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An Innovative Model to Estimate Fracture Extensions.

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Abstract

Hydraulic fracturing is a Well intervention program, designed to create fracture(s) within a reservoir system and hopefully, extend the volumes of these fractures, to facilitate improved recovery of in-situ fluid(s). This paper presents mathematical equations in dimensionless forms, to rapidly estimate the fracture extension and efficiency during a hydraulic fracturing operation.

The fracture extension profiles over time are captured in generated plots. The results are consistent with theory established in this work, and provide an innovative method of rapidly estimating the extensions of fracture lengths.

Nomenclature

$a(t)$	= fracture area, ft	$\Psi =$	Dimensionless parameter
λ	= Fluid leaking coefficient	$q_i =$	Injection rate, volume/time
$v(t) =$	Fluid flow velocity into reservoir	$w =$	Fracture width
f =	fracture	r =	reservoir
$t =$	Time	$\eta =$	Efficiency
$\Gamma =$	Gamma function		

1.0 Introduction

Energy is a key requirement to sustain and drive development worldwide. Petroleum is a major part of the energy chain, and the petroleum industry is continuously looking for ways to improve the recovery of in-situ fluids from reservoirs. This is particularly necessary in view of the greater demand for energy, especially natural gas.

A major part of the effort to improve the recovery of in-situ fluids, is through Well intervention programs, particularly, Well stimulation. Well stimulation programs are broadly classified into:

- Hydraulic Fracturing of the immediate environment of the reservoir around the Well, and
- Matrix Acidizing of the immediate environment of the reservoir around the Well.

The lithology of the reservoir system – sandstone(s), carbonates, shale, etc., determines the stimulation program that should be used. This facilitates the right fluid (acid) formulation for executing the program – injection into the reservoir. In this paper, we concentrate on hydraulic fracturing, which is basically, the splitting of a rock, thereby creating openings – fractures, by a fluid under pressure. Our emphasis is on the length(s) of the created fractures.

A classical in-situ fluid is natural gas, which is increasingly becoming a very important part of the energy mix. We have the so-called:

- Conventional Gas, and
- Un-conventional Gas.

Under un-conventional gas, we have such categories as:

- Tight gas, and
- Shale gas.

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The reservoir systems containing these gases, are characterized by their low values of permeability, which must be enhanced by “induced” openings, to effect any and/or improved recovery of the natural gas, thus, hydraulic fracturing. Generally, the purpose of hydraulic fracturing is to improve well productivity beyond the natural capabilities of the reservoir. This applies to both natural gas and crude oil reservoirs, where there is a need to create additional “openings” for the in-situ fluid(s), to flow to the Well.

Designing fracturing jobs to improve production and optimize returns is an optimization and exact mathematical science. A recent paper [1] on Well performance improvement on a Well in a Texas gas field, describes how fracturing has opened numerous unconventional frontiers that would have been uneconomical to develop without hydraulic fracturing.

This paper presents a mathematical model that can be used to estimate a fractured length, obtained during the period of injection of the fracturing fluid after the fracture initiation. Also, we report the results of the analysis of parameters that can be controlled so that effective fracture length will be increased. Fracture length is related to the fracture conductivities. A long fracture length leads to an infinite conductivity fracture. This is a fracture that is so conductive compared to the reservoir itself [2].

Mathematical Formulation - Model

For natural rock formations, the theory of fracture extension indicates that the injection pressure should be twice the confining stress and the tensile strength, however, field evidence does not support this [3]. For instance, an injection pressure twice the magnitude of confining pressure seldom occur in extending horizontal fracture, while vertical fracture extension pressure is two-third of the overburden stress. The extension length of a fracture depends on the volume of fluid that leak into the reservoir and the volume retained in the fracture, the sum of which is the total volume injected over an interval of time, which can be expressed as:

$$q_i = q_f + q_r \tag{1}$$

Equation 1 describes the material balance nature of fluid conservation, which is the basis of the proposed mathematical model. In the fracture, the injected fluid will split into two parts: one part enters into the reservoir under the influence of pressure differential $P_i - P_r$ between the fracture and the external reservoir boundary. The remaining volume subsequently moves into the fracture opening and thereby increases the length of the fracture.

