of annual maxima in the sample. The upper and lower confidence limits were calculated for the selected return periods using estimated values of Q_T from equation 3.6 [14] given by;

$$Q_T \pm t_{\alpha,\nu}\, SE(Q_T) \tag{3.6}$$

where t_{α},ν are values of the t – distribution obtained from standard statistical tables with α the probability limit required and ν the degree of freedom.

In the application of the Log Pearson Type III probability distribution to the observed hydrometric data, the following steps suggested in [11] and [16] were followed:

(i) The annual flood series (X_i) were assembled

(ii) The base 10 logarithms of the annual flood series were calculated as $y_i = \log x_i$ and utilized to obtain mean, $\overline{y}(\overline{\log X})$, standard deviation, $\sigma_y(\sigma_{\log x})$ and Skew coefficient $C_{sy}(G)$

(iii) The mean $\overline{y}(\overline{\log x})$, The standard deviation $\sigma_y\ (\sigma \log_x)$ and Skew coefficient C_{sy} (G) of the logarithms y_i were calculated using equations (2.29),(2.30) and (2.31) respectively.

(iv) The logarithms of the flood discharge (log Q_i) for each of the chosen probability level P_j were calculated using the following frequency formular:

$\log Q_j = y + K_j S_y$ (3.7)

where K_j is the frequency factor, a function of the probability P_j and Skewness coefficient Csy. The frequency factor (K) for Pearson Type III distribution for ten probability levels in the range from 0.5 to 95% and Skewness coefficient in the range from -3.0 to 3.0 are provided in Table 4.6.The flood discharge Q_j for each probability level (P_i), return period (T_c) is obtained by taking antilogarithm of the log Q_j values.

(v) The flood discharge (Q_T) for associated with each probability level (P_j) or return period (T_j) are listed.

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(vi) The flood discharge (Q_T) associated with each probability level or return periods obtained by application of alternative probability distribution methods were compared.

4.0 PRESENTATION, ANALYSIS AND DISCUSSION OR RESULTS

The annual peak flow data for Ikpoba river by water year obtained from the analysis of daily discharge data of the river from 1989 to 2000 as measured by the Benin Owena River Basin Development Authority is presented in Table 4.1.

Table 4.1: Annual Peak Flow Data for Ikpoba River (1989-2000)

Water Year	Stream Flow, Annual max (m^3/s)
1989	43.89
1990	28.25
1991	55.00
1992	38.30
1993	38.80
1994	50.00
1995	52.10
1996	43.89
1997	43.89
1998	43.89
1999	65.40
2000	65.10

Source: BORBDA (2005)

The various statistical parameters were computed as outlined in the methodology. The results are presented in Table 4.2.

Table 4.2: Computation of the Statistical Parameters

Rank (m)	Water Year	Peak Discharge (m^3/s)	Q_p^2	Return Period (T)	P = 1/T	100P
		Q_P				
1	1999	65.40	4277.16	13.00	0.0769	7.69
2	2000	65.10	4238.01	6.50	0.1538	15.38
3	1991	55.00	30.25	4.30	0.2326	23.26
4	1995	52.10	2714.4	3.25	0.3077	30.77
5	1994	50.00	2500	2.60	0.3846	38.46
6	1989	43.89	1926.33	2.17	0.4608	46.08
7	1996	43.89	1926.33	2.17	0.4608	46.08
8	1997	43.89	1926.33	2.17	0.4608	46.08
9	1998	43.89	1926.33	2.17	0.4609	46.09
10	1993	38.80	1505.44	1.30	0.7692	76.92
11	1992	38.30	1466.89	1.18	0.8475	84.75
12	1990	28.25	798.06	1.08	0.9259	92.59
		$\sum = 568.51$ $\overline{Q} = 47.38$	$\sum = 28{,}230.3$ $\sigma = 10.84$			

As shown in the Table 4.2, the mean peak discharge Q_p during the period is equal to $47.38 m^3/s$ while the standard deviation (σ) is equal to $10.84 m^3/s$. The return period for the annual peak discharge were determined by the use of Weibull plotting position formula $T = \frac{n+1}{m}$

The results obtained by the application of extreme value Type 1 (Gumbel) distribution to the annual series discharge data of Ikpoba River are shown in Table 4.3.

