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Analytical Solutions To Describe Juxtaposed Sands

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Abstract

Mathematical (linear diffusion) equations are presented for two pseudo-reservoir regions intersected by fault that describe the effects of partial communicating fault on pressure transient behaviour for each fault block. Green's and source function technique solve these equations. A two-well system is considered for the reservoir, on one block is the active well, which is producing at constant rate and the second well, the observation well on the other side of the fault, is shut in at all times. An analytical solution is presented for each block, and parametric group that uniquely determines specific transmissibility ratio of the fault to reservoir is identified.

They are initially developed for well testing with measured sand face flow rate, but can be extended, using convolution integral that can be deconvolved by Laplace transformation, to correct for storage capacity of the well bore and near well bore complexities. These solutions can improve design and analysis of interference testing. Type curves are presented to characterize flow regime as predicted by Boudet et al. from which reservoir parameter can be estimated.

Symbols

b = distance of active well to the fault
ε = Ratio of porosity compressibility product of region 1 to region 2
α_L= Specific transmissibility ratio with respect respect to the to the interwell distance distance plane
α_A = Specific transmissibility ratio with respect to the distance between the active well and the fault plane
$\lambda = M\beta$
k_f = fault permeability
l_f = effective fault of the fault zone. fault
ΔP = pressure drawdown
t = time
ϕ = Porosity
s = laplace variable
y = distance from active well parallel to the fault the fault.
$G = G(m, m', \tau)$
EI (x) = exponential integral solution, - $\text{EI}(-x) = \int_x^{\infty} \frac{e^{-U}}{u}$

c_t = total compressibility, psi^{-1} (Pa^{-1})
M= Mobility ratio
α_A = Specific transmissibility ratio with between the active well and the fault
η = Coefficient of diffusion
β = Thickness ratio
K= formation permeability
μ= Fluid viscosity
L = interwell distance perpendicular to the
P_D = dimensionless pressure
t_D = dimensionless time
ω = Fourier transforms variable
x = distance perpendicular to the fault
V_x = volume leakage rate perpendicular to

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$$erfc\left(x\right)=\frac{2}{\sqrt{\pi}}\int_{x}^{\infty}e^{-u^{2}}du$$; Complementary error function

Subscript

A = active well

i = Initial

f = fault

L = inter-wells distance

D = dimensionless

x = linear x-axis

b = distance from the active well to the fault

1.0 Introduction

Fractures in rocks are classified as joints or faults: joints are those fractures that have merely opened, without appreciable offset of the rock along the fracture; faults are those showing definite offsets. Many faults have displacement of several thousands of meters [1]. Yaxley [2] described the nature of this system and developed model of the pressure equations: A fault is an anomaly, which could cause the low permeability of the cap rock to be interrupted in a local area of the reservoir. Fault in hydrocarbon structure may be either sealing or non-sealing. A sealing fault separates permeable sand from non-permeable shale; it will impede lateral fluid flow and may actually form part of fluid trapping mechanism for hydrocarbon accumulation. On the contrary, a non-sealing fault bisects two permeable strata, which could be different strata or a homogeneous reservoir bisect into two regions by the fault. Then it is physically obvious that sand to shale contact on each side of the fault can effectively act as seal that prevent fluid flow across the fault, and sand-to-sand contact at the fault will transmit fluid readily.

A question frequently arises in the developmental planning of oil and gas fields is to what extent a fault that has been identified by seismic and geologic studies will act as fluid barrier; the juxtaposed sands properties. However, it is normal to be apprehensive over the presence of a fault for they are potential source of fluid leakage should the throw, or, vertical movement exceed the thickness of the continuous cap rock as shown in fig. 1. This is important because it has major impact on the numbers of well required to exploit a discovered field.

It should also be noted that, while the throw of a sealing fault is such that a permeable stratum on one side of the fault completely juxtaposed against impermeable stratum on the other side, a non sealing fault usually has insufficient throw to cause complete separation of permeable strata on opposite side of the fault. A sealing fault will completely prevent the flow of fluid laterally, while a non-sealing fault on the other hand will always allow appreciable amount of fluid flow across it. Because of various mechanical processes, such as grain crushing, bed deformation, and clay smearing, however, the transmissibility of the fault zone may be much lower than the transmissibility of the adjacent strata.

In-situ knowledge of juxtaposed sands and the transmissibility of the fault can be used as hard data in simulation model, eliminating the need for fault transmissibility as history matching parameter.

A sealing fault is usually generated when the throw of the fault plane is such that a permeable stratum on one side of the fault plane is completely juxtaposed against an impermeable stratum on the other side of the fault. It is also generated as a result of the precipitation and crystallization of mineral within the fault plane before oil migrates into the reservoir.