To derive the basic flow equation, q_i, we make the following assumptions:

- The fracture is of uniform width,
- The flow of fracturing fluid into the reservoir is governed by Darcy flow equation or any linear flow model,
- The flow of fluid into the reservoir is through the fracture surfaces and at any point on the fracture face, the velocity of flow is a function of time at which the fracture fluid reaches it., and
- The pressure in the fracture is equal to the sand face injection pressure.

The third assumption simplifies the flow rate of fracture fluid into the reservoir as product of area open to flow and velocity of flow. Thus, the integral of flow velocity over the present area and over the fracture is:

$$q_r = \int_o^{a(t)} v(t-\tau)\, da(\tau) \tag{2}$$

Since the extension of a fracture increases with time as long as the fracture keeps on accepting the injected fluid, for a given area formed, there is a lag in time for the leak off into the reservoir.

By superposition theorem, the velocity of flow into the reservoir, $v(t-\tau)$, is the response to a unit impulse created at time t at a point that arrives at another point at time $(t-\tau)$ in a system that is infinite. Since fracture area also increases with time, then

$$da = \frac{da}{d\tau} d\tau \tag{3}$$

Substituting Equation (3) into Equation (2), the flow rate into the reservoir from the fracture, becomes:

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$$q_r = \int_o^t v(t-\tau)\frac{da(\tau)}{d\tau}d\tau \quad (4)$$

The flow rate into the fracture is equivalent to the (injected) volume (into fracture) per unit time. This volume is the product of area created over time and the width measured at entry point, which is:

$$q_f = w\frac{da}{dt} \quad (5)$$

Equation 1 can now be re-written as:

$$q_i = \int_o^t v(t-\tau)\frac{da(\tau)}{d\tau}d\tau + w\frac{da}{dt} \quad (6)$$

This is a convolution equation, which can easily be solved in the transform space.

Method of Solution

The time variable t can be transformed into a space of complex variable by Laplace transformation [3], defined as:

$$f(s) = \int_0^\infty f(t)e^{-st}dt \quad (7)$$

$$s > 0$$

where $f(t)$ is applied to represent any variable that is time-dependent.

The Laplace transform of a convoluted integral is given as the product of each variable involved in the superposition. By this definition, the transformation of Equation (6) is given by:

$$\frac{q_i}{s} = v(s)\int_0^\infty \frac{da}{dt}e^{-st}dt + w\int_0^\infty e^{-st}\frac{da}{dt}dt \quad (8)$$

And by rearrangement, we can rewrite Equation (8) after an evaluation of the integral. Through integration by parts, the transform of the derivatives, is:

$$\int_o^\infty e^{-st}\frac{da(t)}{dt}dt = sa(s) \quad (9)$$

$$\frac{q_i}{s} = (v(s)+w)sa(s) \quad (10)$$

Thus,

$$a(s) = \frac{q_i}{s^2(2v(s)+w)} \quad (11)$$

To recover the area in time dependence, we define the Cauchy integral:

$$f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+} f(s)e^{st}ds \quad (12)$$

The form of the velocity profile determines the function $v(s)$. If the velocity profile follows the form:

$$v(t) = \frac{\lambda}{\sqrt{t}} \quad (13)$$

With the transformation

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$$v(s)=\lambda\int_0^\infty t^{-\frac{1}{2}}e^{-st}dt \tag{14}$$

By making a change of variable:

$$\sqrt{t}=\frac{y}{\sqrt{s}}\text{, then } dt=\frac{2}{s}ydy.$$

Equation 14 becomes:

$$v(s)=\frac{2\lambda}{s^{1/2}}\int_0^\infty e^{-y^2}dy \tag{15}$$

By transformation into radial and azimuthal coordinates, we have:

$$v(s)=\lambda\sqrt{\frac{\pi}{s}} \tag{16}$$

$$a(s)=\frac{q_i}{s^2\left(2\lambda\sqrt{\frac{\pi}{s}}+w\right)} \tag{17}$$

To recover the fracture area in real time variable, our construction proceeds from the Bromwich integral:

$$f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}f(x)e^{st}dt \tag{18}$$

Making intuitive construction, Equation (17) is expanded by Laurent series as:

$$a(s)=\frac{q_i}{s^{\frac{3}{2}}\left(2\lambda\sqrt{\pi}+ws^{\frac{1}{2}}\right)} \tag{19}$$