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Table 4.3: Computation of Predicted discharge for selected return periods assuming Gumbel distribution

T(yrs) (1)	Reduced variate (y) (2)	K(T) from Table 2.1 (3)	Q_P (from Table 4.1) (4)	σ (From Table 4.2) (5)	K(T) σ (6)=(3)X(5)	$Q_T = Q_P$ + K(T) σ (7)=(4) +(6)
2	0.3667	-0.16	47.38	10.84	-1.7344	45.645
5	1.50	0.72	47.38	10.84	7.8048	55.1848
10	2.250	1.30	47.38	10.84	14.092	61.47
25	3.1985	2.04	47.38	10.84	22.1138	69.49
50	3.902	2.61	47.38	10.84	28.2924	75.67
100	4.60	3.14	47.38	10.84	34.0376	81.417
200	5.295	3.68	47.38	10.84	39.8912	87.27
1000	6.906	4.94	47.38	10.84	53.5496	100.92

Table 4,3 shows the predicted discharge values for selected T of 2, 5, 10, 25, 50, 100, 200 and 1000 years commonly used for the engineering design of hydraulic structures. The results indicate that for the stated return periods, the predicted peak discharge are 45.65m^3/s, 55.18m^3/s, 61.47m^3/s, 69.49m^3/s, 75.67m^3/s, 81.42m^3/s and 100.92m^3/s respectively. The plots of the predicted discharge against reduced variates (return periods) are shown in Figures 4.1. Confidence limits (95%) about the fitted straight line relationship between the predicted annual maxima and the reduced variate for the Gumbel probability fit is constructed

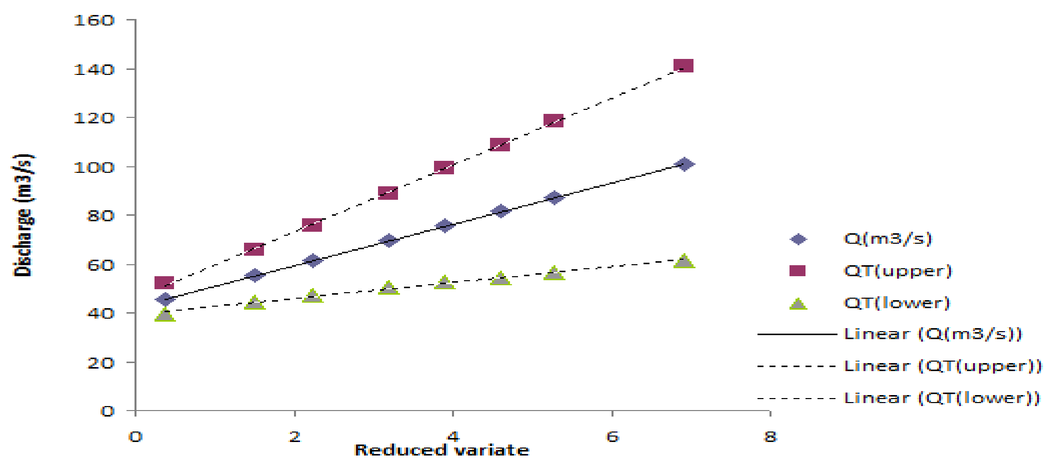

Figure 4.1: Plot of Predicted discharge against reduced variate for Gumbel model (95% confidence limits)

The calculations for the 95% confidence limits for the predicted data are set out in Table 4.4.The value of t-statistic is 2.2 for α = 100-95% = 5% and γ =12-1=11.

Table 4.4: Calculation of 95% confidence limits (Gumbel distribution)

($t_{5,11}$ = 2.2) from t – tables

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T(yrs)	2	5	10	25	50	100	200	1000
Reduced variate(y)	0.3667	1.500	2.250	3.1985	3.902	4.60	5.295	6.906
K(T)	-0.16	0.72	1.3	2.04	2.61	3.14	3.68	4.94
Q_T(m3/s)	45.65	55.18	61.47	69.49	75.67	81.42	87.27	100.92
SE(Q_T)(m3/s)	2.877	4.838	6.5199	8.7973	10.597	12.29	14.026	18.10
$t_{5,11}$ SE(Q_T) m3/s 2.2 x SE(Q_T)	6.33	10.64	14.34	19.35	23.31	27.03	30.86	39.82
Upper Q_T (m3/s)	51.96	65.82	75.81	88.84	98.98	108.45	118.13	140.74
Lower Q_T (m3/s)	39.32	44.54	47.13	50.14	52.36	54.39	56.41	61.1

The curves of the 95% confidence limits are also plotted on Fig. 4.1 for the range of selected T values.
The computations of the statistical parameters for the Log-Pearson Type III probability distribution were carried out using the procedure outlined in section 3.0 (methodology). The results of the computations are shown in Table 4.5.

