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Flow Through A Horizontal Porous Channel With A Harmonic Pressure Gradient

O. T. Gideon

Department of mathematics, statistics and computer science
Kaduna polytechnics, kaduna

Abstract

In this research work we provide a finite element solution to the problem of the flow through a horizontal channel with a harmonic pressure gradient. Results obtained shows that the velocity and temperature increases with time and that a turning point occurs in the temperature profile due to the viscous dissipation effect.

Keywords: - Asymptotic techniques, Maxwell fluid, constantly accelerating plates and velocity fields.

1.0 Introduction

The phenomenal growth of research interest in the field of flow through porous media stems from the fact that heat and mass transfer occurs in many engineering applications, geophysical and biological applications in which porous media plays a vital role.

Many researchers have showed varied interest in the field of porous media flow. For example Raptis (1984) studied the viscous boundary through a very porous medium bounded by a horizontal semi-infinite plate. Kafoussias (1989) studied the flow through a porous media in the presence of heat transfer but neglected the viscous dissipation effect. Gideon and Eletta (2008) further studied the viscous dissipation effect on the flow through a very porous media. Recently Gideon and Eletta (2009) studied the flow in a horizontal channel with a temperature dependent fluid viscosity in the presence of viscous dissipation.

The focus of this present paper is to study the effects of viscous dissipation and a harmonic pressure gradient on the flow through a porous horizontal channel using the finite element method which has not yet been studied.

Problem Formulation:

Consider a laminar flow of an incompressible fluid contained between two horizontal parallel walls, under the influence of a harmonically varying pressure gradient in the x-direction and the fluid oscillates harmonically with a frequency w. The differential equation describing the fluid motion and heat transfer is

$$\ell \frac{\partial u}{\partial t} = \chi \cos wt + \mu \frac{\partial^2 u}{\partial x^2} + \frac{\mu}{k} u \qquad (1.1)$$

$$\ell c_p \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \mu \left(\frac{\partial u}{\partial x} \right)^2 \qquad (1.2)$$

Subject to

Corresponding author: E-mail; okedayo@yahoo.com Tel +2348037052324, +2348028700813

$$\begin{bmatrix} u(0,t) = 0, \\ u(l,t) = u_0 \\ u(x,0) = 0 \\ T(0,t) = 0 \\ T(l,t) = T_0 \\ T(x,0) = 0 \end{bmatrix} \quad (1.3)$$

Introducing the following dimensionless quantities

$$u = \frac{u}{u_0}, \tau = \frac{t}{t_0}, X = \frac{x}{a}, \theta = \frac{T}{T_0} \quad (1.4)$$

We have the following dimensionless equation

$$\frac{\partial u}{\partial \tau} = q \cos w\tau + \frac{1}{\mathrm{Re}} \frac{\partial^2 u}{\partial X^2} + \frac{u}{Da} \quad (1.5)$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta}{\partial X^2} + \lambda \left(\frac{\partial u}{\partial X} \right)^2 \quad (1.6)$$

with

$$\begin{pmatrix} u(0,\tau) = \theta(0,\tau) = 0 \\ u(1,\tau) = \theta(l,\tau) = 1 \\ u(X,0) = \theta(X,0) = 0 \end{pmatrix} \quad (1.7)$$

Where,

u = Velocity of flow.
x = Amplitude of oscillation.
w = Frequency.
τ = Time.
μ = Viscosity.
k = Permeability.
e = Density.
C_p = Specific heat capacity.
T = Temperature.
X = Distance.
D_a = Darcy Number.
R_e = Reynolds Number.
$P_r = Prandtyl\ Number.$
λ = Brinkman Number.

Problem Solution: We shall now proceed to solve (1.5), (1.6) and (1.7) using the Galerkin finite element procedure. We shall apply the finite element discretization in space and the crank Nicolson finite difference discretization in the temporal domain as follows.

Let $$\begin{matrix} U = u^{(e)}(t) N^{(e)} \\ \theta = \phi^{(e)}(t) N^{(e)} \end{matrix} \quad (1.8)$$

Where $N^{(e)}$ are the linear lagrange interpolation polynomials and $u^{(e)}(t)$ and $\phi^{(e)}(t)$ are nodal element values.

Corresponding author: E-mail; okedayo@yahoo.com Tel +2348037052324, +2348028700813

Substituting (1.8) into (1.5) & (1.6) and carrying out the necessary integrations and algebraic simplifications we arrived at the following

$$\frac{l}{6}\begin{pmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{pmatrix}=\frac{ql\cos w\tau}{2}\begin{pmatrix}1\\ 2\\ 1\end{pmatrix}-\frac{1}{lR_e}\begin{pmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{pmatrix}\begin{pmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{pmatrix}+\frac{D_a l}{6}\begin{pmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{pmatrix}\begin{pmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{pmatrix}$$

Where l is the length of the element. If we take node i as the origin we have

$$\frac{l}{6}\left(\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right)=ql\cos w\tau+\frac{1}{lR_e}(u_{i-1}+u_i+u_{i+1})+D_a(u_{i-1}+4u_i+u_{i+1}) \tag{1.9}$$