Based on this analysis fluid flow may occur across the fault plane laterally from one stratum to another; if the width of the fault zone is small compared with the distance between the fault plane and the producing well. The idealized nature of flow of fluid from one medium across the fault to the producing region of the reservoir may thus be described as one in which the observation region feeds it fluid across the fault into the active well region and resistance to flow is relatively determined. For this idealised approach it is frequently met that the fault width is less than its distance to the active region.

Green's function offers analytical convenience to obtain solution to the equations derived for the juxtaposed semi-infinite regions. [3], [4] and [5] have demonstrated extensively on the effectiveness of the use of Green's function for solving problem relating fluid flow through porous medium. These papers reduced the problem into seeking the appropriate source function from which the solution to the diffusivity equation can be written directly, albeit in an integral form. [5] extended this method to solve boundary value problems by seeking the Green's function, which is the summation of fundamental Green's function that is singular at the source at the initial time, zero everywhere else, and the function that is solution to the diffusion equation at point away from the boundary, and is zero at initial time(complementary function). This paper presented how kernel function was obtained for a reservoir with plane barrier using Liouville-Neuman-type series expansion – an approximation technique. In the present paper, we applied the classical integral transforms in seeking these two functions. [6] employed Green's function technique for a situation of pressure field in a well near fracture with flow occurring along the fault plane in

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addition. They obtained solution using the method of superimposition of complex linear diffusion boundary value problem by taking the fault plane as additional source in the reservoir and couple the flow by descretisation of the reservoir and fault flow solution, summing up all the resistance to flow. This paper presented analytical solution to the flow of fluid across the fault plane based on problem posed in reference 2. The reservoir properties are different at the two adjacent blocks so that juxtaposed formations can be identified. In general, the storage capacity of the well bore and near well bore damage affects transient behaviour of a well. During the analysis of pressure-time data, each of these and its duration must be recognised for the application of the semi log and type curve techniques to determine flow capacity (kh) and damage skin. Although this paper prepared analytical solution for middle time period, but it is extended for the early time data

Theory

Statement of Problem And Assumptions

The problem being considered is the pressure transient behaviour resulting from constant rate production from a well in reservoir(s) that contains a linear vertical fault. Fig.1 shows schematic of a typical faulting system in the actual juxtaposed reservoir(s). The idealised model is shown in Fig.2

The fault plane is of thickness l_f and permeability k_f. It is located a distance b from the active well which acts as the source of strength q_w. Reservoir properties on the active well region are considered to be different from the observation region. The assumptions made on the reservoirs are as follows:

1) Two porous media of different properties are juxtaposed at the fault plane. Each reservoir is isotropic with respect to Permeability, homogeneous with respect to their rock properties, and both flowing a slightly compressible fluid of constant viscosity.
2) Each reservoir is initially at same constant pressure, P_i.
3) The physical properties of the fluid remain constant at all pressure.
4) Reservoir pressure remains constant and equal to the initial pressure, as distance from the fault is infinitely large.
5) The well could be approximated by infinite line source assumption.
6) The semi-permeable barrier is infinitely long and has negligible capacity.
7) The fluid leakage rate through the semi-permeable barrier is proportional to the instantaneous pressure different across the fault.
8) The well fully penetrated the whole reservoir thickness that is; sand-face pressure is independent of depth.
9) The fault width is small compared to its distant to the producing well.

Statements 1-5 are generally the basic assumption for transient problems, while statements 6 and 7 allow the partially communicating fault to be approximated by vertical plane. Statement 7 expresses that the resistant effect of the fault, then the leakage rate per unit time per unit length of the fault can be expressed as

$$V_x = \frac{k_f h_f}{l_f \mu_f}\left(p_2 - p_1\right) \ \ while \ \ V_x = -\frac{k_2 h_2}{\mu}\frac{dp_2}{dx}$$

Statement 8 states that none of the constants of the system or physical conditions vary with depth, so that the problem becomes 2 dimensional. Then the flow in horizontal section of each region bisected by the fault needs be considered and then couple at the fault line.