Then we make some substitution and expand the result by binomial theorem. That is:

Let $Z=\frac{w}{2\lambda}\sqrt{\frac{s}{\pi}}$. This implies that $\sqrt{s}=\frac{2\lambda z}{w}\sqrt{\pi}$

If we set $\Psi=\frac{2\lambda\sqrt{\pi}}{w}$, then,

$$a(s)=\frac{q_i}{s^{\frac{3}{2}}w\Psi\left(1+\frac{s^{\frac{1}{2}}}{\Psi}\right)} \tag{20}$$

Expanding the term in the bracket and taking note of analytical theorem of complex variable s:

$$R_1(s)=\frac{q_i\left(1-\frac{s^{\frac{1}{2}}}{\psi}+\frac{s}{\Psi^2}-\frac{s^{\frac{3}{2}}}{\Psi^3}+\ldots\right)}{w\Psi s^{\frac{3}{2}}} \tag{21}$$

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$$a(s)=\frac{q_i}{\Psi w}\left(\frac{1}{s^{\frac{3}{2}}}-\frac{1}{\psi s}+\frac{s}{\Psi^2 s^{\frac{1}{2}}}-\frac{1}{\Psi^3}+\ldots\right) \quad (22)$$

Also putting $\Psi+s^{\frac{1}{2}}=\zeta$, this implies that $s^{\frac{1}{2}}=\zeta-\psi$. Substitution into Equation (20) and then expanding by binomial theorem, we get:

$$a(s)=\frac{q_i}{(\zeta-\Psi)^3 w\Psi\zeta} \quad (23)$$

$$R_2(s)=\frac{q_i}{w}\left(\frac{1}{\Psi^4\zeta}+\frac{3}{\psi\Psi^5}+\frac{12\zeta}{\Psi^6}+\frac{60\zeta^2}{\Psi^7}+\ldots\right) \quad (24)$$

$$R_2(s)=\frac{-q_i\left(1+\frac{3\zeta}{\psi}+\frac{12\zeta}{\Psi^2}+\frac{\zeta^2}{\Psi^3}+\ldots\right)}{w\Psi^4\zeta} \quad (25)$$

Since the integral contains no pole other than the branch point, Equation (25) becomes:

$$R_2(s)=\frac{q}{w}\left(\frac{1}{\Psi^4\left(s^{\frac{1}{2}}+\Psi\right)}\right) \quad (26)$$

Combining Equations (22) and (26) we have

$$a(s)=\frac{q}{w}\left(\frac{1}{\Psi^4\left(s^{\frac{1}{2}}+\Psi\right)}+\frac{1}{\Psi s^{\frac{3}{2}}}-\frac{1}{\Psi^2 s}+\frac{1}{\Psi^3 s^{\frac{1}{2}}}\right) \quad (27)$$

Expanding further:

$$a(s)=\frac{q_i}{\Psi w}\left(\frac{1}{s^{\frac{3}{2}}}-\frac{1}{\psi s}+\frac{s}{\Psi^2 s^{\frac{1}{2}}}-\frac{1}{\Psi^3}+\ldots\right) \quad (28)$$

Inversion to Time

Equation (28) will be inverted back to the time variable using Equation (12). The nature of the integral requires a construction of a cut plane that will accommodate each branch point to the left of the origin. For the first term we have:

$$a_1(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{st}}{s^{\frac{1}{2}}}dt \quad (29)$$

By analytical continuation, we set $s=re^{i\pi}$ and $s=re^{-i\pi}$ along the two paths. Thus:

$$a_1(t)=\frac{1}{2\pi i}\left(\int_0^\infty\frac{e^{-rt}}{r^{\frac{1}{2}}e^{\frac{i\pi}{2}}}e^{i\pi}dr+\int\frac{e^{-rt}}{r^{\frac{1}{2}}e^{\frac{-i\pi}{2}}}e^{-i\pi}dr\right) \quad (30)$$

which simplifies to:

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$$a_1(t)=\frac{1}{2\pi i}\left(\int_0^\infty \frac{e^{-rt}}{r^{\frac{1}{2}}} i\sin\left(\frac{\pi}{2}\right)dr\right) \tag{31}$$