Table 4.5: Computation of Statistical parameters for Log-Pearson Type III distribution

Rank (m)	Water year	Flood flow (m3/s)	y=logx	$y-\bar{y}$	$(y-\bar{y})^2$	$(y-\bar{y})^3$	T=(n+1)/m	P=100/T
1	1999	65.40	1.82	0.155	0.02419	0.00375	13	7.69
2	2000	65.10	1.81	0.1535	0.0235	0.0036	6.5	15.38
3	1991	55	1.74	0.0803	0.00645	0.000512	4.33	23.08
4	1995	52	1.72	0.0560	0.00313	0.00018	3.25	30.77
5	1994	50	1.70	0.0389	0.00151	0.0000587	2.60	38.46
6	1989	43.89	1.64	-0.0176	0.000310	-0.0000113	2.17	46.15
7	1996	43.89	1.64	-0.0176	0.000310	-0.0000113	1.86	53.85
8	1997	43.89	1.64	-0.0176	0.000310	-0.0000113	1.63	61.54
9	1998	43.89	1.64	-0.0176	0.000310	-0.0000113	1.44	69.23
10	1993	38.80	1.59	-0.0711	0.00506	-0.000438	1.30	76.92
11	1992	38.30	1.58	-0.0768	0.005898	-0.00054	1.18	84.62
12	1990	28.25	1.45	-0.2089	0.0436	-0.000977	1.08	92.31
mean		47.38	1.66					

Standard deviation (σ) =0.102, Skewness coefficient (G) = -0.3560, Mean ($\bar{x}$) =47.38, Mean ($\bar{y}$) = 1.66

The results indicate that the mean $\left(\bar{x}\right)$ of the annual peak discharge for the period is 47.38m^3/s. The mean $\left(\bar{y}\right)$, standard deviation $\sigma_{\log x}$, and Skewness coefficient (G) of the logarithms of the annual peak discharge values for the period were obtained as 1.66m^3/s, 0.102m^3/s and -0.3560 respectively.

Table 4.6: Frequency Factors k for Pearson III Distribution

Return Period T(y)

	1.05	1.11	1.25	2	5	10	25	50	100	200
	Probability of exceedence P (percent)									
C_s	95	90	80	50	20	10	4	2	1	0.5
3.0	-0.665	-0.660	-0.636	-0.396	0.420	1.180	2.278	3.152	4.051	4.970
2.8	-0.711	-0.702	-0.666	-0.384	0.460	1.210	2.275	3.114	3.973	4.847
2.6	-0.762	-0.747	-0.696	-0.368	0.499	1.238	2.267	3.071	3.889	4.718
2.4	-0.819	-0.795	-0.725	-0.351	0.537	1.262	2.256	3.023	3.800	4.584
2.2	-0.882	-0.844	-0.752	-0.330	0.574	1.284	2.40	2.970	3.075	4.444
2.0	-0.949	-0.895	-0.777	-0.307	0.609	1.302	2.219	2.912	3.605	4.398
1.8	-1.020	-0.945	-0.799	-0.282	0.643	1.318	2.193	2.848	3.499	4.417
1.6	-1.093	-0.994	-0.817	-0.254	0.675	1.329	2.163	2.780	3.388	3.990
1.4	-1.168	-1.041	-0.832	-0.225	0.705	1.337	2.128	2.706	3.271	3.828
1.2	-1.243	-1.086	-0.844	-0.195	0.732	1.340	2.087	2.626	3.149	3.661
1.0	-1.317	-1.128	-0.852	-0.164	0.758	1.340	2.043	2.542	3.022	3.489