On both sides of the fault the pressure obeys diffusivity equation, and for the present situation it is best expressed in Cartesian coordinates since the flow across the fault is linear. As shown in Fig.2 the semi barrier lies along the y-coordinate. The partial differential equations describing flow of viscous fluid in this heterogeneous and isotropic system are described as

$$\frac{\partial^2 \Delta p_1}{\partial x^2} + \frac{\partial^2 \Delta p_1}{\partial y^2} = \frac{1}{\eta_1}\frac{\partial \Delta p_1}{\partial t}\Big|_{\infty < x \le -b} \tag{1}$$

$$\frac{\partial^2 \Delta p_2}{\partial \ x^2} + \frac{\partial^2 \Delta p_2}{\partial y^2} = \frac{1}{\eta_2}\frac{\partial \Delta p_2}{\partial t}\Big|_{-b < x \le -\infty} \tag{2}$$

With the initial and boundary conditions prescribed below as

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$$\Delta p_n(x,y,t\to 0)=0\Big|_{n=1,2} \qquad (3)$$

$$\Delta p_n(x\to\infty,y,t)=0\Big|_{n=1,2} \qquad (4)$$

$$\Delta p\,(x\to\infty)=0_{n=1,2}. \qquad (5)$$

$$\left(\frac{k_1h_1}{\mu}\right)\frac{d\Delta p_1}{dx}=\left(\frac{k_2h_2}{\mu}\right)\frac{d\Delta p_2}{dx}\Big|_{x=-b}. \qquad (6)$$

$$\left(\frac{k_2h_2}{\mu}\right)\frac{d\Delta p_2}{dx}=\left(\frac{k_fh_f}{\mu}\right)(p_1-p_2)\Big|_{x=-b}. \qquad (7)$$

For convenience the following dimensionless variables are defined.

$$p_{Dn}=2\pi k_1h_1/q\mu(\Delta p_n)\Big|_{n=1,2}. \qquad (8)$$

$$t_{Dl}=\frac{\eta t}{(-b-x)^2}. \qquad (9)$$

Note $t_{DL}=f(t,x)$ and putting –b-x=L. L is the distance between active well and a point in the observation region

$$X_D=\frac{x}{L}. \qquad (10)$$

$$b_D=\frac{b}{L}. \qquad (11)$$

$$\alpha_L=\left(\left(\frac{k_fh_f}{\mu l_f}\right)/\left(\frac{k_2h_2}{\mu L}\right)\right). \qquad (12a)$$

$$l_{fD}=\frac{l_f}{L}. \qquad (12b)$$

$$\beta=\frac{h_1}{h_2}. \qquad (12c)$$

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$$M = \frac{\frac{k_1}{\mu}}{\frac{k_2}{\mu}}. \qquad (12d)$$

Method of Solution

We are now in position to give logical and inductive procedure in establishing relationships between pressure and time in environment of parameters controlling flow through porous materials.
The governing equations describing flow in the two regions, in dimensionless form are defined below:

$$\frac{\partial^2 p_D}{\partial x_D{}^2} + \frac{\partial^2 p_D}{\partial y_D{}^2} = \frac{\partial p_D}{\partial t_D}\Big|_{-\infty < x_D < \infty} \qquad (13)$$

$$p_D = 2\pi \int_0^{t_{DL}} G(p, m, \tau) d\tau. \qquad (14)$$

$$\frac{d^2 p_{D1}}{dx_D{}^2} + \frac{\partial^2 p_{D1}}{\partial y_D{}^2} = \frac{\partial p}{\partial t_{DL}}\Big|_{-b_D \le x_D < \infty}. \qquad (15)$$

$$\frac{\partial^2 p_{D2}}{dx_D{}^2} + \frac{\partial^2 p_{D2}}{\partial y_D{}^2} = \frac{\partial p_{D2}}{\partial t_D}\Big|_{-b_D \le X_D < -\infty}. \qquad (16)$$

$$p_n(x_D, y_D, t_{DL} \to 0) = 0\Big|_{n=1,2}. \qquad (17)$$

$$p_n(x_D \to \infty, y_D, t_{DL}) = 0\Big|_{n=1,2}. \qquad (18)$$

$$p_n(x_D, y_D \to \infty, t_{DL}) = 0\Big|_{n=1,2}. \qquad (19)$$

$$\beta M \frac{dp_{D1}}{dx_D} = \frac{dp_{D2}}{dx_D}\Big|_{x_D = -b_D}. \qquad (20)$$

$$\frac{dp_{D2}}{dx_D} = \alpha_L (p_{D1} - p_{D2})\Big|_{x_D = -b_D}. \qquad (21)$$

The Green's function for the Equations (13) & (15) are easily found using Neuman product method [3, 4], which says that the Green's function for two-, or three-dimension variables is equal to the product of the corresponding one dimensions. Applying this we seek the solution to the one dimensional diffusivity equation in x_D that is infinite at x_D =0 and is zero at $(x_D \neq 0)$, i.e. we request for the solution of the differential equation

$$\frac{\partial^2 p_{D1}}{\partial x_D{}^2} = \frac{\partial P_{D1}}{\partial t_{DL}}\Big|_{-\infty < X_D < \infty}. \qquad (22)$$