Now it remains to evaluate the semi-infinite integral. This is best confronted by making the transformation $r=y^2 \Rightarrow dr=2ydy$ in Equation (31), yielding:

$$a_1(t)=\frac{1}{\pi}\left(\int_0^\infty \frac{e^{-y^2t}}{y}2ydy\right) \tag{32}$$

Another change of variable and introduction of radial coordinates, the solution to Equation (32) is:

$$a_1(t)=\frac{1}{\pi\sqrt{t}}\left(\int_0^{2\pi} d\varphi\int_0^\infty e^{-r^2}rdr\right) \tag{33}$$

Simplified to:

$$a_1(t)=\frac{1}{\sqrt{t\pi}} \tag{34}$$

To invert the second term in Equation (30), we invoke the property of transform of integral that is:

$$a_2(t)=\frac{1}{s}\left(\int_0^\infty \frac{e^{st}}{s^{1/2}}dy\right)=\int_0^t \frac{1}{\sqrt{u\pi}}du \tag{35}$$

Simplifying further, we have:

$$a_2(t)=\sqrt{\frac{t}{\pi}} \tag{36}$$

The inversion of inverse of s has a simple solution:

$$a_3(t)=1 \tag{37}$$

The last term in the Laplace space in Equation (28), by analytical continuation has the form:

$$a_4(t)=\frac{1}{2\pi i}\left(\int_0^\infty \frac{e^{-rt}}{z+r^{\frac{1}{2}}e^{\frac{i\pi}{2}}}e^{i\pi}dr+\int_\infty^0 \frac{e^{-rt}}{z+r^{\frac{1}{2}}e^{\frac{-i\pi}{2}}}e^{-i\pi}dr\right) \tag{38}$$

By rationalization we have:

$$a_4(t)=\frac{2i}{2\pi i}\left(\int_0^\infty \frac{r^{1/2}e^{-rt}}{z^2+r}dr\right) \tag{39}$$

whose solution is:

$$a_4(t)=\sqrt{1/t\pi}+e^{Zt}\left(1-erf\left(z\sqrt{t}\right)\right) \tag{40}$$

Combining, $a_i(t), i=1,2,3,4$, we have:

$$a(t)=\frac{q}{w\Psi^2}\left(2\Psi\sqrt{\frac{t}{\pi}}+\frac{1}{\Psi^3\sqrt{t\pi}}-1-\frac{1}{\Psi^3}\sqrt{1/t\pi}+e^{\Psi^2t}\left(1-erf\left(\Psi\sqrt{t}\right)\right)\right) \tag{41}$$

Thus, Equation (41) is an expression for fracture area created. Replacing $\Psi=\dfrac{2c\sqrt{\pi}}{w}$:

$$a(t)=\frac{qw}{4\pi\lambda^2}\left(4\frac{\lambda\sqrt{\pi}}{w}\sqrt{\frac{t}{\pi}}+e^{\frac{4\pi}{w^2}C^2t}\left(erfc\left(\frac{\lambda}{w}\sqrt{\pi t}\right)\right)-1\right) \tag{42}$$

Equation (42) is the model that describes the fracture area as a function of time of injection and fluid leaking coefficients, λ. The term erfc can be fully simplified by finding the asymptotic expansion of a familial function defined in form of definite integral using method of integration by part. We choose the incomplete Gamma function defined by:

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$$\mathrm{H}(u,x)=\int_0^x e^{-s}t^{u-1}dt \qquad (43)$$

By repeated integration by parts, a suitable series representative can be found for Equation (43), when variable x is small:

$$\mathrm{H}(u,x)=\sum_0^{\infty}\frac{(-)^n x^{n+u}}{(u+n)n!} \qquad (44)$$

When variable x is small, we define complementary incomplete Gamma function as:

$$\Gamma(u,x)=\int e^{-t}t^{u-1}dt \quad = \quad \Gamma u-\mathrm{H}(u,x) \qquad (45)$$

By repeated integration by parts, we have

$$\Gamma(u,x)=\left[\frac{\Gamma u}{\Gamma(u-n)}\right]e^{-x}x^{u-n-1}; \qquad x > u-1 \qquad (46)$$

Hence as $x \longrightarrow \infty$,

$$\Gamma(u,x)\approx\left[\sum_{r=1}^{\infty}\frac{\Gamma u}{\Gamma(u-r+1)}e^{-x}x^{u-r}\right] \qquad (47)$$

Therefore, erfc is an example complementary Gamma function and is of the form:

$$erfc(x)=\frac{1}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}{}'x^2\right)=e^{-x^2}\sum_{r=1}^{\infty}\Gamma\left(r-\frac{1}{2}\right)\frac{(-1)^{r-1}}{x^{2r-1}} \qquad (48)$$

We note that this representation of the erfc term is for computational purposes, remembering that our objective is equation 42, which describes the fracture area.