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0.8	-1.388	-1.166	-0.856	-0.132	0.780	1.336	1.993	2.453	2.891	3.312
0.6	-1.458	-1.200	-0.857	-0.099	0.800	1.328	1.939	2.359	2.755	3.132
0.4	-1.524	-1.231	-0.855	-0.66	0.816	1.317	1.880	2.261	2.615	2.949
0.2	-1.586	-1.258	-0.850	-0.033	0.830	1.301	1.818	2.159	2.472	2.763
0.0	-1.645	-1.282	-0.842	0.000	0.842	1.282	1.751	2.054	2.326	2.576
-0.2	-1.700	-1.301	-0.830	0.033	0.850	1.258	1.680	1.945	2.178	2.388
-0.4	-1.750	-1.317	-0.816	0.066	0.855	1.231	1.606	1.834	2.029	2.201
-0.6	-1.797	-1.328	-0.800	0.099	0.857	1.200	1.528	1.720	1.880	2.016
-0.8	-1.839	-1.336	-0.780	0.132	0.856	1.166	1.448	1.606	1.733	1.837
-1.0	-1.877	-1.340	-0.758	0.164	0.852	1.128	1.366	1.492	1.588	1.664
-1.2	-1.910	-1.340	-0.732	0.195	0.844	1.086	1.282	1.379	1.449	1.501
-1.4	-1.938	-1.337	-0.705	0.225	0.832	1.041	1.198	1.270	1.318	1.351
-1.6	-1.962	-1.329	-0.675	0.254	0.817	0.994	1.116	1.166	1.197	1.216
-1.8	-1.981	-1.318	0.643	0.282	0.799	0.945	1.305	1.069	1.087	1.097
-2.0	-1.996	-1.302	-0.609	0.307	0.777	0.895	0.959	0.980	0.990	0.995
-2.2	-2.006	-1.284	-0.574	0.330	0.752	0.844	0.888	0.900	0.905	0.907
-2.4	-2.011	-1.262	-0.537	0.351	0.725	0.795	0.823	0.830	0.832	0.833
-2.6	-2.013	-1.238	0.499	0.368	0.696	0.747	0.764	0.768	0.769	0.769
-2.8	-2.010	-1.210	-0.460	0.384	0.666	0.702	0.712	0.714	0.714	0.714
-3.0	-2.003	1.180	-0.420	0.383	0.836	0.660	0.666	0.666	0.667	0.667

The result of the application of the Log Pearson type III probability distribution model to the observed data for the specified return periods is summarized in Table 4.7.

Table 4.7 Application of Log- Pearson Type III to Observed data

Return Period(T) (years)	Probability (P %)	Frequency factor(K) for G= -0.356	$y_i = \overline{y} + KSy$	x_i = Q (m3/s)
2	50	0.059	1.666	46.34
5	20	0.854	1.747	55.84
10	10	1.237	1.786	61.09
25	4	1.622	1.8254	66.89
50	2	1.859	1.8496	70.73
100	1	2.062	1.870	74.18
200	0.5	2.242	1.888	77.38
1000	0.1	-	-	89.63

Standard deviation (σ) =0.102, Skewness coefficient (G) = -0.3560, Mean ($\overline{y}$) = 1.66

The results indicate that for the specified return periods of 2yrs, 5yrs, 10yrs, 25yrs, 50yrs, 100yrs and 200yrs the predicted peak discharges are 46.34m^3/s, 55.84m^3/s, 61.09m_3/s, 66.89m^3/s, 70.73m^3/s, 74.18m^3/s and 77.38m^3/s respectively. To obtain the value for the predicted peak discharge for return period of 1000yrs for which K values were not provided in Table4.6 the equation, **y= 6.5506ln(x) +44.44.386(R^2=0.9781)** obtained by application of Log Pearson Type III to the data was utilized to obtain a predicted peak discharge of 89.63m^3/s

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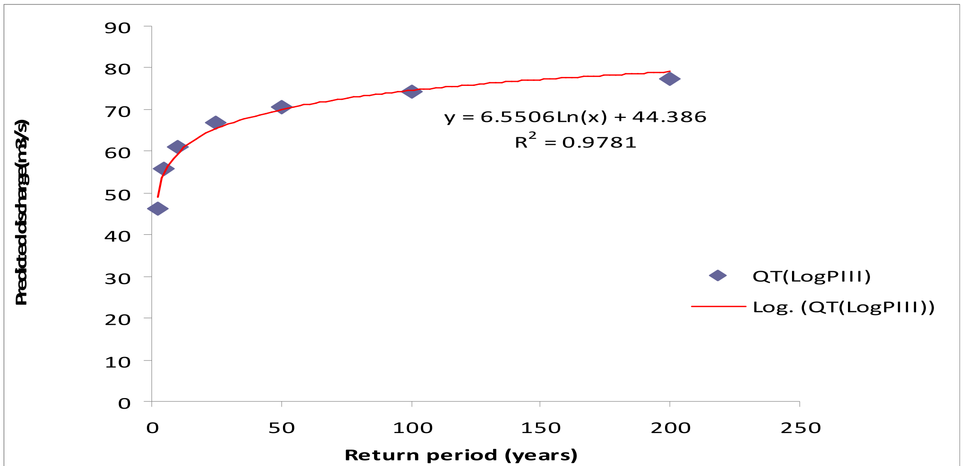

Figure 4.2: Plot of predicted discharge vs return period (Log-Pearson III)

The predicted peak discharge (Q_T) obtained for the specified return periods by application of the two different probability distribution methods to the annual peak discharge of Ikpoba River at Benin City for the period 1989 to 2000 are presented in Table 4.7

Table 4.7: Comparison of Predicted Discharge Values for different return periods using Log Pearson and Gumbel Probability distributions