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For an infinite reservoir, subject to the condition that

$$p_D(x_D, t_D = 0) = \delta(x_D). \quad (23)$$

Applying Fourier integral transform defines as

$$\Im(w) = \int_{-\infty}^{\infty} e^{-ixt_D} p(x_D) dx_D. \quad (24)$$

And inversion formula

$$p(x_D, t_{DL}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ixt_{DL}} \Im(w, t_{DL}) dw. \quad (25)$$

It is readily shown that transformation reduces the partial differential equation to the ordinary differential equation

$$-\omega^2 \Im = \frac{d\Im}{dt_{DL}}. \quad (26)$$

The transformation of the initial condition is

$$\Im(\omega, t_{DL} \to 0) = \int_{\infty}^{\infty} e^{-i\omega x} \delta(x_D) dx_D. \quad (27)$$

Then,

$$\Im(\omega, t_{DL}) = Ae^{-\omega^2 t_{DL}}. \quad (28)$$

Substituting the initial condition we have
$A = 1$, Then

$$\Im(\omega, t_{DL}) = e^{-\omega^2 t_{DL}}. \quad (29)$$

Introducing eqn. 29 into the inversion formulae given above gives

$$p(x_D, t_{DL}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\omega x_D} e^{-\omega^2 t_{DL}} dw. \quad (30)$$

Since the integral is analytic function we can integrate under the integral sign to easily solve Equation (30) as follows
set

$$I(x_D, t_{DL}) = \int_{-\infty}^{\infty} e^{-i\omega x_D} e^{-\omega^2 t_{DL}} dw. \quad (31)$$

Differentiating $I(x,t)$ with respect to x, we get

$$\frac{dI(x, t_{DL})}{dx_D} = \int_{-\infty}^{\infty} -i\omega e^{-i\omega x} e^{-\omega^2 t_{DL}} dw. \quad (32)$$

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$$\frac{dI\left(x_D,t_{DL}\right)}{dx_D}=\left|i\frac{e^{-i\omega x_D}e^{-\omega^2 t_{DL}}}{2t_{DL}}\right|_{-\infty}^{\infty}-i\int_{-\infty}^{\infty}\frac{-ix_D e^{-i\omega a}e^{-\omega^2 t_{DL}}}{2t_{DL}}. \tag{33}$$

The first term in Equation (31) vanishes identically, therefore

$$\frac{dI\left(x_D,t_{DL}\right)}{dx_D}=-\frac{x_D}{2t_{DL}}I\left(x_D,t_{DL}\right). \tag{34}$$

Then by direct integration we have

$$\log_e I\left(x_D,t_{DL}\right)=-\frac{x_D{}^2}{4t_{DL}}+C'. \tag{35}$$

where C is the constant of integration, this implies that

$$I\left(x_D,t_{DL}\right)=Ce^{-\frac{x_D{}^2}{4t_{DL}}}. \tag{36}$$

then, we seek the constant C by setting x_D=0, I(t_{DL}) = C.
And

$$I\left(t_{DL}\right)=\int_{-\infty}^{\infty}e^{-\omega^2 t_{DL}}dw. \tag{37}$$

Equation (37) is symmetrical about the origin and making a substitution $z^2=\omega^2 t_D$, it is transformed into

$$I\left(t_D\right)=\frac{2}{\sqrt{t_D}}\int_0^{\infty}e^{-Z^2}dz. \tag{38}$$

the integral term is solved in the polar coordinate, we arrive at

$$I^2\left(t_{DL}\right)=\frac{2}{\sqrt{t_{DL}}}\int_0^{\frac{\pi}{2}}\int_0^{\infty}e^{-r^2}rdr\,d\theta. \tag{39}$$

$$I\left(t_{DL}\right)=\frac{2}{\sqrt{t_{DL}}}\frac{\sqrt{\pi}}{2}=C. \tag{40}$$

Putting Equation (40) to Equation (36) and finally into Equation (30) we have

$$p\left(x_D,t_{DL}\right)=\frac{1}{2\sqrt{\pi t_{DL}}}e^{-X_D{}^2/4t_{DL}}. \tag{41}$$

.