Fracture Efficiency

The efficiency of any hydraulic fracturing process is the ratio of its output response to the quantity of injection input. Here, the input is the total volume (e.g. in gallons) of injected fracturing fluid and the response is the volume of fracture created, a function of the fracture area. The total volume injected is the product of injection time and injection rate, in which the output is the product of fracture width and the area created. It follows that:

$$\eta=\frac{w*a(t)}{q_i t} \qquad (49)$$

By substituting the equation for fracture area, the fracture efficiency can be represented by Equation (50).

$$\eta(t)=\frac{w^2}{4\pi\lambda^2 t}\left(4\frac{\lambda\sqrt{\pi}}{w}\sqrt{\frac{t}{\pi}}+e^{\frac{4\pi}{w^2}\varepsilon^2 t}\left(erfc\left(\frac{\lambda}{w}\sqrt{\pi t}\right)\right)-1\right) \qquad 50$$

Defining the dimensionless variable x as:

$x=\frac{2\lambda\sqrt{t\pi}}{w}$, therefore Equation (50) becomes:

$$\eta(t)=\frac{1}{x^2}\left(2x\sqrt{\frac{1}{\pi}}+e^{x}\ erfc(x)-1\right) \qquad 51$$

Equation (51) is the expression for fracture efficiency. The erfc term is represented by Equation (48).

Validation of Model

Equation (42), the objective of this paper, describes the area of a fracture as a function of time, with fluid properties, reservoir properties and fracture width, as parameters. Table 1 shows our sample data used for the

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estimation of the fracture area, a, in this work. Using these data, we present in Table 2, the estimated values of the fracture area, a, at various injection times. Table 3 shows the efficiency of the fracturing fluid as a function of time. Figures 1 and 2 with fracture widths of 0.2 and 0.4 inches, demonstrate the importance of a fracturing fluid on created fracture length. We note that fracture area is a function of fracture length. The leaking fluid coefficient is representative of a fracturing fluid. The lower the value of this parameter, the longer the extended length of the fracture created. The input data are synthetic data generated by FORTRAN, to indicate the arbitrariness of the parameters that can be used for investigations, and subsequently, the effective design of a hydraulic fracturing job. The plots in the Figures 1 and 2, describe the area-time profiles for variations in the widths of the created fractures. It is seen in Figures 1 and 2 that a fracture fluid with a leaking coefficient greater than one is ineffective as a fracturing fluid. Table 3 generated from equation 51, clearly shows that the efficiency of a fracturing job depends greatly on the leaking fluid coefficient. From Figure 3, it can be seen that low leaking coefficient of about 0.001, will give an efficiency of about 100%. Since injection pressure is constant the fracture length is longer for shorter injection time as the plot shown. This will be a major input in designing hydraulic fracturing jobs.

Conclusions

The ability to have effective control of a fracturing program can be an invaluable asset to the petroleum engineer. This helps to optimizing reservoir surveillance and production. The proper selection of a fracturing fluid with the right leaking coefficient as well as knowledge of the fracture width, are the parameters by which the fracture length, which can be confirmed by well test, will be pre-determined. From this paper, we conclude as follows:

- A mathematical model is presented to provide an innovative method of rapidly estimating the extensions of fracture lengths from hydraulic stimulation jobs.
- The effectiveness of a fracture fluid depends on the area of the fracture produced.
- The fracturing fluid leaking coefficient controls the profile of the fracture extension-time plot, with lower coefficient producing the most extension.
- The fracture efficiency can be rapidly modeled to provide information about the loss of the injected fluid and to detect thief zones.

Table 1: Sample Data for Estimating Fracture Area, a.