Applying the crank Nicolson procedure to (1.20) we have

$$2u_i^{m+1}-2u_{i-1}^m+8(u_i^{m+1}-u_i^m)+2u_{i+1}^{m+1}-u_{i+1}^m=12\Delta tq\cos wt+\Delta t(D_a+\frac{6}{l^2R_e})(u_{i-1}^{m+1}+u_{i-1}^m)+$$
$$\Delta t(4D_a-\frac{12}{l^2R_e})(u_i^{m+1}+u_i^m)+\Delta t(D_a+\frac{6}{l^2R_e})(u_{i+1}^{m+1}+u_{i+1}^m) \tag{1.20}$$

Equation (1.20) is the finite element solution of the momentum equation. Similarly we have the following finite element solution for the energy equation.

$$(1-\alpha)\theta_{i-1}^{m+1}-(1+\alpha)\theta_{i-1}^m+(4+2\alpha)\theta_i^{m+1}-(4-2\alpha)\theta_i^m+(1-\alpha)\theta_{i+1}^{m+1}-$$
$$(1+\alpha)\theta_{i+1}^m=\frac{3\lambda\Delta t}{2l^2}((u_{i-1}^m)^2-2u_i^m u_{i+1}^m+2(u_i^m)^2-2u_i^m u_{i+1}^m+(u_{i+1}^m)^2) \tag{1.21}$$

Where $\alpha=\frac{3\Delta t}{3l^2\Pr}$

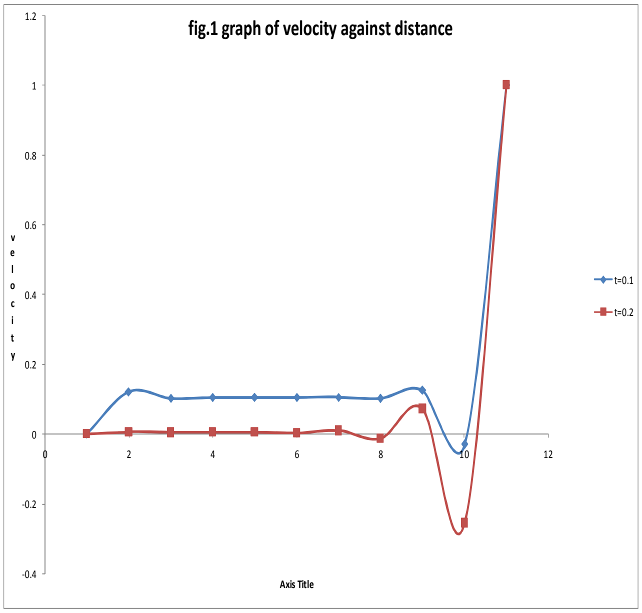
Numerical Results and Discussion:

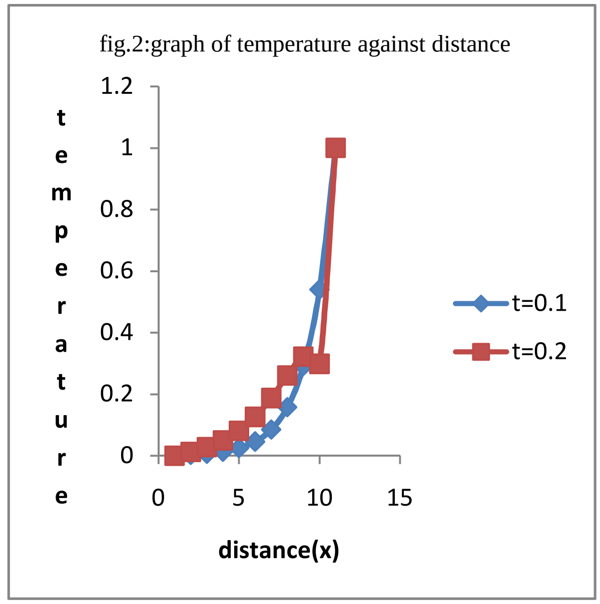
We shall now provide a numerical simulation of equations (1.20) and (1.21) for various parameters in the flow equations. Using Microsoft excel software.

In fig.1 we have $\Delta\tau=0.1, \mathrm{D_a}=1, \mathrm{l}=0.1, \mathrm{R_e}=100$ for $\tau=0.2$ for the momentum equation. It is also observed that the velocity profile increases from zero then a steady state is almost achieved before oscillation then set in.

In fig 2 we have the graph of the temperature against distance, at t = 0.1and 0.2, Pr = 0.7, $\Delta t=0.1, L=0.1, \lambda=0.001$. It is observed that for the series at t 0.1 the temperature increased steadily when the viscous dissipation effect was zero. And at t = 0.2 a turning point occurs in the temperature profile due to the influence of the viscous dissipation term. The temperature was observed to increase with time. .

Corresponding author: E-mail; okedayo@yahoo.com Tel +2348037052324, +2348028700813





Summary & Conclusion:

In summary we have demonstrated the utility of the finite element method to the problem of flow in a horizontal porous channel with a harmonic pressure field using the linear Lagrange interpolation polynomials. The study has provided a tool for the influence of various parameters on the momentum and energy equations.
Furthermore, finite element in space and finite difference in time resulting in the crank Nicolson scheme was used to provide a model for predicting the momentum and energy changes in a porous media flow.

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Corresponding author: E-mail; okedayo@yahoo.com Tel +2348037052324, +2348028700813