This is the fundamental green function from which solutions to other problem are obtained by summation with the complementary function. We then seek the Green's function of the form [3]

$$G_n(x_D,t_{DL})=f_N\left(x_D,t_{DL}\right)+H_N(x_D,t_{DL}). \tag{42}$$

Where

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$$f\left(x_D, t_{DL}\right) = \frac{1}{2\sqrt{\pi t_{DL}}} e^{-X_D^2/4t_{DL}}. \tag{42a}$$

For region $x_D < -b_D$ and zero for $x_D < -b_D$; and H is the solution to the one dimensional diffusivity equation for $x_D >$ or $< b_D$ and is zero at $t_{DL} = 0$ such that G satisfies the prescription at the fault. We then have:

$$\frac{\partial^2 H_{D1}}{\partial x_D^{\,2}} = \frac{\partial H_{D1}}{\partial t_{DL}}\bigg|_{-b_D < x_D < \infty}. \tag{43}$$

$$\frac{\partial^2 H_{D2}}{\partial x_D^{\,2}} = \frac{M}{\varepsilon}\frac{\partial H_{D2}}{\partial t_{DL}}\bigg|_{-b_D < x_D < -\infty}. \tag{44}$$

The solutions to Equations (43) & (44) in the two semi-infinite regions $x_D < -b_D$ are approached using Laplace transformation with respect to dimensionless time. The Laplace transform is defined as

$$H_n(x_D, s) = \int_0^\infty e^{-st_D} H_n(x_D, t_{DL}) dt_{DL}. \tag{45}$$

and

$$L\left(\frac{\partial H_n}{\partial t_{DL}}\right) = \int_0^\infty e^{-st_{DL}} \frac{\partial H_n}{\partial t_{DL}} dt_{DL}. \tag{46}$$

$$= \left| e^{-st_{DL}} p_{DL} \right|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st_{DL}} H_n dt_{DL}. \tag{47}$$

$$\equiv -H(0) + \frac{1}{s} H(x_D, s). \tag{48}$$

$H(0)=0$

Also

$$L\left(\frac{\partial^2 H_n}{\partial x_D}\right) = \int_0^\infty e^{-st_{DL}} \frac{\partial^2 H_n}{\partial x_D} dt_{DL}. \tag{49}$$

$$\equiv \frac{d^2 H(x_D, s)}{dx_D}. \tag{50}$$

On applying this, the solutions to Equations (31) and (32) are

$$H_1(x_{D,}, s) = \mathrm{A} e^{-\sqrt{s} x_D}. \tag{51}$$

$$H_2(x_{D,}, s) = B e^{\sqrt{s\frac{M}{\varepsilon}} x_D}. \tag{52}$$

Also transformation of fundamental function f is

$$F\left(x_D, s\right) = \frac{1}{2\sqrt{s}} e^{-\sqrt{s}|x|}. \tag{53}$$

Substituting Equations (41) – (42) into Equation (30) for active region and observation region separately, we have

$$G_1(x_D, b_D, s) = \frac{1}{2\sqrt{s}} e^{-\sqrt{s}|x_D|} + \mathrm{A} e^{-\sqrt{s} x_D}\bigg|_{-b_D < x_D < \infty}. \tag{54}$$

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In the observation region

$$G_2(x_D, s) = \mathrm{B}e^{\sqrt{\frac{Ms}{\varepsilon}}x_D}\Big|_{-b_D < x_D < -\infty} . \tag{55}$$

And

$$\beta M \frac{dG_1}{dx_D} = \frac{dG_2}{dx_D}\Big|_{x_D = -b_D} . \tag{56}$$

$$\frac{dG_2}{dx_D} = \alpha_L \left(G_1 - G_2\right)\Big|_{x_D = -b_D} . \tag{57a}$$

Setting

$$q_1 = \sqrt{s}. \tag{57b}$$

$$and$$

$$q_2 = \sqrt{s\frac{M}{\varepsilon}}. \tag{57c}$$

We solve Equations (56) and (57a) simultaneously to yield

$$\frac{\beta M}{2} e^{-q_1|b_D|} - A\beta M q_1 e^{q_1 b_D} = B q_2 e^{-q_2 b_D} . \tag{58}$$

$$\frac{\alpha_L}{2q_1} e^{-q_1 b_D} + A\alpha_L e^{-q_1 b_D} = Be^{-q_2 b_D}\left(q_2 + \alpha_L\right). \tag{59}$$

Multiply Equation (58) by $\left(q_2 + \alpha_L\right)$ and Equation (59) by q_2 we have

$$\frac{\beta M\left(q_2 - \alpha_L\right)}{2} e^{-q_1 b_D} - Aq_1\beta M\left(q_2 + \alpha_L\right)e^{q_1 b_D} = Bq_2\left(q_2 + \alpha_L\right)e^{-q_2 b_D}. \tag{60}$$

$$\frac{\alpha_L q_2}{2q_1} e^{-q_1 b_D} + A\alpha_L q_2 e^{q_1 b_D} = Be^{-q_2 b_D} q_2 \left(q_2 + \alpha_L\right).. \tag{61}$$