Return Period(T) (yrs)	Probability (P %)	Predicted discharge (m3/s) (**Log Pearson III**)	Predicted discharge (m3/s) **(Gumbel**)	% Deviation
2	50	46.34	45.65	1.48
5	20	55.84	55.18	1.18
10	10	61.09	61.47	-0.62
25	4	66.89	69.49	-3.88
50	2	70.73	75.67	-6.98
100	1	74.18	81.42	-9.76
200	0.5	77.38	87.27	-12.78
1000	0.1	89.63	100.92	-12.59

The table shows that for low return periods less than 5 years, the Log Pearson Type III distribution predicted discharge values less than that predicted by the Gumbel distribution. While for higher return periods (10 yrs and above) the values predicted by the Gumbel distribution were lower than that predicted by the log-Pearson Type III. The percentage deviation of the predicted discharge (Q_T) values for various return periods obtained by utilizing Gumbel distribution from that obtained by Log Pearson distribution ranges from -12.78% to 1.48%. Thus, for higher return periods the Gumbel probability distribution predicts higher values for the same return period. Though, the Log-Pearson Type III distribution is the recommended method adopted for modeling peak flows by federal water agencies in the United States, the Gumbel distribution have been investigated and shows good and satisfactory promise for application to extreme values produced by flood peak discharges.

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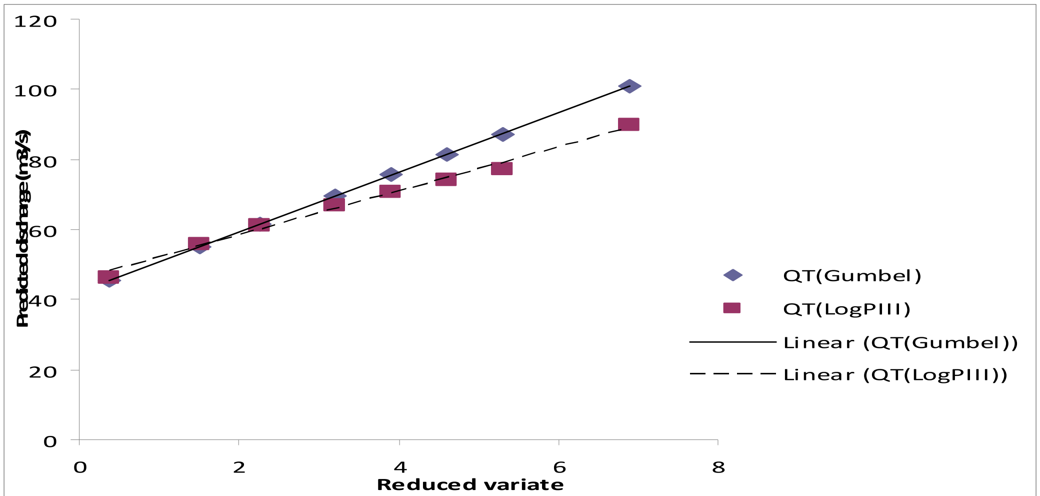

Figure 4.3 : Comparison of Predicted discharge by Gumbel and Log - Pearson Type III distribution models

The larger the recurrence interval (T), the less likely it is for a hydrologic event to be equaled or exceeded in a given year, hence the more critical a project is in terms of potential loss of life, economical damage or adverse environmental effects, the larger is the value of T adopted in design calculations and therefore for larger return periods ($\geq$ 10 years). The Gumbel distribution method which predicted higher discharge values will provide safer design discharge values for engineering design of hydraulic structures in the river or catchment especially as the period of measurements (12 years) is short.
While a dam may be designed to accommodate a 100 year flood in order to reduce the chance of failure or breach of the dam and ensure the protection of human lives and property, it may be expedient to design a local storm drain to handle only the flow from 2 years storm.

5.0 Conclusions

From the flood frequency study of Ikpoba river catchment carried out the following conclusions are made:

(i) That flood frequency studies can be used and serves as guide for determining the capacity of hydraulic structure.

(ii) That Extreme value Type I (Gumbel) and Log-Pearson Type III distribution) can be used satisfactorily to

(iii) alternately model peak flows of Ikpoba River for various return periods. But for return periods up to 5 years, the Gumbel probability distribution gives lower predicted discharge values and for return periods equal to or greater than 10 years, the Gumbel probability distribution predicted higher values.

Arising from the findings in this study it is recommended that the Gumbel distribution be utilized to model peak flood flows in the Ikpoba river catchment considering the limited hydrometric measurements used for the study. In evaluating the frequency of a given flood magnitude for a particular river availability of data and the amount of discharge details required are important considerations .

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