Injection Rate, q (bbls/min)	Fluid Leaking Coefficient λ	Fracture Width, w (inches)
5	$10^{-4} – 10^{2}$	2 - 10

Table 2: Estimated Fractures Areas, a, in dimensionless form

Fluid Leaking Coeff, λ, =	1.00E+02	1.00E+01	1.00E+00	1.00E-01	1.00E-02	1.00E-03	1.00E-04
Injection Time (min)	Area	Area	Area	Area	Area	Area	Area
1	1.59E-02	1.58E-01	1.51E+00	1.02E+01	2.20E+01	2.47E+01	2.48E+01
2	2.25E-02	2.24E-01	2.17E+00	1.62E+01	4.19E+01	4.91E+01	5.02E+01
3	2.76E-02	2.75E-01	2.68E+00	2.10E+01	6.07E+01	7.33E+01	7.47E+01

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4	3.18E-02	3.18E-01	3.10E+00	2.51E+01	7.85E+01	9.74E+01	9.91E+01
5	3.56E-02	3.55E-01	3.48E+00	2.87E+01	9.56E+01	1.21E+02	1.25E+02
6	3.90E-02	3.89E-01	3.82E+00	3.20E+01	1.12E+02	1.45E+02	1.49E+02
7	4.21E-02	4.20E-01	4.13E+00	3.51E+01	1.28E+02	1.69E+02	1.74E+02
8	4.50E-02	4.49E-01	4.42E+00	3.79E+01	1.44E+02	1.93E+02	2.00E+02
9	4.77E-02	4.77E-01	4.70E+00	4.06E+01	1.59E+02	2.16E+02	2.24E+02
10	5.03E-02	5.02E-01	4.95E+00	4.32E+01	1.74E+02	2.40E+02	2.50E+02

Table 3: Estimated Fracture Efficiency, η (t)

Fluid Leaking Coeff, λ, =	1.00E+02	1.00E+01	1.00E+00	1.00E-01	1.00E-02	1.00E-03	1.00E-04
Injection Time (min)	Efficiency	Efficiency	Efficiency	Efficiency	Efficiency	Efficiency	Efficiency
1	6.36E-04	6.33E-03	6.08E-02	4.08E-01	8.81E-01	9.87E-01	9.93E-01
2	4.50E-04	4.49E-03	4.35E-02	3.24E-01	8.39E-01	9.81E-01	1.00E+00
3	3.67E-04	3.66E-03	3.58E-02	2.80E-01	8.09E-01	9.77E-01	9.96E-01
4	3.18E-04	3.18E-03	3.11E-02	2.51E-01	7.85E-01	9.74E-01	9.91E-01
5	2.85E-04	2.84E-03	2.79E-02	2.30E-01	7.65E-01	9.71E-01	9.97E-01
6	2.60E-04	2.59E-03	2.55E-02	2.14E-01	7.48E-01	9.68E-01	9.95E-01
7	2.41E-04	2.40E-03	2.36E-02	2.01E-01	7.33E-01	9.66E-01	9.94E-01
8	2.25E-04	2.25E-03	2.21E-02	1.90E-01	7.19E-01	9.63E-01	9.98E-01
9	2.12E-04	2.12E-03	2.09E-02	1.81E-01	7.07E-01	9.61E-01	9.96E-01
10	2.01E-04	2.01E-03	1.98E-02	1.73E-01	6.95E-01	9.59E-01	9.98E-01

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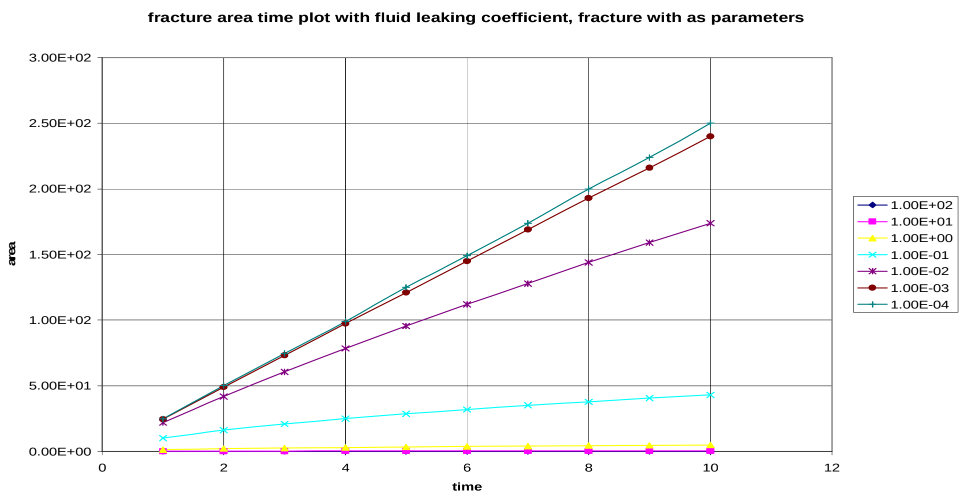