Subtract Equation (60) from Equation (61) to get

$$\left(\beta M q_2 - \beta M \alpha_L - \frac{\alpha_L q_2}{q_1}\right)\frac{e^{-q_1 b_D}}{2} - A\left(q_1 q_2 \beta M + q_1 \alpha_L \beta M + \alpha_L q_2\right)e^{-q_1 b_D} = 0.$$

$$\left(q_1 + \frac{q_1}{q_2}\alpha_L - \frac{\alpha_L}{\beta M}\right)\frac{\beta M q_2 e^{-q_1 b_D}}{2q_1} - A\left(q_1 + \frac{q_1 \alpha_L}{q_2} + \frac{\alpha_L}{\beta M}\right) q_2 M \beta e^{-q_1 b_D} = 0. \tag{62}$$

then

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$$\frac{\left(q_1+\frac{q_1}{q_2}\alpha_L-\frac{\alpha_L}{\beta M}\right)}{\left(q_1+\frac{q_1\alpha_L}{q_2}+\frac{\alpha_L}{\beta M}\right)}\frac{e^{-2q_1b_D}}{2q_1}=A. \quad (63)$$

then for q_1 and q_2

$$A=\left(\frac{q_1+\alpha_L\sqrt{\frac{\varepsilon}{M}}-\frac{\alpha_L}{\lambda}}{\left(q_1+\alpha_L\left(\sqrt{\frac{\varepsilon}{M}}\right)+\frac{\alpha_L}{\lambda}\right)}\right)\frac{e^{-2q_1b_D}}{2q_1}. \quad (64)$$

and

$$B=\frac{\lambda}{2q_2}e^{-q_1b_D+q_2b_D}-\left(\frac{q_1+\alpha_L\sqrt{\frac{\varepsilon}{M}}-\frac{\alpha_L}{\lambda}}{2q_1\left(q_1+\alpha_L\left(\sqrt{\frac{\varepsilon}{M}}\right)+\frac{\alpha_L}{\lambda}\right)}\right)\sqrt{\frac{\varepsilon}{M}}e^{-q_1b_D+q_2b_D}. \quad (65)$$

Where

$$\lambda=M\beta,\quad q_1=\sqrt{s}\ and\ \ q_2=\sqrt{s\frac{M}{\varepsilon}}$$

where β = the ratio of active well thickness to the observation well thickness.
Defining a new parameter h as the effective transmissibility, then

$$h=\alpha\sqrt{\frac{\varepsilon}{M}}+\frac{\alpha_L}{\lambda}$$

We therefore obtain, by substituting, the green functions as

$$G_1(x_D,q)=\frac{1}{2q_1}e^{-q_1x_D}+\left(\frac{q_1+\alpha_L\sqrt{\frac{\varepsilon}{M}}-\frac{\alpha_L}{\lambda}}{2q_1(q_1+h)}\right)e^{-q_1(2b_D+x_D)}. \quad (66)$$

$$G_2(x_D,q)=\frac{\lambda}{2\sqrt{\frac{M}{\varepsilon}}q_1}e^{-q_1\left(b_D-\sqrt{\frac{M}{\varepsilon}}(b_D+x_D)\right)}-$$

$$\lambda\left(\frac{q_1+\alpha_L\sqrt{\frac{\varepsilon}{M}}-\frac{\alpha}{\lambda}}{2q_1(q_1+h)}\right)\sqrt{\frac{\varepsilon}{M}}e^{-q_1\left(b_D-\sqrt{\frac{M}{\varepsilon}}(b_D+x_D)\right)}. \quad (67)$$

These two equations. are inverted using the Bromwich integral formula defined by

$$G_n(x_D,t_{DL})=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}e^{st_D}G(x_D,s)ds \quad (68)$$

we have, after analytical evaluation of the integral

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$$G_1(x_D, y_D, t_{DL}) = \frac{1}{4\pi t_{DL}} e^{-\frac{(x^2+y^2{}_D)}{4t_{DL}}} + \frac{1}{4\pi t_{DL}} e^{-\frac{z^2+y^2}{4t_{DL}}} - \frac{\alpha_L}{2\lambda\sqrt{\pi t_{DL}}} e^{h^2 t_{DL} + hz - \frac{y^2{}_D}{4t_{DL}}} erfc\left(h\sqrt{t_{DL}} + \frac{z}{2\sqrt{t_{DL}}}\right). \qquad (69)$$

In the same vein the Green's function for y-axis is found to be

$$G(y_D, t_{DL}) = \frac{1}{2\sqrt{\pi t_{DL}}} e^{-{y^2{}_D}/{4t_{DL}}}. \qquad (70)$$

And by Newman product scheme, the equivalent G in the reservoir is product of the corresponding one-dimensional coordinates in the horizontal plane, which is written below

$$G_1(x_D, y_D, t_{DL}) = \frac{1}{4\pi t_{DL}} e^{-\frac{(x^2+y^2{}_D)}{4t_{DL}}} + \frac{1}{4\pi t_{DL}} e^{-\frac{z^2+y^2}{4t_{DL}}} - \frac{\alpha_L}{2\lambda\sqrt{\pi t_{DL}}} e^{h^2 t_{DL} + hz - \frac{y^2{}_D}{4t_{DL}}} erfc\left(h\sqrt{t_{DL}} + \frac{z}{2\sqrt{t_{DL}}}\right) \qquad . \qquad (71)$$

$$G_2(x_D, y_D, t_{DL}) = \alpha_L \sqrt{\frac{\varepsilon}{M}} \frac{1}{2\sqrt{\pi t_{DL}}} e^{h^2 t_{DL} + h\left(b_D - \sqrt{\frac{M}{\varepsilon}}(b_D + x_D)\right) - \frac{y^2{}_D}{4t_{DL}}} erfc\left(h\sqrt{t_{DL}} + \frac{\left(b_D - \sqrt{\frac{M}{\varepsilon}}(b_D + x_D)\right)}{2\sqrt{t_{DL}}}\right) \qquad (72)$$

Substituting Equations (71) and (72) into Equations (13) & (14) respectively we have

$$P_{D1} = \frac{-1}{2}\mathrm{EI}\left(-\frac{x^2+y^2}{4t_{DL}}\right) - \frac{1}{2}\mathrm{EI}\left(-\frac{z^2+y^2}{4t_{DL}}\right) - \sqrt{\pi}\frac{\alpha_L}{\lambda}\int_0^{t_{DL}} \frac{1}{\sqrt{u}} e^{h^2 u + hz - \frac{y^2{}_D}{4u}} erfc\left(h\sqrt{u} + \frac{z}{2\sqrt{u}}\right) du. \qquad (73)$$

$$P_{D2} = \alpha_L \sqrt{\pi \frac{M}{\varepsilon}} \int_0^{t_{DL}} \left(e^{h^2 u + h\left(b_D - \sqrt{\frac{M}{\varepsilon}}(b_D + x_D) - \frac{y_D{}^2}{4u}\right)} \right) erfc\left(h\sqrt{u} + \frac{b_D - \sqrt{\frac{M}{\varepsilon}}(b_D + x_D)}{2\sqrt{u}}\right) \frac{du}{\sqrt{u}} \qquad (74)$$

Where variable 'u' is a dummy variable of integration and
$z = b_D + x_D$ in Equation (73)

Derived Model Equations

The dimensionless pressure distribution for the active well region, as derived above, is

$$p_{D1}(x_D, y_D, t_{DL}) = \frac{-1}{2}\mathrm{EI}\left(-\frac{x^2+y^2}{4t_{DL}}\right) - \frac{1}{2}\mathrm{EI}\left(-\frac{z^2+y^2}{4t_{DL}}\right) - \sqrt{\pi}\frac{\alpha_L}{\lambda}\int_0^{t_{DL}} \frac{1}{\sqrt{u}} e^{h^2 u + hz - \frac{y^2{}_D}{4u}} erfc\left(h\sqrt{u} + \frac{z}{2\sqrt{u}}\right) du \qquad (73)$$

And for the observation region we have

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$$p_{D2}\left(x_{D},y_{D},t_{DL}\right)=\sqrt{\pi}\alpha_{L}\sqrt{\frac{M}{\varepsilon}}\int_{0}^{t_{DL}}\frac{1}{\sqrt{u}}e^{h^{2}u+hb_{D}-\sqrt{\frac{M}{\varepsilon}}\left(b_{D}+x_{D}\right)-\frac{y^{2}_{D}}{4u}}erfc\left(h\sqrt{u}+\frac{b_{D}-\sqrt{\frac{M}{\varepsilon}}\left(b_{D}+x_{D}\right)}{2\sqrt{u}}\right)du \qquad (74)$$

Equation (73) shows the application of principle of superposition to pressure transient response as function of space. The first term represents pressure distribution in a homogeneous reservoir, while the second term is an indication of sealing fault response and the third term describes the characteristic of the fault. The transmissibility across fault is parameter describing the magnitude of the third term. Figure 3 describes these responses profile for active well region, which is called type curve. Type curves are quite useful for identifying and analysing composite systems. The most noticeable feature, characteristic of pressure profile for this system, is the present of two straight lines or curves. The first of these curves is the straight line response for estimating the parameters of active well region, and the second curve demonstrates the transmissibility nature of the observation well region. Figure 4 is a type curve that can be used to match the interference response of a well on the other side of the active or flowing well plotted for various values of transmissibility of the fault.

Conclusion

A very important factor in this method of solution is its flexibility in seeking solution to partial differential equations for complex boundary conditions albeit in integral form, and the derivation of the fundamental equation is straightforward. The Kernel function is sought by employing classical integral transforms method.