Figure 1: Area-Time Plot with Fracture Width of 0.2 inches

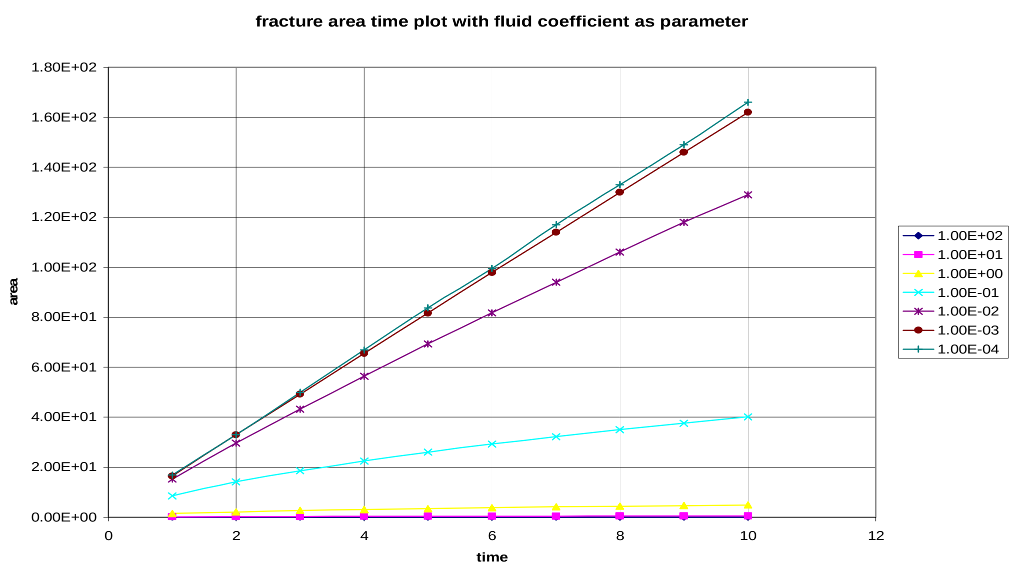

Figure 2: Area-Time Plot with Fracture Width of 0.4 inches

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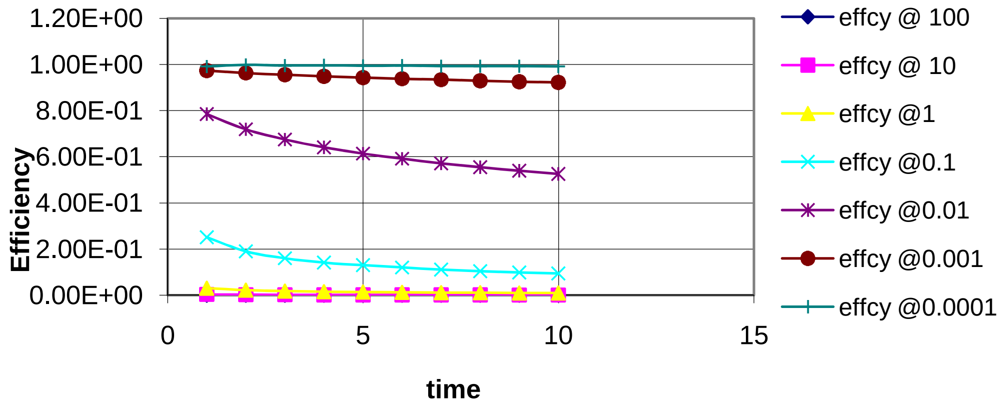

Figure 3: Efficiency – Time Plot

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