The purpose of this study is to provide analytical solution that could be applied to identify juxtaposed formation across a fault using well test. This is achieved by developing simultaneously pressure and pressure derivative type curve for the partially communicating fault across the pseudo-reservoirs. The analytical solution obtained in this study could be used to improve the design and analysis of interference tests between wells separated by the communicating fault. The information generated by these solutions will yield separate estimate of formation transmissibility and the transmissibility of the fault itself.

An explicit solution for draw down at the active well offers the possibility of deriving the fault transmissibility from the draw down and build up behaviors of the active well alone. One could use this method if convenient observation well has not been drilled to determine the fault transmissibility theoretically.

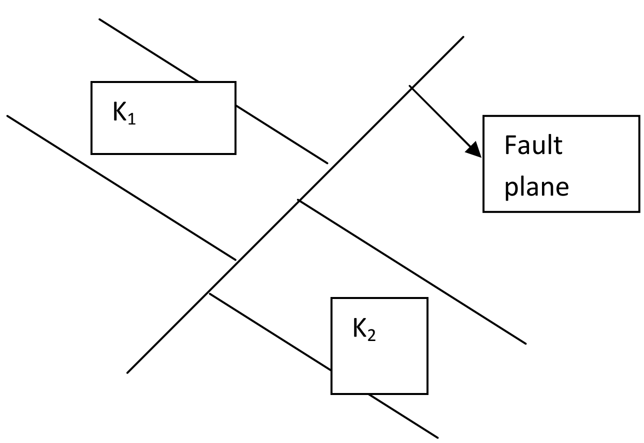

Fig. 1: typical faulted Reservoir

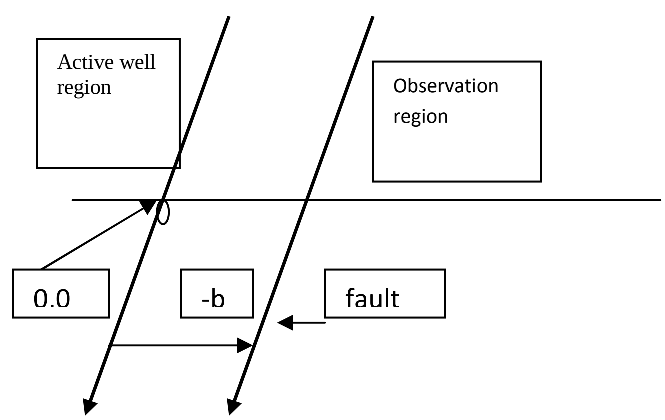

Fig 2: modelling the fault plane as vertical linear barrier

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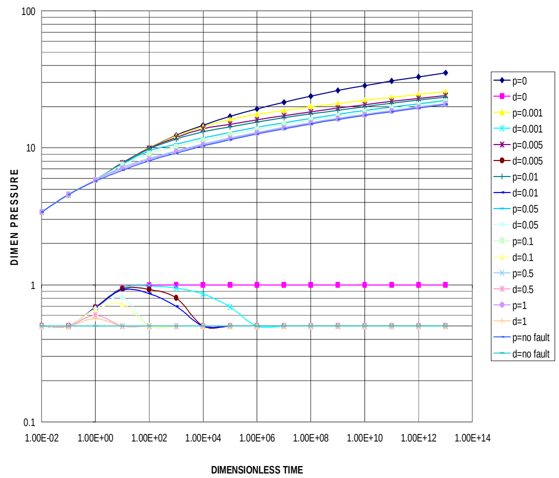

Fig.3: Dimensionless Pressure response and derivative plot.

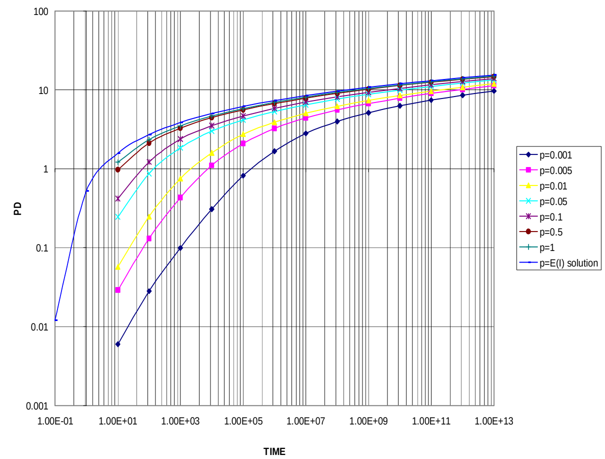

Fig.4:dimensionless Log – Log plot of Observation well response

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